Enhanced tomography resolution by a fat ray technique

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Summary

We propose a tomography algorithm to enhance the resolution of 3D velocity model building. Our algorithm uses a fat ray instead of the conventional asymptotic ray. Compared to conventional ray tomography, the resulting tomography matrix is far less sparse. Thus, while the new matrix may still be ill-conditioned, much less model regularization is required to produce a well-posed system. In turn this should lead to a higher resolution tomographic update. A subsalt velocity analysis test on the Sigsbee 2a model shows promising result; we also apply the fat ray technique to well-tie tomography that estimates anisotropy parameters.

Introduction

In complex structural imaging, the quality of 3D prestack depth migration depends on the accuracy of the velocity model. Thus velocity model building/refining is a crucial step for seismic data processing, and tomography is probably the best production tool for achieving that goal. In prestack depth imaging, the models are conventionally represented by grids/cells. The objective of tomography is to invert a linear system and find a velocity model which best fits certain picked seismic events. Different tomography algorithms may fit the picked events in different domains. For instance, in the prestack time domain, traveltime tomography uses the picked traveltime curves (Bishop et al. 1985) while stereo-tomography uses local coherent traveltime events (Billette et al. 1998). In the migrated depth domain, residual curvature analysis (RCA) uses the picked residual moveout in offset gather (Liu and Bleistein, 1995) or in subsurface angle gathers (Chauris et al., 2002). Mathematically, the problem is to minimize an objective (cost) function to obtain the velocity:

$$ Cost = \sum \left\| S_{cal} - S_{pick} \right\|_d, $$  \hspace{1cm} (1)

where \( \left\| \cdot \right\|_d \) is some norm, typically \( L^2 \), \( S_{pick} \) equals the picked seismic events, and \( S_{cal} \) is the output of the modeling. Here, seismic events are represented by traveltime curves for traveltime tomography or depth moveout for RCA. \( S_{cal} \) can be written:

$$ S_{cal} = A m, $$  \hspace{1cm} (2)

where \( m \) is the initial model and \( A \) is the tomography matrix which is constructed by ray-tracing in the initial model. Substitution of (2) into (1) yields:

$$ Cost = \sum \left\| A m - S_{pick} \right\|_d, $$  \hspace{1cm} (3)

and for the least squares norm, \( m \) is obtained by applying the pseudo-inverse of \( A \) to the picked data. Unfortunately this inversion is typically ill-conditioned and may yield undesired solutions having spiky values or unexpected oscillations in the shallow part of the model. These phenomena normally arise because the 3D reflection tomography matrix is sparse and may be badly ill-conditioned resulting in a fairly large null space.

To avoid the problem of a large null space, one can add \( a \) priori information which regularizes the ill-conditioned system. The regularization of the model normally consists of a smoothing operator such as the Laplace operator applied to \( m \):

$$ Cost = \sum \left\| A m - S_{pick} \right\|_d + w \left\| V^2 m \right\|_m, $$  \hspace{1cm} (4)

where \( \left\| \cdot \right\|_m \) denotes a norm on the model, and \( W \) is a factor used to weight the influence of the data space versus the model regularization. Good examples for the norms are either the \( L^1 \) or Cauchy norm (Zhou et al. 2002; Zhou et al. 2003). The choice of a value for \( W \) is something of an art. Too small a value will not reduce the impact of the null space on the model and, as the number of iterations grows, oscillatory solutions occur in the shallow part of the model. Bad spike values will also occur in the overall model. On the other hand, too large a value will destroy the resolution and produce an overly smoothed velocity model. Compromising between the instabilities of an ill-conditioned system and model regularization is a dilemma. In this paper, we propose using as little model regularization as possible by building the tomography matrix \( A \) using “fat rays”, which should result in a less sparse and hence better conditioned matrix.

With this fat ray technique, we have developed TTI well-tie tomography, and implemented it in one land TTI velocity model building project, characterized with complex overthrust geologic structures. With 165 sonic well logs and a good coverage on the entire survey, after TTI well-tie tomography, Kirchhoff TTI prestack depth migration result shows image with depth errors of 1% or less.

