Scaled physical modelling of anisotropic wave propagation: multioffset profiles over an orthorhombic medium

R. James Brown, Don C. Lawton and Scott P. Cheadle*, Department of Geology and Geophysics, The University of Calgary, Calgary, Alberta, Canada T2N 1N4
Accepted 1991 June 19. Received 1991 June 14; in original form 1991 January 18

Summary

In this paper we report further results of scaled physical modelling experiments in the laboratory in which ultrasonic elastic waves are propagated through an anisotropic medium of orthorhombic symmetry. Whereas our earlier experiments consisted for the most part in sending and receiving on opposite faces of a small cube of phenolic laminate, these new results are from multioffset profiles run parallel and at 45° to principal directions on a larger slab of phenolic.

The variation of NMO velocity with offset (or angle of incidence) has been determined for compressional and transverse shear waves along profiles in the two principal directions on the 3-face (parallel to laminations) of the slab. These observed group velocities differ from the exact theoretical values by a maximum of about 1 per cent or less and also compare favourably with the theoretical velocities calculated from Thomsen's first-order equations, with maximum differences of about 2 per cent. Differences between the observed and theoretical velocities are attributed to some combination of finite transducer size (geometrical or effective path length effects, array attenuation effects, or interference with the otherwise free surface), sample inhomogeneity and/or anelasticity, and experimental error.

The transmission shot gathers acquired for propagation in symmetry planes, and for source-receiver pairs with the same polarization, are similar in form to records acquired over a transversely isotropic medium. The effect of the shear-wave window and the variation of the hyperbolic NMO parameter with offset are clearly seen. Transmission records were also acquired in off-symmetry planes, namely along profiles at 45° to principal directions. On these records, which include all nine possible pairs of source-receiver polarizations, we see clear shear-wave splitting at and near zero offset and more complicated wave effects with increasing offset, such as one or another wave phase dying out. This could be due to cusping of wave surfaces or rapid changes of amplitude and/or polarization with ray direction, possibly as consequences of nearby shear-wave singularities.

Key words: anisotropy, NMO velocity, orthorhombic symmetry, physical modelling, shear-wave splitting, singularities.

INTRODUCTION

Using as a medium the anisotropic phenolic laminate described below and by Cheadle, Brown & Lawton (1991), physical seismic experiments have been continuing at the University of Calgary with scaled-down models (1:5000) and scaled-up frequencies (5000:1). In this earlier work we looked primarily at body waves propagating along principal (or symmetry) directions, corresponding to zero-offset transmission through each of three principal faces (symmetry planes) of the phenolic. The only departure from this simplest case, necessary to enable determination of the nine stiffnesses of the material, was propagation along paths at 45° to each of two principal directions and contained within the principal plane thereby subtended. Such records correspond to relatively far-offset shots (angle of incidence=45°) along profiles oriented in principal directions.

This paper presents the results of laboratory experiments in which transmission shot gathers have been recorded for a wide range of offsets and for two fundamentally different profile orientations: (a) along principal directions, for which the sagittal or propagation planes are principal or symmetry planes, and (b) at 45° to two principal directions, for which the sagittal planes are no longer principal planes. For this latter case we are reporting our first experimental results for cases where the propagation paths contain components of all three principal directions. Our earlier experiments (as well as some reported upon in this paper) for propagation within symmetry planes yielded results that are considerably less complex than for the case of an arbitrary direction of propagation. For example, the variation of phase velocity with direction in a symmetry plane of an orthorhombic medium for one particular wave phase is of the same algebraic form as in the transverse-isotropy (TI) case, although the interrelationship of these variations between different wave phases is not, in general, the same as in the TI case.

Experimental results obtained on synthetic materials that are essentially transversely isotropic have been reported by Ebrom et al. (1990) and Rathore et al. (1990), among others. Similar laboratory experiments using ultrasonic waves on real rock samples have been carried out by Jones & Wang (1981), Lin (1985) and Sayers (1988, 1990), for example. However, we are not aware of any other published results to date of the kind reported here of physical seismic modelling using orthorhombic materials. We document our results in two related areas, the variation of stacking or NMO velocity with offset for cases of sagittal symmetry and the analysis of shot gathers obtained for various combinations of shot and receiver polarization (vertical, radial, transverse) in both symmetry and off-symmetry planes.

