Orthorhombic anisotropy: A physical seismic modeling study
Scott P. Cheadle*, R. James Brown‡, and Don C. Lawton‡

ABSTRACT
An industrial laminate, Phenolic CE, is shown to possess seismic anisotropy. This material is composed of laminated sheets of canvas fabric, with an approximately orthogonal weave of fibers, bonded with phenolic resin. It is currently being used in scaled physical modeling studies of anisotropic media at The University of Calgary. Ultrasonic transmission experiments using this material show a directional variation of compressional- and shear-wave velocities and distinct shear-wave birefringence, or splitting. Analysis of group-velocity measurements taken for specific directions of propagation through the material demonstrates that the observed anisotropy is characteristic of orthorhombic symmetry, i.e., that the material has three mutually orthogonal axes of two-fold symmetry. For P waves, the observed anisotropy in symmetry planes varies from 6.3 to 22.4 percent, while for S waves it is observed to vary from 3.5 to 9.6 percent.

From the Kelvin-Christoffel equations, which yield phase velocities given a set of stiffness values, expressions are elaborated that yield the stiffnesses of a material given a specified set of group-velocity observations, at least three of which must be for off-symmetry directions.

INTRODUCTION
Studies of anisotropy and shear-wave splitting are gaining importance as part of the ongoing effort to enhance seismic data interpretation and reservoir exploitation. Several authors have dealt with the relationships among anisotropy, shear-wave polarization and fracture patterns (e.g. Keith and Crampin, 1977; Crampin, 1981, 1984, 1985; Lewis et al., 1989; Yale and Sprunt, 1989). Liu et al. (1989) used numerical modeling results to outline the potential and limitations of shear-wave splitting analysis for the crosswell configuration. Both compressional- and shear-wave anisotropy impact on velocity analysis for multicomponent seismic imaging and on methods of estimating subsurface stress based on the Vp/Vs ratio (Thomsen, 1986, 1988). Banik (1984) reported errors in depth estimates of between 150 and 300 m in areas of the North Sea basin due to anisotropy within some shaly units. Ensley (1989) described anisotropy values of between -40 and +40 percent for “sand-, shale-, and carbonate-prone” units in the Prudhoe Bay area.

Physical seismic modeling can be extremely useful in bridging the gap between theory and the complexities observed in field seismic data, and this seems particularly true in the context of seismic anisotropy. Many theoretical predictions of wave-propagation phenomena can be tested in sealed laboratory experiments. Ultrasonic modeling using phenolic laminate is ideally suited to the study of velocity anisotropy because the ambiguities inherent in field data are absent. Tatham et al. (1987), Sayers (1988), Ebrom et al. (1990) and Rathore et al. (1990) have described physical modeling experiments simulating fracture-induced anisotropy. This type of work is being carried further within the CREWES Project (Consortium for Research in Elastic Wave Exploration Seismology) at The University of Calgary.

This paper describes the results of experiments to determine the anisotropic elastic properties of Phenolic CE. We first determine the variation of the body-wave group velocities with direction by measuring travel times over a selection of paths. The theory of wave propagation in anisotropic media (Appendix) is then used to relate these observed group velocities to the nine (in the orthorhombic case) elastic coefficients or stiffnesses, permitting us to compute the details of elastic-wave propagation in any direction through the phenolic and, in particular, to compute the variation of quantities such as Thomsen’s (1986) anisotropy parameters and the phase velocity.

PHYSICAL MODEL EXPERIMENTS
Laboratory set-up
We are using piezoelectric P-wave and S-wave transducers as both sources and receivers in our multicomponent physical modeling. Both types are flat-faced cylindrical contact transducers with an active element 12.6 mm in diameter. With reference to a horizontal profile, the compressional or P-wave transducer (Panametrics V103) is vertically polarized, with the maximum sensitivity normal to the contact face; the shear-wave transducer (Panametrics V153) is horizontally polarized, with the maximum sensitivity parallel to a line across the contact face. During operation, these contact faces are coupled to a selected flat

Presented at the 60th Annual International Meeting, Society of Exploration Geophysicists. Manuscript received by the Editor August 24, 1990; revised manuscript received March 21, 1991.

* Formerly Department of Geology & Geophysics, The University of Calgary; presently Veritas Seismic Ltd., 200, 615-3rd Ave. SW Calgary, Alberta, Canada T2P OG6.
‡ Department of Geology & Geophysics, The University of Calgary, Calgary, Alberta, Canada T2N 1N4.
surface of the phenolic and, for a particular experiment, a profile direction and sagittal plane are established. To record the radial component, the shear-receiver transducer is used with its polarization parallel to the direction of the profile (inline), whereas for the transverse component, it is rotated so that the polarization is perpendicular to the azimuth of the profile and to the sagittal plane (cross-line).

The source transducer is driven with a 28-volt square wave tuned to produce a broadband wavelet with a central frequency of 600 kHz. Amplified data are sampled using a Nicolet digital oscilloscope connected through an IBM-XT, which controls the experiments, to a Perkin-Elmer 3240 seismic processing system for storage. Traces of up to 4096 samples are recorded sequentially and stored on tape or disk in SEG-Y format.

The CE-grade phenolic laminate is composed of layers of a woven canvas fabric saturated and bonded with a phenolic resin, and has a density of 1364 kg/m³. In one direction of the fabric the fibers of the warp run more or less straight, like the fixed threads on a loom; in the orthogonal direction the fibers of the woof run back and forth across the warp. Initial tests with the material showed a directional dependence of the velocity for both P and S waves, suggesting its suitability for physical modeling of an anisotropic medium. Shear-wave splitting was observed during transmission tests when the sample was rotated between two shear-wave transducers. The polarizations of the split shear waves were approximately parallel to the orientations of the orthogonal weave of fibers in the canvas fabric. For this reason, subsequent experiments were conducted on pieces of phenolic that were cut with faces parallel or orthogonal to the observed fiber directions as well as to the plane of the canvas layers. A sample of the phenolic with the faces labeled with the convention used in this study is shown in Figure 1. The factory-machined surface of the laminate sheet parallel to the fabric layers was designated Face 3, consistent with the conventional choice of x₃ as the vertical direction and with a horizontal attitude for the layering of the medium. Since the 3-direction turned out to be slowest for P-wave propagation, the other two principal (or symmetry) directions were labeled such that the 1-direction (parallel to the woof) is fastest and the 2-direction (parallel to the warp) intermediate for P-wave propagation.

