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Joint Low-rank and Sparse Inversion for Multidimensional Simultaneous Random/Erratic Noise Attenuation and Interpolation

R. Sternfels* (CGG), G. Viguier (CGG), R. Gondoin (CGG) & D. Le Meur (CGG)

SUMMARY

Recent developments in the field of compressed sensing enable us to take a new look on Cadzow / Singular Spectrum Analysis (SSA) filtering and its robust and interpolation derivatives. We formulate the problem of simultaneous random plus erratic noise attenuation and interpolation as a well-posed Joint Low-Rank and Sparse Inversion (JLRSI) convex optimization program. The JLRSI problem consists in a joint minimization of a nuclear norm term and a L1 norm term to recover the low-rank signal component from incomplete and noisy data. Numerical results on field data illustrate the effectiveness of our approach at recovering missing data and increasing the signal-to-noise ratio.

Introduction

Several methods have been proposed to improve the signal-to-noise ratio by attenuating incoherent noise, including prediction error filtering (Canales 1984), projection filtering (Soubaras 1995), and more recently rank reduction filtering. In this last category, we can differentiate eigenimage filtering (Trickett 2003), Cadzow / Singular Spectrum Analysis (SSA) filtering (Trickett 2009, Sacchi 2009) and tensor methods (Kreimer and Sacchi 2012, Trickett 2013, Da Silva and Herrmann 2014). Also, these latter methods have been extended to robust noise attenuation to deal with erratic noise, or data interpolation for binned data within a defined grid (Trickett et al. 2010, 2012, Oropeza and Sacchi 2011, Chen and Sacchi 2013). Here, we propose a systematic formulation of the simultaneous random plus erratic noise attenuation and data interpolation problem as a convex optimization program, which can be solved efficiently. We model the coherent signal via its low-rank trajectory matrix in the spirit of Cadzow/SSA filtering, and the erratic noise as a sparse component of the input data. The signal component is recovered by solving a Joint Low-Rank and Sparse Inversion (JLRSI) thanks to a joint minimization of the nuclear and L1 norms of the low-rank and sparse components respectively.

Theory

For a given spatiotemporal window, we work in the temporal frequency domain and form the following trajectory matrix for each frequency, in this case for a spatial window of five traces and a frequency slice $D = (D_1,\ldots,D_5)$:

$$H(D) = \begin{pmatrix} D_1 & D_2 & D_3 \\ D_2 & D_3 & D_4 \\ D_3 & D_4 & D_5 \end{pmatrix}$$

The trajectory matrix $H(D)$ is a Hankel matrix, and rank reduction of this matrix via its Singular Value Decomposition (SVD) is carried out in Cadzow/SSA filtering in view of random noise attenuation (Trickett 2009, Sacchi 2009). The denoised signal is then recovered by averaging the resulting rank-reduced matrix along its anti-diagonals. If the input data is free of noise, then the rank of the trajectory matrix is equal to the number of dipping events in the data (Trickett 2009, Sacchi 2009). We can generalize this process in higher dimensions by organizing the data D into a block Hankel matrix with Hankel blocks (Trickett 2009):

$$H(D) = \begin{pmatrix} H_1 & H_2 & H_3 \\ H_2 & H_3 & H_4 \\ H_3 & H_4 & H_5 \end{pmatrix}$$

Here, each H_i submatrix is itself a Hankel matrix in the case of two spatial dimensions, or a block Hankel matrix with Hankel blocks for higher spatial dimensions. Again, rank reduction of $H(D)$ via its SVD followed by averaging along the (block) anti-diagonals is performed for Cadzow denoising and multichannel SSA (Oropeza and Sacchi 2011).

Assuming a given level of random noise δ in the input data D, we can reformulate the Cadzow/SSA filtering problem as follows to recover the signal S:

$$\begin{array}{ll} \text{minimize} & \text{rank } H(S) \\ \text{such that} & \| D - S \|_2 \le \delta \end{array}$$

Now, let's assume that the recorded data D is both incomplete and corrupted by erratic noise such as spike bursts or localized phase distortions. Applying Cadzow/SSA filtering directly to such data would yield incorrect results because the SVD is optimal only in the least-squares sense, thus very sensitive to missing data and outliers. Variants of Cadzow/SSA filtering have been previously proposed to address the problems of either robust denoising or interpolation (Trickett et al. 2010, 2012, Oropeza and Sacchi 2011, Chen and Sacchi 2013); we develop here a systematic approach based on compressed sensing and convex programming to solve both problems simultaneously. Furthermore, our approach enables us to apply theoretical developments from the mathematical literature and state-of-the-art, efficient convex optimization algorithms.

We can extend our reformulation of Cadzow/SSA filtering as follows to better address our problem; still denoting the input data and the signal component respectively by D and S, we assume that the available information is encoded by the sampling operator $P_{\Omega}[.]$ (zero when data are unavailable, identity otherwise) and we introduce an erratic noise component E:

$$\begin{array}{ll} \text{minimize} & \operatorname{rank} H(S) + \| E \|_0 \\ \text{such that} & \| P_{\Omega}[\, D - S - E \,] \|_2 \leq \delta \end{array}$$

This constrained minimization problem can be interpreted as follows:

- The trajectory matrix formed from the denoised and interpolated signal component of the input data must be low-rank;
- The sparsity of the erratic noise component is measured by its number of non-zero components $\|E\|_0$;
- The low-rank and sparse components must fit the available input data up to an assumed level δ of random noise.