LABORATORY SET-UP

The set-up of the laboratory equipment used in these experiments is very similar to that described by Cheadle et al. (1991). Flat-faced piezoelectric transducers are used as sources and receivers, both types having an active element 12.6mm in diameter. The compressional-wave transducer (Panametrics V103) is sensitive to displacement normal to

* Now at Veritas Seismic Ltd, 200, 615-3rd Ave SW, Calgary, Alberta, Canada T2P OG6.
Scaled physical modeling of anisotropic wave propagation

the contact face, whereas the shear-wave transducer (V153) is sensitive to displacement tangential to the transducer face, in a direction parallel to the cable connector. A source pulse is obtained by driving the source transducer (P or S) with a single cycle of a 28 V square wave. The width of the pulse is adjusted manually to produce an approximately symmetrical broadband wavelet with a central frequency of about 600 kHz, corresponding to minimum wavelengths of about 2.5 mm and 5 mm for SH and qP, respectively. Data are recorded using a Nicolet digital oscilloscope at a sampling interval of 50 ns. The transducers are coupled to the surface of the model using a viscous pharmaceutical wax and a uniform pressure is applied to each transducer to ensure consistent coupling between the transducers and the model.

![Figure 1. Photograph of a slab of Phenolic CE with the experimental arrangement used in transmission experiments, i.e. with source and receiver transducers on opposite surfaces of the slab. The slab thickness is 104 mm (520 m at 1:5000 scaling).](image)

![Figure 2. Schematic diagram of wave propagation paths and transmission-experiment design as seen in section.](image)

Figures 1 and 2 are, respectively, a photograph of the apparatus used for the experiments described in this paper and a schematic diagram of the system. As illustrated in these figures, we carried out transmission experiments in which the source and receiver transducers were on opposite sides of a phenolic slab. This reduces the number of unwanted arrivals (e.g. converted reflections, direct waves, additional head waves) that would accompany reflection experiments, wherein both transducers would be on the same surface. During an experiment, the source transducer was fixed below the slab while the receiver transducer was moved incrementally, on a calibrated worm drive, across the top of the slab (Figs 1 and 2).

The material used, Phenolic CE, is constructed of canvas laminae that are bonded together with a phenolic resin. Each lamina is woven from fine cotton fibers, roughly 0.1 mm in diameter, and consists of a warp (the set of straight parallel fibers that are stretched tight) and a woof (the parallel set of fibers that are orthogonal to and curl up and down over the warp). The density of fibers is about 8 per cm in the warp and 9 per cm in the woof, and there are about 25 laminae per cm. Phenolic CE is available commercially and is commonly used in applications requiring both electrical insulation and mechanical strength (e.g. in transformers) as well as for machining components such as gears, pulleys and sheaves.

**SHOT GATHERS WITHIN SYMMETRY PLANES OF THE PHENOLIC**

The physical modelling capability described in the preceding section and by Cheadle et al. (1991) has been applied to generate several shot gathers along different profiles on the 3-face of the phenolic slab (taken as horizontal; i.e. the 3-direction is taken as the vertical). Profiles were initially shot at 0° (the 1-direction) and 90° (the 2-direction) for vertical, radial and transverse shot-receiver polarizations (Fig. 3). Fig. 2 also shows how an SV-to-P converted phase, termed the local SP wave by Booth & Crampin (1985), will be generated at the critical angle. For the corresponding critical offset and beyond, the shear-wave arrivals will suffer interference from this head wave. This critical offset defines the limits of the so called shear-wave window (Evans 1984; Crampin, Evans & Ücer 1985; Booth & Crampin 1985). One might expect the polarization of this local SP wave to be within the sagittal plane and perhaps closer to horizontal than vertical. Hence we might expect it to be strongest on radial-receiver records and the weakest on transverse-receiver records. [These conventional (isotropic) expectations should be applied with caution in the anisotropic case, especially for ray paths in off-symmetry planes.]

For the 0° and 90° records (Fig. 3) there is sagittal symmetry (i.e. the sagittal planes are symmetry planes) and the wave phenomena that one observes in the records are similar to what would have been seen for a transversely isotropic medium. This is true also for SH propagation in a symmetry plane even though the polarization then is not in this sagittal plane (Crampin & Kirkwood 1981). In Fig. 3, only the V-V record (vertically polarized source and receiver) is shown for the 90° profile as this family of records is very similar in nature to the 0° records. The records in Fig. 3 also show a significant broadening (lower frequency content) of the wavelet with increasing source-receiver offset. This is primarily due to array effects of the source and receiver transducers caused by their finite size and possibly partly due to anelastic attenuation.