The apparatus used for studying split shear waves is shown in Figure 2. The cube of material is placed between two fixed shear-wave transducers which are aligned with parallel polarizations. The cube is rotated between the transducers, and a pointer on the cube is used to determine the azimuth of the sample with respect to a fixed circular protractor. A similar experimental procedure was described by Tatham et al. (1987) for a study of fracture-induced shear-wave splitting.

**Experimental results**

Shear-wave splitting experiments were conducted using cubes of the phenolic as described above. Figures 3, 4, and 5 show the transmission records through Faces 1, 2, and 3, respectively, of an approximately 9.6 cm cube of phenolic. Each trace records the signal transmitted through the cube at 5-degree intervals of rotation with respect to the polarization direction of the shear-wave transducers. The 0-degree direction was chosen to correspond to the polarization azimuth of the amplitude maximum of the faster of the two shear-wave arrivals. The sample interval used in this study was 50 nanoseconds, and the arrival times are shown in microseconds. The faster shear arrival is designated S₁ and the slower mode S₂. While it is more correct to refer to the split shear waves and the compressional waves under most conditions as quasishear and quasicompressional modes, except for special cases, such as propagation in one of the principal directions, that prefix will usually be implied rather than explicitly stated. [Crampin (1989) and Winterstein (1990) have provided authoritative manuals of terminology for seismic anisotropy.] In Figures 3, 4, and 5, the weakly coupled P-wave arrival is barely visible. The compressional velocities were determined separately using the P-wave transducers.

In Figures 3 and 4, for propagation in the 1- and 2-directions, respectively, the polarization (particle motion) at the S₁ amplitude maximum is, in each case, parallel to the "bedding plane" of the canvas layers, whereas for the S₂ amplitude maxima, the polarizations are perpendicular to this plane. In Figure 5, for propagation in the 3-direction, the polarization of the S₁ amplitude maximum is parallel to the 1-direction (the woof), while the S₂ amplitude maximum is parallel to the 2-direction (the warp). A plot of amplitude versus polarization direction for a record

![FIG. 1. Phenolic CE laminate is composed of layers of a canvas weave fabric bonded with phenolic resin. The faces are labeled as used in this study.](image-url)
FIG. 2. The apparatus used for shear-wave splitting experiments clamps the cube of phenolic between two shear-wave transducers with mutually parallel polarization. The cube is rotated while the transducers remain fixed. A circular protractor scale is used to determine the azimuth of rotation.

FIG. 3. The record through Face I of a 9.6 cm cube of phenolic, showing the faster $S_1$ arrival at 1665 m/s and the slower $S_2$ arrival at 1602 m/s. The compressional velocity in the 1-direction is 3576 m/s. The polarization direction of the $S_1$ amplitude maximum is parallel to the “bedding plane” of the canvas layers, while that of the $S_2$ amplitude maximum is perpendicular to that plane.

FIG. 4. The record through Face 2, showing the faster $S_1$ arrival at 1658 m/s and the slower $S_2$ arrival at 1506 m/s. The compressional velocity in the 2-direction is 3365 m/s. The polarization directions of the $S_1$ and $S_2$ amplitude maxima are parallel and perpendicular respectively to the canvas layering, as in Figure 3.
Orthorhombic Physical Seismic Modeling

through Face 2 is shown in Figure 6. This and other transmission records through the phenolic show that the $S_1$ mode generally has a greater maximum amplitude than the $S_2$ arrival, indicating greater attenuation for the $S_2$ mode. The ratios of the amplitudes of the $S_1$ arrivals to those of the $S_2$ arrivals, measured at their maxima, have ranged from 1.1 to 1.4 for the samples tested.

The $P$, $S_1$, and $S_2$ velocities measured along the principal axes are summarized in Figure 7 and those along the 45-degree diagonals in Figure 8. Fast, medium, and slow directions through the cube (1, 2, and 3, respectively) were defined on the basis of the $P$-wave velocities (Figure 7). The values quoted are group velocities based on the transit time measured with respect to the onset of the pulse. The velocities are the averages of values measured through 10- and 8-cm cubes. The measured velocities for the phenolic cubes were repeatable to within $\pm 15$ m/s ($=0.5$ percent) for $P$-waves and $\pm 4$ m/s ($=0.25$ percent) for shear waves. The variations are likely related to small inconsistencies in the thickness of the coupling agent used to bond the transducers to the phenolic. Velocity variations between different samples of phenolic ranged up to 2 percent. The time picks used to calculate the velocities were made directly on the digital oscilloscope for maximum accuracy.

For the following discussion, the velocities will be labeled with 2 subscripts indicating, respectively, the directions of propagation and polarization with respect to the three symmetry axes (Figure 7). For example, $V_{11}$ is the group velocity for propagation and particle motion in the 1-direction (a $P$ wave) while $V_{12}$ indicates propagation in the 1-direction with polarization in the 2-direction (an $S$ wave). The six shear-wave velocities measured in the principal directions were paired as follows: $V_{23} = V_{32}$; $V_{31} = V_{13}$; $V_{12} = V_{21}$; indicating, with very small error (Table 1), only three independent values.