Algorithm and method

Most tomography algorithms use two point ray-tracing, so that the tomography matrix is intimately connected to the underlying model. For instance, the matrix for a layered model is much less sparse than that for a grid/cell model which is the concern of this paper. In a grid/cell model (Zhou et al., 2003), the associated picked data results in a velocity perturbation on those cells through which the rays pass. In a high resolution model, the cell size is small, and a
Enhanced resolution tomography by fat ray techniques

Ray passes through relatively few cells. This produces a sparse matrix, the higher the desired resolution, the sparser the matrix. A better model representation such as a cubic B-spline model (Billette and Lambaré, 1998; Chauris et al., 2002) gives a less sparse matrix because the update will affect more B-spline coefficients than the number of traversed cells. This is equivalent to a ray becoming “fatter” than a simple ray path, and results in solutions becoming more stable.

This paper addresses a grid/cell model, and proposes a method of using fat rays directly to build the tomography matrix. This means that the velocity update occurs in a certain neighborhood centered about the ray. In Figure 1, the black line shows a traced ray in the Sigsbee 2A velocity model; the red area around the ray is that region which will be affected by the tomography. The fat ray has a starting width (initial fatness) and the fatness is allowed to increase gradually as its distance from the initial location increases. Thus the deep data will give average velocity update effects to the shallow part of velocity model, effectively avoid the oscillation perturbation as the number of iterations increases. We also propose an additional weight which is a B-spline based taper depending on distance to the actual ray. Therefore smooth illumination gives more regularity to the tomography matrix.

The proposed algorithm yields a tomography matrix having many more non-zero entries than conventional rays, thereby requiring less regularization. Still, some regularization is required because it does not completely resolve the null space problem. In practice we use the smallest regularization weight which provides a stable solution.

The fatness of the ray is an artificially defined parameter. It relates to the model definition and the number of seismic event picks, and must be adapted for different applications. The spread-out of fatness is another artificial parameter. Theoretically we should employ the same sort of width appearing in Gaussian beams but in practice a simple artificial linear divergence seems sufficient.

Applications and costs

Our algorithm can be applied to different tomography applications, e.g. traveltime tomography, migration velocity analysis (MVA), sub-salt velocity model building, well-tie velocity model building etc. The only difference is our tomography matrix is better conditioned and requires less regularization for a stable solution than conventional tomography. On the other hand, solving the system for a less sparse matrix typically requires more iterations. Compared to conventional tomography, greater computer resources are thus required, and the implementation is therefore parallelized over a PC cluster.

Subsalt model building

Subsalt imaging is always challenging. Traditional single-arrival Kirchhoff migration fails to yield good images because of multi-pathing. On the other hand, wave-equation migrations provide high fidelity images only if the velocity model is known accurately enough (Zhang and Zhang, 2005). The sediment velocity model above the salt can generally be accurately obtained using RCA (Zhou et al. 2003). Salt boundaries can be achieved with salt flood techniques. However, accurate subsalt velocity estimated remains one of the greatest challenges for seismic imaging. We applied the fat ray algorithm to subsalt velocity analysis using following strategy:

- The velocity above the bottom of salt is assumed to be correct and fixed;
- Starting with the best possible subsalt model, perturb only the subsalt part of the model over a range of percentages, normally 90%-110%;
- Migrate each subsalt velocity model using an accurate WEM scheme (Zhang and Zhang, 2005) and pick the velocity percentage which yields the best stacked image;
- Formulate the subsalt velocity scanning as a 3D tomography problem and solve it with the fat ray algorithm.