The effect of the shear-wave window is seen in Fig. 3 where the local SP phase (Fig. 2) (labeled P head wave in Fig. 3) arrives before and interferes with the SV phase. Note that this head wave does indeed have greater amplitude on the radial-radial (R-R) record (Fig. 3, bottom left) as expected. Thus, outside the shear-wave window (from about -250 to 250 m on this scaled section) the SP head wave totally obscures the SV. On the V-V sections (Fig. 3, top) the interference is apparently not nearly as great but the picking of accurate SV onset times is still made practically impossible. For the T-T case (Fig. 3, bottom right) the shear-wave window is not a factor because of the transverse polarizations of source and receiver and the sagittal symmetry. The fact that these are transmission and not reflection records probably also contributes to their simplicity of appearance.
Scaled physical modeling of anisotropic wave propagation

Figure 3. Transmission shot gathers recorded across the 3-face of the Phenolic CE slab. Traveltimes and offsets are scaled by 5000:1. The nominal trace spacing is 25 m, although this value varies slightly with position because of transducer finiteness (see text). Top left: profile in the 1-direction (0°) with vertically polarized source and receiver (V-V). Top right: profile in the 2-direction (90°) with vertically polarized source and receiver (V-V). Bottom left: profile in the 1-direction (0°) with radially polarized source and receiver (R-R). Bottom right: profile in the 1-direction (0°) with transversely polarized source and receiver (T-T). Hollow arrows identify various direct and refracted arrivals (P' is the first P-wave multiple through the slab). Solid arrows marked Vv, Vh and Vsv indicate hypothetical onset times on the far trace assuming the vertical P-wave velocity, horizontal P-wave velocity and axial SV-wave velocity, respectively, to persist isotropically throughout the slab.

THEORY OF VELOCITY VARIATION IN SYMMETRY PLANES

Introduction

The variation in velocity (group and phase) with direction of propagation is one of the fundamental properties of seismic anisotropy. The actual directional dependence of velocities for various symmetries has been studied by Backus (1965), Crampin (1977, 1981) and Crampin & Kirkwood (1981), for example. The specific dependence of NMO velocity on angle of incidence for the case of transverse isotropy (TI) has been examined by Postma (1955), Daley & Hron (1977), Levin (1979), Berryman (1979), Thomsen (1986) and Uren, Gardner & McDonald (1990), among others. Winterstein (1986) has studied the depth discrepancies resulting from the application of observed NMO (or stacking) velocities, particularly for SH, in TI media.

The exact expressions for seismic velocities as functions of direction in an arbitrary off-symmetry plane and for an arbitrary symmetry system can be extremely complicated and not easily manipulated. However, within symmetry planes— for any symmetry system - the expressions simplify to the same algebraic form as those that apply for the TI system (Postma 1955; Crampin & Kirkwood 1981; Crampin 1981). These exact expressions may further be simplified to approximate expressions that are valid for cases in which the anisotropy is weak (Backus 1965; Crampin 1977; Thomsen 1986). Such simplified approximate expressions can provide reasonable estimates of the velocity variations in symmetry planes for all symmetry systems (Crampin & Kirkwood 1981) and are also important in providing intuition into the physical nature of the directional variation of a particular wave velocity (Thomsen 1986). For most computational purposes, of course, the exact expressions should be used.
Phase velocity

We have derived the exact expressions for the phase velocities as functions of phase angle (wavefront-normal direction), $\psi_{SH}(\theta)$ and $\psi_{P}(\theta)$, in a vertical symmetry plane of an orthorhombic medium. We assume the 3-direction ($\theta = 0$) to be aligned with the vertical and consider profiles in both the 1- and 2-directions (propagation in the 1-3 and 2-3 planes). Starting with equations (A-13) and (A-14) of Cheadle et al. (1991) (see also Musgrave 1970, chapter 9), we obtain for $SH$ and $P$ waves in the 1-3 plane:

\[
\psi_{SH}^2(\theta) = \frac{C_{33}}{\rho} \cos^2 \theta + \frac{C_{44}}{\rho} \sin^2 \theta 
\]

