Spectral Analysis of Post-imaging Seismic Data

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SUMMARY

A migrated seismic image represents the spatially variable reflectivity of the medium where migration effectively rotates the wavelet to be normal to the imaged reflectors. While this is the general case, it is often disregarded, and one-dimensional spectral analysis of the vertical coordinate is commonly used. We show that spectral processing of post-imaging data in the direction normal to the reflectors provides accurate results for steep and complex structures. We introduce the concept of Orthogonal-Image-Gathers (OIGs) which facilitate this approach while providing a platform for handling spatially variable spectral distortions due to the velocity field and other medium-related properties.
Introduction

Seismic data processing can be broadly divided into three stages: pre-imaging, imaging, and post imaging. Spectral analysis plays an important role within the pre-imaging and post-imaging steps and is the basis of many applied processes. All filtering and convolution-based techniques, which can be as simple as a low-cut filter or elaborate as an elastic inversion, are founded on spectral theory and correct analysis is essential for success.

Pre-imaging seismic data is defined in terms of surface and time coordinates; in this case temporal one-dimensional spectral analysis is truly representative of the recorded energy. The spatial wave-number components are intrinsically coupled to the temporal frequency by a dispersion relation, that is, they follow the wave equation.

On the other hand, post-imaging seismic data is defined in terms of 3D space coordinates. The seismic image represents the spatially variable reflectivity of the medium where the migration process effectively rotates the seismic wavelet to be normal to the imaged reflectors. While this is the general case, it is often disregarded, and one-dimensional spectral analysis of the depth coordinate is commonly used. This is performed after a 1D depth-to-time conversion.

The 1D convolutional model approximation is valid for simple flat events but in the presence of complex geologies and steep dips, this analysis is not representative of real spectral changes. The harmonic components are stretched and squeezed according to the structural dip of the image and distort the true reflectivity of the seismic image. Lazaratos and David (2009), present the benefit of spectrally shaping the data in the pre-imaging domain for this specific reason.

We show that spectral processing of post-imaging data in the direction normal to the reflectors provides accurate results for steep and complex structures. We introduce the concept of Orthogonal-Image-Gathers (OIGs) which facilitate this approach while effectively providing a platform for handling spatially variable spectral distortions due to the velocity field and other medium-related properties.

Theory

Seismic data are recorded at the surface of the acquisition survey where amplitude variation with respect to time is measured. Temporal spectral analysis is then fully representative of the data as it characterises the total energy of the recorded wavefield and is intrinsically coupled to the spatial components by the dispersion relation. This relation is derived from the governing wave equation. In the acoustic case, the dispersion relation is simply

\[ f = \frac{1}{2\pi} \sqrt{v^2 k_x^2 + k_y^2 + k_z^2} \]

where \( f \) is the temporal frequency, \( v \) is the velocity and \( k_x, k_y, \) and \( k_z \) are the spatial wave-numbers.

Imaging algorithms map the energy recorded on the surface back to the subsurface location which generated the reflection. The wavelet essentially becomes three-dimensional. In net effect, the wavelets are orientated towards the direction which is normal to the reflector. Figure 1 shows the wavelets for seismic data before and after imaging. Wavelets are aligned with the time axis for the
pre-imaging case, where 1D spectral analysis is valid, while for the post-imaging case the wavelets are orthogonal to the reflector and depth variations are stretched by the dip.

To perform correct spectral analysis of post-imaging data, the analysis should be performed in the direction normal to the reflectors. This can be achieved by utilizing the total wave-number in multi-dimensional Fourier domain where it by definition points to the reflectors’ direction. A mapping is then required to link this wave-number to the temporal frequency which is used to design and apply filters.

We introduce Orthogonal-Image-Gathers (OIGs) by applying orthogonal displacements to the seismic image. This is achieved in wave-number domain thanks to the Fourier translation property. Figure 2 shows a schematic of this operation.

\[
I(k; \zeta) = I_o(k)e^{-i|k|\zeta},
\]

where \( I \) are the generated OIGs, \( k \) is the wave-number vector, \( \zeta \) is the orthogonal displacement, and \( I_o \) is the input seismic image. OIGs are then inverse-transformed back into space domain.

While \( \zeta \) describes the seismic image in the direction normal to the reflectors, it is defined in terms of distance. Mapping to time is required such that temporal filtering may be performed.

Similar to Khalil et al. (2013), a transformation is introduced to locally stretch OIGs from displacement to time.

\[
I(x; \tau) = I(x; \zeta(\tau)) , \quad \zeta(\tau) = \frac{1}{2} c(x) \tau ,
\]

where \( x \) is the space coordinates vector, the \( \tau \) axis of \( I(x; \tau) \) denotes the data in the direction normal to the reflectors at each imaged point stretched to time, and \( c(x) \) is the spatially variable velocity of the medium.

Spectral analysis using the \( \tau \) axis is correct for dipping and non-dipping structures. In addition non-stationary and spatially variable effects as from the velocity field are now accessible and can be easily handled. Theoretically, the incidence angle is assumed to be zero in Equations 3. This is a typical assumption for migrated stacks. For angle gathers, the incidence angle needs to be considered in the stretching function.

**Figure 2** (Left) a dipping reflector and the direction of orthogonal displacement. (Right) Orthogonal Image gathers.
Example 1: Frequency scale splitting

The post processing sequence involves many practices that require the data to be split into different bands, normally octaves. Figure 3 shows three of the decomposed bands of a seismic image. At the top of the figure, the scale decomposition is based on the depth axis with the 1D assumption. At the bottom, the scale decomposition is based on the direction normal to the reflector. For the one-dimensional case, steep events leak into lower scales masking the true nature of the data and cause false interpretation of the true spectral character. Residual migration noise is relatively amplified and leaks into lower bands due to its curvature. With the decomposition based on the normal direction, the different scales look more physical and spectral components are mapped to their intended scale.

Example 2: Coloured inversion

In this example we present the result of applying coloured inversion (CI) (Lancaster and Whitcombe, 2000) using the usual depth axis based method (after time conversion) and the direction normal to the reflector. Local depth-to-time conversion using the migration velocity model is performed to map the wave-number of the orthogonal displacement to temporal frequency. Figure 4 shows the application of the two approaches on the seismic image. The steeply dipping events and the migration noise are overly boosted with the conventional approach, while when correctly applying the CI operator in the direction normal to the reflector, the inversion results look physical and realistic without noise or steep dip amplification.

Figure 3 Frequency scale splitting of imaged data. (Top) Splitting based on the depth axis (Bottom) Splitting based on the direction normal to the reflector.
Conclusions

For post-imaging data, conventional depth axis based spectral processing is not valid for dipping events (with or without time-to-depth conversion). Spectral processing in the direction normal the reflector is the correct way. This can be achieved in 3D wave-number domain for stationary processes which are space-independent. For non-stationary effects, as from velocity or absorption, orthogonal-image-gathers can be used to extract the spectral content variations in the direction normal to the reflector in a spatially variable manner. The examples we show are not limiting and the same concept is applicable to other methods, for example: elastic inversion and 4D time-shift estimation.

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Figure 4 Applying coloured inversion to a seismic image (Left) using the conventional approach (Right) based on the direction normal to the reflector.

References