However, this is a very difficult problem, infeasible in most cases. Therefore, we seek a surrogate, solvable, problem. This can be achieved using a convex relaxation, that is, we “convexify” the objective function by modifying it as follows to give the JLRSI problem:

$$\begin{array}{ll} \text{minimize} & \| H(S) \|_* + \lambda \, \| E \|_1 \\ \text{such that} & \| P_{\Omega}[\, D - S - E \,] \|_2 \leq \delta \end{array}$$

Where: $\|.\|_*$ is the nuclear norm (sum of singular values), $\|.\|_1$ is the entry-wise L1 norm (sum of absolute values), and λ is a regularization parameter. This is a convex optimization problem, and although non-smooth, it can be solved efficiently: in our case, we apply an efficient proximal Alternating Direction Method of Multipliers (ADMM) algorithm (Parikh and Boyd 2013), with a fast SVD kernel based on fast Hankel matrix-vector products (Gao et al. 2012) achieving an $O(n \log(n))$ complexity in floating point operations, where n is the number of traces per frequency slice.

We point out that this problem is an extension of the principal components pursuit problem (Candès et al. 2009); and that the nuclear and L1 norms are workhorses of compressed sensing theory (Herrmann et al. 2012), enabling strong results based on weak assumptions on the input data and its structure.

Example

We applied the proposed JLRSI method to a land dataset arranged in thirty-six Common Offset-Vector (COV, Cary 1999) volumes with inline and crossline increments of 15 meters. We used a total of four spatial dimensions in this example: inline, crossline, and both components of the offset vector. Figure 1 shows the original central COV volume and the stack of the 36 original COV volumes. As we can see, pre-stack data exhibits a rather poor signal-to-noise ratio. Figure 2 shows a stack of the raw, fully populated, data before and after multi-dimensional Cadzow/SSA filtering with no apparent signal leakage. Then, we randomly decimated the input data to keep 25% of all original traces in the input pre-stack volume, and we randomly add high-amplitude spikes acting as erratic noise. We show the central COV volume in Figure 3, with all original data, then with decimation and randomly added high-amplitude spikes, and finally after application of JLRSI. In Figure 4, we show the stack of the 36 decimated COV volumes, the stack of all COV volumes after JLRSI recovery, and finally a difference with the stack of the original undecimated, uncorrupted COV volumes shown in the left-hand panel of Figure 2. Slightly less aggressive parameters were chosen for the JLRSI process (Figures 3-4) than for Cadzow/SSA filtering (Figure 2): JLRSI being a slower iterative optimization process, it is safer to be conservative at first then apply an additional pass of random noise attenuation if needed. In this example, the choice of parameterization provides effective noise attenuation while still preserving the character of the data.

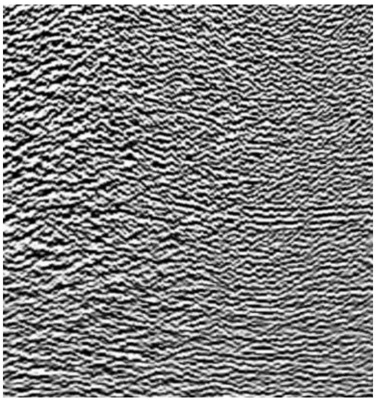
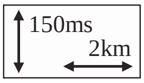
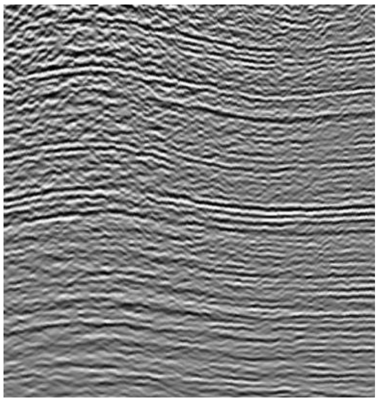


***Figure 1**:*
Left) Original central COV volume.
Right) Stack of the original 36 COV volumes.





Conclusion

We propose to formulate the problem of simultaneous random plus erratic noise attenuation with interpolation as a convex optimization problem, using benefits from the latest developments in compressed sensing theory. The resulting well-posed convex program, Joint Low-Rank and Sparse Inversion (JLRSI), is then solved thanks to an efficient ADMM algorithm. Finally, we are able to exploit the full dimensionality of seismic data to efficiently and accurately recover the signal component from incomplete and corrupted data with a poor signal-to-noise ratio, as it is demonstrated on our field example.