(1a)

and

\[
\psi_{P}^2(\theta) = \frac{1}{2 \rho} [C_{33} + C_{55} + (C_{11} - C_{33}) \sin^2 \theta + D(\theta)]
\]

(1b)

where

\[
D(\theta) = (C_{33} - C_{55})^2 + 2[2(C_{11} + C_{55}) - (C_{33} - C_{55})] \times (C_{11} + C_{33} - 2C_{55}) \sin^2 \theta + [C_{11} + C_{33} - 2C_{55}]^2 - 4(C_{11} + C_{33}) \sin^4 \theta
\]

(1c)

For a profile in the 2-direction (propagation in the 2-3 plane), the index changes are: 4$\rightarrow$5 (for $SH$), 1$\rightarrow$2 and 5$\rightarrow$4 (for $P$). The expressions for $\psi_{SH}(\theta)$ are as given for $\psi_{P}(\theta)$ but with a change of sign on $D(\theta)$.

Equations (1), valid in symmetry planes of an orthorhombic medium, are of the same algebraic form as those given by Daley & Hron (1977) and Thomsen (1986), for example, for TI media (and conform to Thomsen's notation). They reduce exactly, of course, to the TI equations upon degeneracy of TI media (and conform to Thomsen's notation). They reduce exactly, of course, to the TI equations upon degeneracy of the orthorhombic system described by: $C_{11} = C_{22}$; $C_{13} = C_{23}$; $C_{33} = C_{55}$ and $C_{12} = C_{11} - C_{33}$. (Going in the opposite direction, however, from the TI to the more general orthorhombic expressions cannot be done in an immediately obvious manner without some additional knowledge).

By utilizing the Thomsen anisotropy parameters $\gamma$, $\delta$ and $\epsilon$ for each vertical symmetry plane, equations (1) may be written in the form

\[
\psi_{SH}(\theta) = \psi_{SH}(0) \left( 1 + 2\gamma \sin^2 \theta \right)^{1/2}
\]

(2a)

and

\[
\psi_{P}(\theta) = \psi_{P}(0) \left[ 1 + \epsilon \sin^2 \theta + D^*(\theta) \right]^{1/2}
\]

(2b)

where

\[
D^*(\theta) = [E + 2E(2\delta - \epsilon) \sin^2 \theta \cos^2 \theta + \epsilon(2E + \epsilon) \sin^4 \theta]^{1/2}
\]

(3a)

in which we introduce $E$ (not a first-order small quantity):

\[
E = \frac{1}{2} \left[ 1 - \frac{\psi_{SH}^2(0)}{\psi_{P}^2(0)} \right]
\]

(3b)

and where, for the 1-3 plane,

\[
\gamma = \frac{C_{66} - C_{44}}{2C_{44}}
\]

(4a)

\[
\delta = \frac{(C_{11} + 55)^2 - (C_{33} - C_{55})^2}{2C_{33}(C_{33} - C_{55})}
\]

(4b)

\[
\epsilon = \frac{C_{11} - C_{33}}{2C_{33}}
\]

(4c)

and

\[
E = \frac{C_{13} - C_{55}}{2C_{33}}
\]

(4d)

The same changes of index as given above for equations (1) apply for equations (4). Note in the above equations that $\psi_{SH}(0)$ is the same for both profile directions. However, unlike the TI case, $\psi_{SH}(0)$ is not the same for both profiles and for a particular profile $\psi_{SH}(0) \neq \psi_{P}(0)$ in general.

Group Velocity

The group velocity in a symmetry plane can be determined from the phase velocity using well-known relationship (e.g. Postma 1955; Backus 1965; Berryman 1979; Crampin 1981; Radovich & Levin 1982):

\[
V^2[\phi(\theta)] = V^2(\theta) + \left( \frac{dV}{d\theta} \right)^2
\]

(5a)

and

\[
\tan \theta = \frac{\tan \phi}{V} \left( \frac{dV}{d\phi} \right)
\]

(5b)

where $\phi$ is group or ray angle. It can also be shown in straightforward way that

\[
\phi - \theta = \tan^{-1}\left[ \frac{1}{V} \left( \frac{dV}{d\theta} \right) \right] = \tan^{-1}\left[ \frac{1}{V} \left( \frac{dV}{d\phi} \right) \right].
\]

(5c)