For the cases of diagonal raypaths in symmetry planes we adopt, for the purpose of this paper alone, a special index
Orthorhombic Physical Seismic Modeling

convention (Figure 8). For the 23-plane, the direction at 45 degrees to the 2- and 3-directions is denoted by the index 4. The group velocity of the quasi-\(P\) (\(qP\)) wave in this direction is thus designated \(V_{44}\). Polarization quasi-normal to this 4-direction but still within the 23-plane is denoted by the index 4. The velocity of the corresponding \(SH\) wave, with particle motion in the 1-direction, is labelled \(V_{41}\). Similarly, we use the indices 5 and 6 to denote propagation in the 31 - and 12-planes, respectively, at 45 degrees. The \(P\)-, \(SV\)-, and \(SH\)-wave group velocities are thus labeled \(V_{45}\), \(V_{47}\), and \(V_{52}\), in the 31-plane, and \(V_{66}\), \(V_{56}\), and \(V_{63}\), in the 12-plane (Figure 8), in each instance only for the special cases of rays at 45 degrees to the symmetry directions.

Each of the velocities along the diagonal raypaths is the average of two measurements (between the two pairs of opposing edges of the cube) which had equivalent raypaths relative to the principal axes within each of the three principal planes. The two traveltimes for each of the diagonal raypath pairs were virtually identical, differing by two sample points (100 ms) or less in all cases. Four measurements were also recorded for rays from corner to corner of the cube, with similarly small differences in the measurements were also recorded for raypaths from corner to corner of the cube, with similarly small differences in the traveltimes. This symmetry confirmed that the presumed principal planes, chosen to correspond to the planar layering of the canvas fabric and the orthogonal weave of fibers in the phenolic, are indeed the seismic anisotropic symmetry planes.

### 45° AXES

**RAY (GROUP) VELOCITIES**

FIG. 8. The results of transmission measurements between opposing edges of the phenolic cube are summarized. The propagation directions were at 45 degrees to two of the principal axes and perpendicular to the third.

**ORTHORHOMBIC ANISOTROPY**

For the orthorhombic symmetry system, the 3 x 3 x 3 x 3 stiffness tensor \(C_{ijkl}\) (see the Appendix) may be reduced to a 6 x 6 symmetric matrix, namely:

\[
C_{\text{an}} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{12} & C_{22} & C_{23} \\
C_{13} & C_{23} & C_{33} \\
C_{44} & C_{15} & C_{16} \\
C_{55} & C_{26} & C_{36} \\
C_{66} & C_{56} & C_{66}
\end{bmatrix}
\]

Of nine independent coefficients (e.g. Nye, 1985). Using the elastic equations of motion the stiffnesses \(C_{\text{an}}\) may be estimated from the observed body-wave velocities and the density of the phenolic (see the Appendix). The computed stiffnesses are summarized in Table 1. Along the principal axes the phase and group velocities are equal and the stiffnesses were computed directly using equations (A-45) and (A-46). Along the diagonal raypaths, the direction of the wavefront normal (i.e. the slowness direction) is not, in general, the same as the 45-degree direction of the raypath (i.e., of energy transport). The procedure used to compute the slowness directions, the phase velocities, and the related stiffnesses for the diagonal raypaths is described in the Appendix.

Nine independent velocity values, are required to enable complete determination of the stiffness matrix for the case of orthorhombic anisotropy. These could include the three \(P\)-wave velocities along the principal axes, three shear-wave velocities observed along the principal axes, and three \(P\)-wave or \(SV\)-wave velocities, each for a raypath perpendicular to one and at 45 degrees to the other two principal axes. In principal, measurements at other orientations could be used but these would require considerably more complex solutions.

Since we actually observe more than nine velocities, the internal consistency of the orthorhombic symmetry model can be checked. In addition to observations in this context already mentioned above, equation (A-44) was used with the shear-wave velocities observed along the principal axes (Figure 7) to calculate additional independent values for the \(SH\)-wave velocities along the diagonal raypaths (Figure 8). For example,

\[
V'_4 = \sqrt{2V_5V_{12}/(V_{12}^2 + V_{13}^2)^{1/2}} = 1633 \text{m/s},
\]

where \(V_{44}\), the observed value, is 1636 m/s, differing by 0.18 percent from \(V'_4\) the calculated value. Similarly, \(V'_{52}\) =1583m/s, equal to \(V_{52}\), and \(V'_{63}\) = 1559m/s, differing by 0.19 percent from the \(V_{63}\) value of 1556 m/s. Clearly, the \(SH\)-mode velocities observed along the diagonal raypaths conform very well to the assumed orthorhombic symmetry model.

The off-diagonal stiffnesses have been computed (Table 1) using the velocities measured on diagonal raypaths [equations (A-47)] both for \(P\) (giving \(C_{ij}\)) and for \(SV\) (giving \(C_{ij}\)). The differences between pairs (\(C_{ij}\), \(C_{ij}\)) seem
Orthorhombic Physical Seismic Modeling

rather large at first; however, the stiffness values are quite sensitive to small velocity changes. For each wave type we compare the two velocities (one measured and one calculated) corresponding to a stiffness pair \((C_{ij}, C_{ji})\). The following are the relative deviations from the mean of these velocity pairs, with the corresponding deviation for the stiffness in square brackets (all given as percentages): -21-plane: \(\pm 0.3\) \((P)\), \(\pm 1.0\) \((SV)\), \([\pm 2.5]\); 13-plane: \(\pm 0.6\) \((P)\), \(\pm 2.4\) \((SV)\), \([\pm 5.7]\); 23-plane: \(\pm 0.5\) \((P)\), \(\pm 2.0\) \((SV)\), \([\pm 4.4]\).

The results of these experiments indicate that the orthorhombic symmetry system is appropriate to describe the anisotropy of this material. In particular, three distinct compressional velocities were measured in mutually orthogonal directions consistent with the visible structural symmetry of the material; the observed shear-wave velocities along the principal axes are equal in pairs \((V_{ij} = V_{ji})\); the observed diagonal \(SH\) velocities agree well with those calculated independently [equation (2); Table 1]; and the two independent determinations of each off-diagonal stiffness \((C_{23}, C_{31}, C_{12})\), from a \(qP\) and a \(qS\) measurement, are in good agreement (Table 1).