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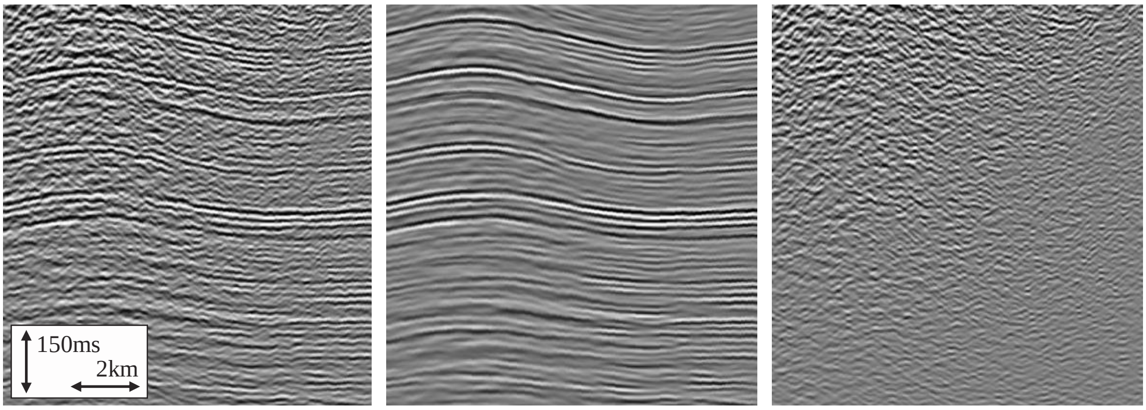

Figure 2 *Left) Stack of the original undecimated, uncorrupted COV volumes. Middle) Resulting stack after application of Cadzow/SSA filtering on the original data. Right) Difference with original stack.*

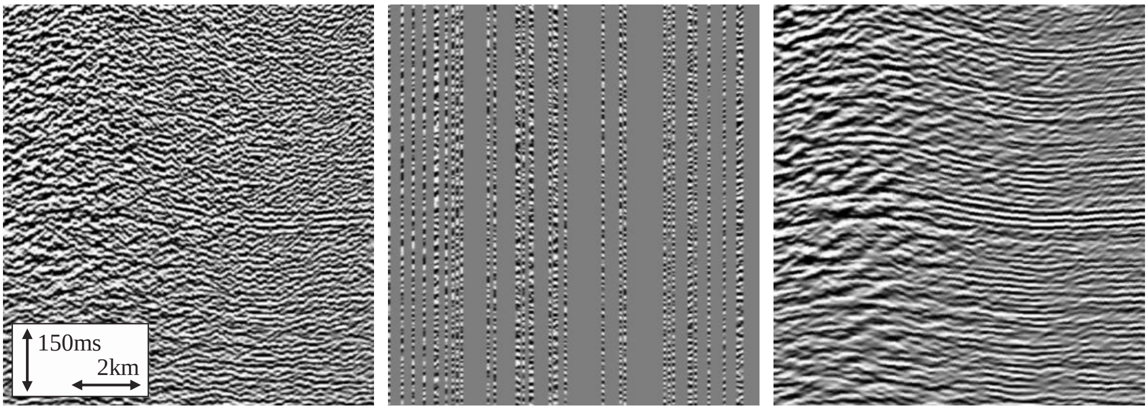

Figure 3 *Left) Original central COV volume, the signal to noise ratio is very low. Middle) Central COV volume, with 25% traces remaining and random high-amplitude spikes added. Right) Decimated central COV volume with added noise spikes after application of the suggested JLRSI process. The signal to noise ratio is improved.*

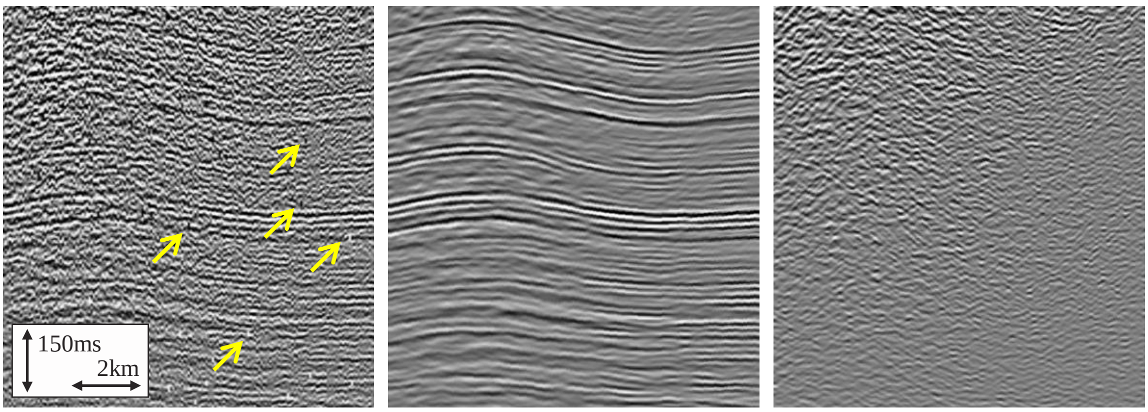

Figure 4 *Left) Stack of the 36 COV volumes with 25% pre-stack traces remaining and randomly added high-amplitude spikes (see yellow arrows). Middle) Stack after simultaneous random and erratic noise attenuation with interpolation of missing traces using JLRSI. Right) Difference with stack of original data in Figure 2, left-hand panel.*