Applying equation (5a) to equations (1) or (2) gives the following group-velocity functions:

\[
V_{SH}^2[\phi(\theta)] = \psi_{SH}^2(\theta) \left[ 1 + \left( \frac{\gamma \sin 2\theta}{1 + 2\gamma \sin^2 \theta} \right)^2 \right]
\]

(6a)

and

\[
V_{P}^2[\phi(\theta)] = \psi_{P}^2(\theta) \left[ 1 + \left( \frac{\epsilon \sin 2\theta + (D^*/d\theta)}{2[1 + \epsilon \sin^2 \theta + D^*(\theta)]} \right)^2 \right].
\]

(6b)

In computing group velocities from these equations for the purpose of comparing with those that we have observed for various angles of incidence, it is essential to distinguish
Scaled physical modeling of anisotropic wave propagation

between θ, the phase angle or slowness direction, and φ, the group angle or angle of incidence of the ray. A computed group velocity must be plotted against the corresponding group angle, φ, and not the phase angle, θ, which may have been inserted into equation (6). Thus we write \( t(\theta) \) and \( V(\phi(\theta)) \) to denote the (in general different) phase and group velocities at one and the same point on a wavefront, for example at point A (Fig. 4). In addition, however, there exists some neighbouring point B (Fig. 4) on the wavefront where the group-velocity direction \( \phi_B \) is equal to \( \theta_A \) the phase-velocity direction at A. We denote such group velocities as \( V(\phi=\theta) \). If one plots the computed group velocities against the related \( \theta \) values, and on the same set of axes plots for comparison observed group velocities versus incidence angles, then one is in fact plotting \( V(\phi_a) \) versus \( \phi_B \).

It can be shown that \( V(\phi(\theta)) \) and \( V(\phi=\theta) \) differ from \( t(\theta) \) by second-order quantities that are of equal magnitude but opposite sign; or in other words, \( V(\phi=\theta) \) differs from \( V(\phi(\theta)) \) by about twice as much as \( t(\theta) \) does.

In view of the above, one should also calculate the group angle, \( \phi \), so that one may plot the pair \( \{ \phi, V[\phi(\theta)] \} \). Applying equation (5b) or (5c) to equations (1) or 2 yields

\[
\phi_{SH} = \theta_{SH} + \tan^{-1} \left( \frac{\gamma \sin 2\theta_{SH}}{1 + 2\gamma \sin^2 \theta_{SH}} \right) \quad (7a)
\]

and

\[
\phi_P = \theta_P + \tan^{-1} \left( \frac{\epsilon \sin 2\theta_P + \frac{dD'}{d\theta_P}}{2[1 + \epsilon \sin^2 \theta_P + D''(\theta_P)]} \right). \quad (7b)
\]

Figure 4. Sketch of an anisotropic wavefront showing the phase velocity \( t(\theta) \) at A, the group velocity \( V[\phi(\theta)] \) at A, and the group velocity \( V(\phi=\theta) \) at B.

Series approximations

Equations (2) may be expanded in series form to any desired order, as was done to first order by Thomsen (1986). Since a first-order expansion is not sufficient to resolve any difference between group and phase velocities, we have expanded these expressions to include the second-order terms, in which a difference between these two velocities first appears. For our test material, maximum anisotropies are about 10 per cent (5H) and 20 per cent (P), so that second-order dimensionless quantities, such as relative differences between group and phase velocities, should be on the order of about 1 to 4 percent and truncated third- and higher order quantities should be on the order of about 0.1 to 0.8 percent. Such accuracy is probably acceptable in most but not all situations and, in any case, the second-order expressions are not so much simpler than the exact ones as to warrant their use routinely. We have therefore used the exact expressions in obtaining our results here. Along the way, however, we compared our observed group velocities with the velocities given by Thomsen’s (1986) expressions and found typical maximum differences of only about 2 per cent. It might also be mentioned that Thomsen (1986) also gives approximate equations for the anisotropy parameters \( \gamma \), \( \delta \) and \( \epsilon \). The approximation for \( \delta \) in particular can be quite bad (out by a factor of about 2 for our data). As Thomsen (1986) stresses, the linearized approximate expressions are intended to aid one’s intuition or understanding of the physics, not for computational purposes.