<table>
<thead>
<tr>
<th>Raypaths in Principal Directions</th>
<th>Mode</th>
<th>Group Velocity (m/s)</th>
<th>Phase Velocity (m/s)</th>
<th>Phase Angle (deg)</th>
<th>Stiffness ((x10^9 N/m^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P) (v_{33})</td>
<td>2925</td>
<td>2925</td>
<td></td>
<td></td>
<td>(C_{33}) 11.670</td>
</tr>
<tr>
<td>(P) (V_{22})</td>
<td>3365</td>
<td>3365</td>
<td></td>
<td></td>
<td>(C_{22}) 15.445</td>
</tr>
<tr>
<td>(P) (V_{11})</td>
<td>3576</td>
<td>3576</td>
<td></td>
<td></td>
<td>(C_{11}) 17.443</td>
</tr>
<tr>
<td>(S) (V_{21})</td>
<td>1658</td>
<td>1662</td>
<td></td>
<td></td>
<td>(C_{66}) 3.768</td>
</tr>
<tr>
<td>(S) (V_{12})</td>
<td>1665</td>
<td>(avg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S) (V_{31})</td>
<td>1610</td>
<td>1606</td>
<td></td>
<td></td>
<td>(C_{55}) 3.518</td>
</tr>
<tr>
<td>(S) (V_{13})</td>
<td>1602</td>
<td>(avg)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S) (V_{32})</td>
<td>1525</td>
<td>1516</td>
<td></td>
<td></td>
<td>(C_{44}) 3.135</td>
</tr>
<tr>
<td>(S) (V_{23})</td>
<td>1506</td>
<td>(avg)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Raypaths at 45 Degrees to Principal Directions</th>
<th>Mode</th>
<th>Group Velocity (m/s)</th>
<th>Phase Velocity (m/s)</th>
<th>Phase Angle (deg)</th>
<th>Stiffness ((x10^9 N/m^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P) (V_{66})</td>
<td>3378</td>
<td>3373</td>
<td>41.6</td>
<td>(C_{21}) 7.104</td>
<td>7.283</td>
</tr>
<tr>
<td>(SV) (V_{5\tau})</td>
<td>1810</td>
<td>1809</td>
<td>47.1</td>
<td>(C_{1\tau}) 7.462</td>
<td>(avg)</td>
</tr>
<tr>
<td>(SH) (V_{63})</td>
<td>1556</td>
<td>(1559)*</td>
<td>1556</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P) (V_{55})</td>
<td>3219</td>
<td>3155</td>
<td>33.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(SV) (V_{5\tau})</td>
<td>1620</td>
<td>1620</td>
<td>45.0</td>
<td>(C_{1\tau}) 7.008</td>
<td>(avg)</td>
</tr>
<tr>
<td>(SH) (V_{52})</td>
<td>1583</td>
<td>(1583)*</td>
<td>1577</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P) (V_{44})</td>
<td>3094</td>
<td>3066</td>
<td>37.1</td>
<td>(C_{23}) 6.097</td>
<td>6.379</td>
</tr>
<tr>
<td>(SV) (V_{4\tau})</td>
<td>1569</td>
<td>1569</td>
<td>45.7</td>
<td>(C_{1\tau}) 6.662</td>
<td>(avg)</td>
</tr>
<tr>
<td>(SH) (V_{41})</td>
<td>1636</td>
<td>(1633)*</td>
<td>1632</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* See equation (2)
 Orthorhombic Physical Seismic Modeling

DISCUSSION

Degree of anisotropy
The conventional measures of anisotropy for the transverse isotropy case are given by Thomsen (1986) as

\[ \varepsilon \approx \frac{V_p(90\text{ degrees}) - V_p(0\text{ degrees})}{V_p(0\text{ degrees})} \]  
\[ \gamma \approx \frac{V_S(90\text{ degrees}) - V_S(0\text{ degrees})}{V_S(0\text{ degrees})} \]

and

At least in the case of transverse isotropy, the \( \varepsilon \) parameter is not always useful in the context of the limited ray angles typical of surface seismic gathers. Thomsen (1986) also defined the parameter and discussed its use in conjunction with moveout-velocity and stress analysis.

Table 2. Measured anisotropy

<table>
<thead>
<tr>
<th>Plane</th>
<th>( \delta )</th>
<th>( \varepsilon )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>21-plane</td>
<td>-0.047</td>
<td>0.063</td>
<td>0.059</td>
</tr>
<tr>
<td>31-plane</td>
<td>0.183</td>
<td>0.224</td>
<td>0.096</td>
</tr>
<tr>
<td>32-plane</td>
<td>0.081</td>
<td>0.150</td>
<td>0.035</td>
</tr>
</tbody>
</table>

The measurements of these velocity ratios determined in the principal planes of the phenolic are shown in Table 2, and fall within the range of the values reported by Thomsen (1986) for a variety of rocks. The \( P \)-wave anisotropy ranges form 6.3 percent in the 21-plane to 22.4 percent in the 31-plane, the \( SH \)-wave anisotropy from 3.5 percent in the 32-plane to 9.6 percent in the 31-plane. The plane of weakest \( P \) anisotropy is not the same as the plane of weakest \( SH \) anisotropy, but the planes of strongest anisotropy do correspond. Anisotropy of the \( SV \) mode is observed along the 45 degrees raypaths. In the 12-plane, \( V_{\sigma} \) is 1810 m/s, 8.9 percent higher than \( V_{31} \). In the 32 plane, \( V_{\sigma} \) is 1569 m/s, 3.5 percent higher than \( V_{32} \). \( SV \) anisotropy in the 31-plane is lowest (0.9 percent) despite this plane exhibiting the strongest \( P \) and \( SH \) anisotropy. Although some of these observations may seem surprising intuitively, they are all quite reasonable since the material has nine independent stiffnesses. In the 23-plane for instance, \( P \) and \( SH \) anisotropies depend on \( C_{22} \) and \( C_{33} \) \((P)\) and \( C_{44} \) \((SH)\) whereas the \( SV \) anisotropy depends not only on \( C_{22} \), \( C_{33} \), and \( C_{44} \), but also on \( C_{23} \).