We tested the algorithm on the Sigsbee 2a synthetic model. Figure 2a shows the initial subsalt velocity model, which can be seen to be quite different from the exact model (Figure 2c). Figure 2b is the velocity model obtained by fat ray velocity scanning. It is quite close to the exact velocity shown in Figure 2c. Figure 3 shows the wave-equation migration results using these three velocity models respectively. The result with initial velocity gives a poor subsalt image. The result from fat ray tomography greatly

Figure 1. A fat ray. The center black line is the real traced ray in Sigsbee 2A model; the color column around the ray denotes the velocity zone affected by this ray.
Enhanced resolution tomography by fat ray techniques

improves the subsalt image quality, and is comparable to the image with exact velocity model.

**TTI model building by well tie tomography**

Seismic wave propagation through the shallow part of the earth generally manifests anisotropic properties; Depth imaging with isotropic velocities usually results in depth error; generally the isotropic images have deeper reflector than the well log shows. Depth errors often exceed 10%. In a simple layer cake model, one can interpret those depth errors by a good interpreter using neighborhood well log; In the complex structures imaging cases, in additional to depth errors, only taking into account isotropic velocity also gives poor focusing.

The anisotropic velocity model building using pure seismic data usually requires very long offset acquisition geometry, with the very far offset data offering the model builder lateral seismic wave propagation information. An alternative procedure is to use sonic well logs, which generally give exact depth location for some major horizons, and the tomography algorithm will tie the seismic reflection on the correct depth with the well logs. We give the algorithm a name: well-tie tomography.

The main objectives of well-tie tomography are

- Build a velocity model that will produce a better depth image, with good fault terminations and no fault shadows.
- Determine the anisotropic velocity model parameters, $V_0$, $\varepsilon$, $\delta$, $\theta$ and $\phi$, that will lead to flat depth migrated

| Figure 2. Sigsbee 2a velocity models. (a) The initial velocity model; (b) the updated velocity model by fat ray tomography; (c) the exact velocity model. |
| Figure 3. Wave equation migration with different velocity model. (a) With initial velocity model; (b) with tomography updated velocity model; (c) with exact velocity model |
Enhanced resolution tomography by fat ray techniques

common image gathers after TTI prestack depth migration. (In our case, the initial $\theta(x,y,z)$ and $\phi(x,y,z)$ are interpreted on the isotropic seismic depth migrated volume).

- Use reflection tomography for the velocity inversion.
- Tie all the well-tops with errors of 1% or less.

The Cupiagua 3D program was acquired in 1995 and 1996 with 4 cables along SE-NW direction. The far offset along the cables was 3160 meters. The Cusiana 3D program was acquired 90° to the Cupiagua program, in SW-NE direction. It was acquired in 1997 and 1998 with 6 cables. Its far offset along the cables was 3585 meters. Both projects have a very short offset range; using pure seismic data is not enough to determine the anisotropic model because lack of lateral seismic wave propagation information. Fortunately there are 165 wells which can provide necessary input for our anisotropic well tie tomography model building.

We implemented our fat ray technique on the well-tie tomography; we converted the well log depth errors to TTI time errors. The new algorithm minimizes the time errors along the TTI ray paths to complete the tomography matrix. Iteratively, our well tie tomography minimizes the depth errors. The final Kirchhoff prestack depth migration result shows that most of depth errors are under 1% after well-tie tomography.

Discussion and conclusions

We have formulated an algorithm using fat rays in tomography. Replacing a regular asymptotic ray by a fat ray gives a less sparse tomography matrix, solving this new tomographic system requires less regularization of the model. This means that higher resolution velocity models can be expected. Tests on Sigsbee 2a synthetic model gives promising subsalt results. Similar to conventional tomography, the fat ray technique can be used in other velocity analysis applications such as well-tie tomography, residual curvature analysis, etc. The computational cost increases with the fat ray method, but it is acceptable using a distributed, parallel PC cluster.

Although fat ray tomography does not make all the promises of waveform tomography (Lailly, 1983; Ravaut et al., 2003; Gao et al., 2004; Sirgue and Pratt, 2004), its computational cost is reasonable. For current computer systems and large 3D velocity model building projects, it is a useful tool.

Reference


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