Table 1. Comparison of parameters determined in two different samples of Phenolic CE: the slab used in this study and the cube used by Cheadle et al. (1991). The 45° velocities (V44 etc. for qP or V55 etc. for qSV) could not be measured accurately for the slab and were taken to be equal to the corresponding cube values. This affects the off-diagonal stiffnesses and the (5 values slightly.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Slab</th>
<th>Cube</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{11} ) (m/s)</td>
<td>3502</td>
<td>3576</td>
</tr>
<tr>
<td>( V_{22} )</td>
<td>3341</td>
<td>3365</td>
</tr>
<tr>
<td>( V_{33} )</td>
<td>2935</td>
<td>2925</td>
</tr>
<tr>
<td>( V_{23} )</td>
<td>1515</td>
<td>1516</td>
</tr>
<tr>
<td>( V_{13} )</td>
<td>1600</td>
<td>1606</td>
</tr>
<tr>
<td>( V_{12} )</td>
<td>1646</td>
<td>1662</td>
</tr>
<tr>
<td>( V_{44} )</td>
<td>3094</td>
<td>3094</td>
</tr>
<tr>
<td>( V_{55} )</td>
<td>3219</td>
<td>3219</td>
</tr>
<tr>
<td>( V_{66} )</td>
<td>3378</td>
<td>3378</td>
</tr>
<tr>
<td>( V_{45} )</td>
<td>1569</td>
<td>1569</td>
</tr>
<tr>
<td>( V_{35} )</td>
<td>1620</td>
<td>1620</td>
</tr>
<tr>
<td>( V_{65} )</td>
<td>1810</td>
<td>1810</td>
</tr>
<tr>
<td>( \gamma ) (2-3)</td>
<td>0.0292</td>
<td>0.0355</td>
</tr>
<tr>
<td>( \gamma ) (1-3)</td>
<td>0.0902</td>
<td>0.1009</td>
</tr>
<tr>
<td>( \gamma ) (1-2)</td>
<td>0.0577</td>
<td>0.0611</td>
</tr>
<tr>
<td>( \delta ) (2-3)</td>
<td>0.1540</td>
<td>0.1656</td>
</tr>
<tr>
<td>( \delta ) (1-3)</td>
<td>0.1166</td>
<td>0.1133</td>
</tr>
<tr>
<td>( \delta ) (1-2)</td>
<td>-0.0975</td>
<td>-0.1131</td>
</tr>
<tr>
<td>( \epsilon ) (2-3)</td>
<td>0.1479</td>
<td>0.1617</td>
</tr>
<tr>
<td>( \epsilon ) (1-3)</td>
<td>0.2118</td>
<td>0.2473</td>
</tr>
<tr>
<td>( \epsilon ) (1-2)</td>
<td>0.0494</td>
<td>0.0645</td>
</tr>
</tbody>
</table>
The three anisotropy parameters of Thomsen (1986) have been discussed and evaluated (exactly) by Cheadle et al. (1991) for a cube of Phenolic CE for each of the three symmetry planes. These values have also been determined separately for the larger slab of phenolic used in the profile experiments described in this paper. These are listed in Table 1 along with a similar comparison of the measured velocities.

**COMPARISON OF OBSERVED AND THEORETICAL GROUP VELOCITIES**

In order to determine experimentally the variation of these NMO velocities with direction (angle of incidence) a geometrical correction first had to be applied. For any particular source-receiver offset for which a traveltime measurement has been made, an initial path length and angle of incidence are determined from basic trigonometry. In doing this, the source and receiver `points` are taken to be the centres of the respective transducers. Preliminary calculation showed that the effective path length was in fact shorter than the nominal distance between transducer centres and was in fact close but not exactly equal to the distance between nearest edges of transducers, which constituted a significant difference. (The transducer diameter of 12.6 mm scales up to 63 m at 1:5000). The correction was determined by repeating the shot profile with precisely the same geometry and the same thickness of material (104 mm), except that we used an isotropic medium (namely Plexiglas,
which has \( P \)-wave and \( S \)-wave velocities of 2740 and 1385 ms\(^{-1} \), respectively. The variations in traveltime with offset for this material were entirely due to geometry, with no anisotropic contribution. We were therefore able, on the basis of measured traveltimes and a known (invariant) velocity, to determine an effective path length, and thus also incidence angle, for each nominal source-receiver offset. This empirical correction was calculated and applied independently for each of the \( P \) and \( SH \) transducer types.