Origin of the anisotropy
The cause of the anisotropy in the phenolic laminate appears to be related to the layering and the weave of the canvas fabric. The material behaves like a stack of nets set in a gel, with different fiber densities and orientations in the directions of the three principal axes. The many causes of anisotropy in rocks range from the microscopic to the macroscopic, including preferred orientation of mineral grains, pores or fractures (Crampin, 1981, 1984, 1985), thin-layer lamination (Helbig, 1983) and regional stress (Nikitin and Chesnokov, 1984). Anisotropy has been recognized in many rocks (Banik, 1984; Thomsen, 1986; Lewis et al., 1989; Ensmley, 1989), but the physical cause and symmetry systems of specific cases of anisotropic media are seldom unambiguously identified. Transverse isotropy can be invoked for horizontal thin-bed layering, for example, in shale sequences, while azimuthal anisotropy may arise in the idealized case of aligned vertical fractures. Both of these examples would be degenerate cases of the more general orthorhombic system. Two or more sources of anisotropy superimposed orthogonally within the same lithologic unit, such as aligned vertical fracturing of a horizontally laminated sequence, could result in orthorhombic anisotropy. The phenolic laminate is being used to simulate media with similar velocity properties regardless of the different physical causes of the anisotropy.

CONCLUSIONS

Ultrasonic modeling with Phenolic CE laminate has demonstrated the anisotropic elastic properties of the material. The patterns of shear-wave splitting observed through each face of a cube of the phenolic, along with the measured compressional-wave velocities, were used to define orthogonal principal (or symmetry) axes related to the fast, medium, and slow directions through the material. Shear- and compressional-wave velocities were also measured in directions between opposing edges of the cube to support the determination of the orientations of the planes of symmetry. Within a principal plane, the \( SV \) wave has equal velocities for propagation in either of the axial directions. Velocities computed (using equations of propagation based on velocities from other directions and assuming the orthorhombic model, closely matched the observed values. Analysis of the data supports the interpretation that the anisotropy conforms very closely to a system of orthorhombic symmetry.

Physical modeling is currently proceeding with the phenolic and involves the recording of shot gathers as well as simulated VSP and crosswell experiments. Observations of the effect of orthorhombic anisotropy on moveout velocities are being reported by the present authors (Brown et al., 1991) and tomographic reconstruction will be described in future publications. Physical model data using phenolic laminate is proving to be a valuable adjunct to numerical studies of the increasingly important topic of seismic anisotropy.

ACKNOWLEDGMENTS

This research is supported by funding provided by the corporate sponsors of the CREWES Project. The authors wish to acknowledge the technical contributions of Mr. Malcolm Bertram of the Department of Geology and
Orthorhombic Physical Seismic Modeling

Geophysics and Mr. Eric Gallant of the CREWES Project, both at The University of Calgary. Finally, we would like to thank Dan Ebrom and Linda Zimmerman for their knowledgeable reviews of this paper and for their constructive suggestions, which have led to an improved final version.

REFERENCES

Ensley, R. A., 1989, Analysis of compressional- and shear-wave seismic data from the Prudhoe Bay Field: The leading Edge, 8, no. 11, 1@13.

APPENDIX

RELATIONSHIPS AMONG VARIOUS ELASTIC-WAVE PARAMETERS IN AN ANISOTROPIC MEDIUM OF ORTHORHOMBIC SYMMETRY

Basic theory and the Kelvin-Christoffel equations

The equations of motion governing wave propagation in a generally isotropic elastic medium are given by many authors (e.g. Bullen, 1963; Fedorov, 1968; Musgrave, 1970- Aki and Richards, 1980; Crampin, 1981, 1984; Nye, 1985) For infinitesimal displacements \( \mathbf{u} \), Cartesian coordinates \( x_i \), density \( \rho \), stress tensor \( \sigma_{ij} \) and body forces per unit mass \( g_i \):

\[
\rho \ddot{u}_i = \sigma_{ij,} + \rho g_j, \tag{A-1}
\]
Orthorhombic Physical Seismic Modeling

where \( \sigma_j \) denotes the partial derivative with respect to \( x_j \) and where the Einstein summation convention (for repeated indices) applies.

The stress tensor, in terms of the strain tensor \( \varepsilon_{ij} \) and the stiffness tensor \( C_{ijkl} \), is given in accordance with Hooke’s law by:

\[
\sigma_j = C_{ijkl} \varepsilon_{ij},
\]

where \( \varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \).

(A-2)

Substituting (A-2) and (A-3) into (A-1), neglecting any body forces, yields:

\[
C_{ijkl} u_{k,ij} - \rho \ddot{u}_i = 0.
\]

(A-4)

These equations of motion, and their solution for monochromatic plane-wave motion, are considered by many authors (e.g., Fedorov, 1968; Musgrave, 1970; Keith and Crampin, 1977; Aki and Richards, 1980; Crampin, 1981, 1984) but here we follow Musgrave's treatment most closely. We assume harmonic plane-wave displacement, expressed as

\[
\ddot{u}_k = A \rho p_k \exp \left( i \omega (s_r x_r - t) \right),
\]

(A-5)

where \( A \) is the amplitude factor, \( p_k \) is the unit polarization (or particle displacement) vector, \( \omega \) is angular frequency, \( s_r \) is the slowness vector, and in this equation only, \( i = \sqrt{-1} \). The slowness vector gives the direction of the wavefront normal and may further be written:

\[
s_r = v^{-1} n_r,
\]

(A-6)

where \( v \) is phase velocity and \( n_r \) is the unit slowness (or wavefront-normal) vector. From equation (A-4), (A-5), and (A-6) one obtains:

\[
\left( C_{ijkl} n_j n_i - \rho v^2 \delta_{ik} \right) p_k = 0.
\]

(A-7)

Thus, the determination of the details of the wave motion has been cast as an eigenvalue problem in which, having specified \( C_{ijkl} \) (the stiffnesses of the medium) and \( n_r \) (the direction of phase propagation), one can solve for \( p_k \) (the particle motion or polarization vector) and three values (in general) for \( v \) (phase velocity).

Due to the well known symmetries involved (see e.g., Musgrave, 1970; Nye, 1985):

\[
C_{ijkl} = C_{jikl} = C_{jikl} = C_{kijl}
\]

(A-8)

and therefore the matrix \( \left( C_{ijkl} n_j n_i - \rho v^2 \delta_{ik} \right) \) is symmetric.

This implies in turn that the three eigenvalues obtained for \( \rho v^2 \) by setting

\[
\left| C_{ijkl} n_j n_i - \rho v^2 \delta_{ik} \right| = 0
\]

(A-9)

will be real. (Throughout this appendix vertical bars denote determinant.)

A further consequence of the symmetries embodied in (A-8) is that there are only 21 independent stiffnesses, \( C_{ijkl} \). Following, e.g., Musgrave (1970), Nye (1985), and Thomsen (1986), the fourth-order stiffness tensor may be written as a second-order (6 x 6) symmetric matrix:

\[
C_{ijkl} \rightarrow C_{ijmn}
\]

where

\[
m = \begin{cases}
1 & \text{if } i = j, \\
9 - (i + j) & \text{if } i \neq j
\end{cases}
\]

(A-10)

and \( n_kl \) are analogous to \( m_i \) and \( ij \).

By introducing the so called Kelvin-Christoffel stiffnesses, given by Musgrave (1970) as:

\[
\Gamma \rightarrow C_{ijkl} n_j n_i
\]

(A-11)

equations (A-7) and (A-9) may be rewritten as:

\[
\begin{bmatrix}
\Gamma_{11} - \rho v^2 & \Gamma_{12} & \Gamma_{13} \\
\Gamma_{21} & \Gamma_{22} - \rho v^2 & \Gamma_{23} \\
\Gamma_{31} & \Gamma_{32} & \Gamma_{33} - \rho v^2
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2 \\
p_3
\end{bmatrix} = 0
\]

(A-12)

and

\[
\begin{bmatrix}
\Gamma_{11} - \rho v^2 & \Gamma_{12} & \Gamma_{13} \\
\Gamma_{21} & \Gamma_{22} - \rho v^2 & \Gamma_{23} \\
\Gamma_{31} & \Gamma_{32} & \Gamma_{33} - \rho v^2
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2 \\
p_3
\end{bmatrix} = 0
\]

(A-13)

Equations (A-12) and (A-13) are known as the Kelvin-Christoffel equations.

In the case of orthorhombic symmetry only nine of the, in general, 21 independent stiffnesses, \( C_{ijmn} \), are nonzero. The six independent Kelvin-Christoffel stiffnesses are then

\[
\begin{align*}
\Gamma_{11} &= n_1^2 C_{11} + n_2^2 C_{44} + n_3^2 C_{55} \\
\Gamma_{12} &= n_1 n_2 (C_{14} + C_{41}) \\
\Gamma_{13} &= n_1 n_3 (C_{15} + C_{51}) \\
\Gamma_{21} &= n_2 n_3 (C_{23} + C_{32}) \\
\Gamma_{22} &= n_2^2 C_{22} + n_3^2 C_{44} \\
\Gamma_{23} &= n_2 n_3 (C_{25} + C_{52}) \\
\Gamma_{31} &= n_3^2 C_{33} \\
\Gamma_{32} &= n_2 n_3 (C_{34} + C_{43}) \\
\Gamma_{33} &= n_3^2 C_{55}
\end{align*}
\]

(A-14)

Propagation along a principal direction

For slowness vector in the 1-direction,

\[
n_j = (1, 0, 0)
\]

(A-15)

and equations (A-14) reduce to:
Orthorhombic Physical Seismic Modeling

\[
\begin{align*}
\Gamma_{11} &= C_{11} \\
\Gamma_{22} &= C_{66} \\
\Gamma_{33} &= C_{11} \\
\Gamma_{23} &= \Gamma_{31} = \Gamma_{12} = 0.
\end{align*}
\]

Equation (A-12) then becomes:

\[
\begin{bmatrix}
\Gamma_{11} - \rho v^2 & 0 & 0 \\
0 & \Gamma_{22} - \rho v^2 & 0 \\
0 & 0 & \Gamma_{33} - \rho v^2
\end{bmatrix}
\begin{bmatrix}
p_1 \\ p_2 \\ p_3
\end{bmatrix}
= 0. \tag{A-17}
\]

For this rather simple case, that of propagation along a principal direction, there are three obvious eigenvalues which will zero the determinant of the 3 x 3 matrix. For each of these, the associated eigenvector \( p_k \) is the polarization (or unit-particle-displacement) vector.

**The P wave.** – Choosing the eigenvalue solution:

\[
\Gamma_{11} - \rho v^2 = 0 \tag{A-18}
\]

reduces the three equation of (A-17) to two, namely:

\[
\begin{bmatrix}
C_{66} - C_{11} & 0 \\
0 & C_{55} - C_{11}
\end{bmatrix}
\begin{bmatrix}
p_2 \\ p_3
\end{bmatrix}
= 0. \tag{A-19}
\]

The only permissible solution to (A-19) is

\[
p_2 = p_3 = 0 , \tag{A-20}
\]

since otherwise at least two of the six independent stiffnesses would have to be equal, violating the assumption of orthorhombic symmetry. It follows from equations (A-16), (A-18), and (A-20) that

\[
p_k = (1,0,0) \quad \text{and} \quad v_{11} = \left( \frac{C_{11}}{\rho} \right)^{1/2} \tag{A-21}
\]

where \( v_{11} \) denotes that \( v \) which applies for propagation (slowness) in the 1-direction with particle motion (polarization) in the 1-direction, that is, the \( P \)-wave velocity.

**The S waves.**–Choosing each of the other two eigenvalue solutions leads to the two solutions:

\[
P_k = (0,1,0) \quad \text{and} \quad v_{12} = \left( \frac{C_{66}}{\rho} \right)^{1/2} \tag{A-22}
\]

and

\[
P_k = (0,0,1) \quad \text{and} \quad v_{13} = \left( \frac{C_{55}}{\rho} \right)^{1/2} , \tag{A-23}
\]

these representing \( S \) waves polarized in the 2- and 3-directions, respectively.

The corresponding velocities and polarizations for the propagation in the 2- and 3-directions are obtained from equations (A-21), (A-22), and (A-23) by cyclic variation of the indices (1,2,3) and the indices (4,5,6). Since, for these axial propagation directions, the wavefront normal and the raypath have the same direction, one could replace \( v \) (phase) with \( V \) (group) in equations (A-21), (A-22), and (A-23).

**Propagation of 45 degrees to two principal axes or “edge to edge”**

Equation (A-21) to (A-23) and their cyclically varied analogs allow one to determine the six stiffnesses along the diagonal of the \( C_{ij} \) matrix from velocities measured along principal directions. In order to determine the three independent off diagonal stiffnesses, one must measure velocities for raypaths along different directions. The next simplest directions to consider would seem to be those in principal planes at 45 degrees to each of two principal directions. We have measured velocities along each such raypath for the three different polarization.

Unfortunately, the raypath or group-velocity direction is not, in general, the same as the wavefront-normal or phase-velocity direction (Figure A-1). So we cannot make simple substitutions for \( n_i \) in equations (A-14). Starting from the geometrical relationship (Figure A-1):

\[
v/V = \cos \Delta = n_i \xi, \tag{A-24}
\]

Musgrave (1970, p.89) gives:

\[
v_i = \frac{1}{2\rho v^2} \left[ \rho v^2 - A \frac{\partial \alpha^2}{\partial n_i} + \alpha^2 \frac{\partial A}{\partial n_i} \right]. \tag{A-25}
\]

Where

\[
\alpha_k = \left[ \frac{\Gamma_{ik} \Gamma_{jk}}{\Gamma_{kk}} \right]^{1/2} , \quad i \neq j \neq k \tag{A-26}
\]

and

\[
A_k = \Gamma_{kk} \quad \text{(no summation).} \tag{A-27}
\]

In equation (A-25), \( p, \alpha, \text{ and } A \) inside brackets should be represented by their \( k \)th components and the products of the brackets are summed. This notation follows Musgrave (1970) except that, whereas Musgrave use \( \xi \) as the group-velocity vector, we use \( V_i \) for group velocity and \( \xi_i \) as its unit vector.

For a medium of orthorhombic symmetry, we get from equations (A-26), (A-27), and (A-14):
Orthorhombic Physical Seismic Modeling

\( \alpha_i^2 = n_i^2 \left( \frac{C_{11} + C_{66}}{C_{22} + C_{44}} \right) \) (A-28)

and

\( A_i = n_i^2 C_{11} + n_i^2 C_{66} + n_i^2 C_{55} \) . (A-29)

and similarly for \( k=2 \) and 3. Substitution into (A-25) results in:

\[ t + \Delta t \]

\[ t \]

\[ \xi_i \]

Figure A-1. Schematic diagrams of a wavefront in an anisotropic medium at times \( t \) and \( \Delta t \), showing the phase and group velocities, \( v \) and \( V \) respectively, their respective unit vectors (and directions), \( n_i \) and \( \xi_i \), and the angle \( \Delta \) between \( n_i \) and \( \xi_i \).

\[ \cos \Delta = \frac{v}{V} \]

\[ \nu = \frac{A_B}{\Delta t} (\text{phase}) \]

\[ V = \frac{A_C}{\Delta t} (\text{group}) \]

\( \rho v V_i = p_i \left[ \left( \rho v^2 - n_i^2 C_{66} - n_i^2 C_{55} \right) n_i^{-1} + n_i C_{11} \right] + n_i \left( p_i^2 C_{66} + p_i^2 C_{55} \right) \) (A-30)

and similarly for \( i=2 \) and 3. If we consider propagation in the 23-plane of symmetry:

\( n_1 = 0 \) and \( V_1 = 0 \) . (A-31)

Thus, from equation (A-30)

\[ p_i^2 n_i^{-1} \left( \rho v^2 - n_i^2 C_{66} - n_i^2 C_{55} \right) = 0 \] (A-32)

and therefore either

\( p_1 = 0 \) (A-33)

or

\[ \rho v^2 = n_2^2 C_{66} + n_3^2 C_{55} \] (A-34)

The quasi-P and \( \delta P \) waves. – Equation (A-33) implies polarization entirely within the 23-plane (the sagittal plane), i.e., \( P-\delta P \) types. From the analogs to equation (A-30) for \( i=2 \) and 3 we then get:

\[ \rho v V_2 = p_2^2 n_2^{-1} \left( \rho v^2 - n_2^2 C_{44} \right) + p_2^2 n_2 C_{44} \]

and

\[ \rho v V_3 = p_3^2 n_3 C_{44} + p_3^2 n_3^{-1} \left( \rho v^2 - n_3^2 C_{44} \right) \].

Further, from equation (A-12) for this case we get:

\[ \frac{p_2}{p_3} = \frac{n_2 n_3 (C_{23} + C_{44})}{\rho v^2 - n_2^2 C_{44} - n_3^2 C_{33}} \]

\[ \frac{p_3}{n_2 n_3 (C_{44} - C_{44})} = \frac{n_2 n_3 (C_{23} + C_{44})}{\rho v^2 - n_3^2 C_{44} + n_1^2 C_{33}} \]

from which

\[ \frac{p_2}{p_3} = \frac{\rho v^2 - n_2^2 C_{44} - n_3^2 C_{44}}{\rho v^2 - n_2^2 C_{44} + n_3^2 C_{44}} \] (A-37)

since the raypath or group-velocity direction is at 45 degrees to the 2- and 3-axes, \( V_2 = V_3 \) and the two right-hand sides of equations (A-35) are equal. From this and equation (A-37) one can eliminate \( p_2 \) and \( p_3 \) and obtain:

\[ \rho^2 \left[ n_3^2 n_2 \left( C_{22} + C_{44} \right) - n_1^2 \left( C_{33} + C_{44} \right) \right] + C_{11} n_3^2 \left( n_2 + n_3 \right) - C_{22} C_{44} n_2^2 \left( n_2 + n_3 \right) = 0 \] (A-38)

It is clear from (A-36) that there are two solutions for \( v_2 \) and thus for \( p_2/p_3 \). One of these solutions is the quasi-P or \( qP \)-wave case, the other the \( qSV \)-wave case. For \( qP \) we denote the phase velocity \( v_{qP} \) and the group velocity \( V_{qP} \). For \( qSV \) these are \( v_{qSV} \) and \( V_{qSV} \), respectively. There is, it is true, a fundamental incongruity between the single- and double-subscript notations for \( V \). However, we do not try to combine the two and thus no problem ever arises here.

Defining \( \theta = \cos^{-1} n_3 \), we can write (figure A-1) that \( \Delta = \theta - 45 \) degrees. Therefore, from equation (A-24) for \( qP \):

\[ v_q = V_q \cos \Delta = \left( \sqrt{2}/2 \right) \left( n_2 + n_3 \right) V_q \] (A-39)

where \( i=44, 44 \) or 41, for \( qP, qSV \), or \( SH \), respectively. Using equations (A-21) to (A-23) and their cyclic variants to eliminate stiffnesses, and (A-39) to eliminate \( v \) from (A-38), we obtain:
Orthorhombic Physical Seismic Modeling

\[ V_{41}'(n_2 + n_1)(n_2^2 - n_1^2) + 2V_{44}'(n_2 + n_1)(n_2^2V_{23}^2 + V_{23}^2) - n_3^2(2V_{33}^2 + V_{23}^2) + 4(n_1V_{33}^2V_{23}^2 - n_2^2V_{22}^2V_{23}^2) = 0 \]

(A-40)

in which all of the \( V_{ij} \) have been measured experimentally.

Now, since \( n_2^2 + n_1^2 = 1 \), equation (A-40) can, in principle, be solved for \( n_2 \) and \( n_3 \). In practice, we determine \( n_2 \) and \( n_3 \) by iterative substitution. Knowing \( n_2 \) and \( n_3 \) and \( v_{44} \) from equation (A-39), equation (A-36) and (A-37) can be solved for \( C_{23} \) and the polarization, \( p_2 / p_3 \). Similarly, using \( V_{44} (qSV) \) in (A-40), we will get different values (in general) for \( n_2 \), \( n_3 \) and \( v_{44} \); but assuming the orthorhombic model to be a reasonable one, we should get the same result for \( C_{23} \).

The SH wave. – Choosing equation (A-34) instead of (A-33) we have, from (A-14) and (A-31):

\[ \rho v_{41}^2 = n_2^2 C_{66} + n_3^2 C_{55} = \Gamma_{11} \]

(A-41)

so that the Kelvin-Christoffel equations (A-12) result in

\[ p_{ik} = (1, 0, 0). \]

(A-42)

Incorporating (A-31) and (A-42) into (A-30) and its cyclic variants yields:

\[ \rho v_{41} V_2 = n_2 C_{66} \quad \text{and} \quad \rho v_{41} V_3 = n_3 C_{55}. \]

(A-43)

Again applying \( V_2 = V_3 \) (for ray at 45 degrees) and equation (A-39), we obtain:

\[ \frac{n_2}{n_3} = \frac{C_{55}}{C_{66}} \quad \text{and} \quad V_{41} = \frac{\sqrt{2} V_{41}' V_{12}'}{(V_{31}'^2 + V_{12}'^2)^{1/2}}. \]

(A-44)

Expression for stiffnesses in terms of group velocities

For completeness, expressions for the nine stiffnesses, for the case of orthorhombic symmetry, are here summarized. These equations follows directly from (A-21) to A-23), (A-36), and (A-39), as well as their cyclic variant.

\[ C_{11} = \rho V_{11}^2 \]
\[ C_{22} = \rho V_{22}^2 \]
\[ C_{33} = \rho V_{33}^2 \]
\[ C_{44} = \rho V_{23}^2 = \rho V_{32}^2 \]
\[ C_{55} = \rho V_{31}^2 = \rho V_{13}^2 \]
\[ C_{66} = \rho V_{12}^2 = \rho V_{21}^2 \]

(A-45)

\[ C_{23} = \frac{\rho}{n_2 n_3} \left[ \frac{1}{2} (n_2 + n_1)^2 V_{44}' - n_1^2 V_{23}' - n_2^2 V_{31}' \right] \]

(A-47a)

\[ C_{31} = \frac{\rho}{n_3 n_1} \left[ \frac{1}{2} (n_3 + n_1)^2 V_{55}' - n_1^2 V_{35}' - n_2^2 V_{31}' \right] \]

(A-47b)

\[ C_{12} = \frac{\rho}{n_1 n_2} \left[ \frac{1}{2} (n_1 + n_2)^2 V_{66}' - n_1^2 V_{12}' - n_2^2 V_{12}' \right] \]

(A-47c)

an in (A-47) \( V_{6i} (i=4,5,6) \) may be replaced by \( V_{ii}' \).