Figure 5 shows the agreement between the experimentally determined NMO velocities and those calculated from equations (6) and (7). Maximum differences are seen to be about 1 per cent or less. Most of the discrepancy for the \( SH \) cases (Figs 5c and d) seems to be associated with a curious rapid group-velocity increase observed at very small offsets (<5°) followed by a decrease with offset around 15°. This behaviour was more pronounced before applying the geometrical correction and is likely a residue of a procedure which, although it accounts for the gross ray path geometry, does not include provision for the non-orthogonal orientation of the anisotropic wavefront to the ray. In addition, our relatively large transducers could be causing problems in at least two other ways: they may be interfering with the free surface in a manner that requires a special free-surface correction and, as can be seen in Fig. 3, they act as arrays and cause high-frequency degradation which increases with offset. Other error sources are measurement uncertainty (±150 ns or, scaled at 5000: 1, ±0.75 ms) and the possibility that the phenolic material may have some measurable inhomogeneity and/or anelasticity.

**ANALYSIS OF THE VECTOR WAVEFIELD IN OFF-SYMMETRY PLANES**

Using an experimental procedure similar to that described above, profiles were shot on the 3-face at 45° to each of the 1- and 2-axes. These were acquired first with radial (R-R) and then transverse (T-T) shot-receiver polarizations (Fig. 6). In these we observe effects, not seen in the 0° (1-direction) or 90° (2-direction) records, that give a hint of the complexity of wave propagation in off-symmetry planes for an orthorhombic medium. In both of these shot records (Fig. 6), split shear waves are clearly visible on the zero-offset traces (identical on the two sections) and for the near offsets. However, as offset increases, one of the \( qS \) arrivals dies out: the \( S_1 \) on the R-R record and the \( S_2 \) on the T-T record. This effect could conceivably be caused by cusping on complex wave surfaces, by rotations of polarizations into the plane perpendicular to the particular receiver polarization of each record, or by a relative- amplitude effect wherein the phases in question do not actually die out but simply become very low-amplitude at larger offsets. (Note that each trace is true-relative- amplitude but traces have been balanced relative to one another simply to maintain a roughly constant visible dynamic range.) The wave phenomena we observe could also be due to some combination of these three effects. All of these are related manifestations of the general complexities of shear-wave propagation in arbitrary anisotropic media, complexities which are most pronounced in the neighbourhood of cusps on the wave or group-velocity surfaces or of singularities on the slowness or phase-velocity surfaces.

Musgrave (1970) discusses these topics in some detail and shows a number of relatively simple theoretical examples of wave and slowness surfaces in symmetry planes. The full extent of the complexities are, however, generally realized in off-symmetry planes. Dellinger & Eigen (1989) show a synthetic example, generated on the basis of a particular orthorhombic medium (namely, cracked Greenhorn shale), of wave surfaces in an off-symmetry plane (Fig. 7). Crampin & Yedlin (1981) and Crampin (1981, 1985, 1989, 1991) discuss the topic of shear-wave singularities and show theoretical examples of the rapid fluctuations in amplitudes and polarizations near such singularities.

In order to provide more information on the polarizations for a particular shot, we decided to record a full 'nine-component' data set. So, for example, instead of just recording with a transversely sensitive transducer in the case of the transverse source (T-T), we recorded also with vertically and radially sensitive transducers (T-V and T-R); and so on for all three source transducers, resulting in nine records for the particular shot location (Fig. 8). It is of ancillary interest to note the manifestation of negative reciprocity, or antireciprocity, in Fig. 8. According to this principle (see e.g. Knopoff & Gangi 1959), interchanging orthogonally polarized transducers, but keeping the shot and receiver locations unchanged, will lead to displacement fields that are equal but of opposite polarity (assuming only an elastic medium and equal shot-impulse magnitudes).

Due to the fact that it can be difficult to replicate the source pulses in different experiments run at intervals of days or more, but not too difficult to maintain a constant source signature for a few hours, we decided to shoot all nine records as one experiment. Comparison of Fig. 6 with the R-R and T-T records of Fig. 8, shows that, although there are some differences in pulse shape and therefore some of the fine details on traces, the overall results in terms of wave phases present, traveltimes, etc., appear to be repeatable between experiments. One can also see indications of split shear waves on the other off-diagonal records of Fig. 8 (V-R and R-V more so than V-T and T-V).

In Fig. 8, the \( S_1 \) arrival that is seen to die out on the R-R record persists to large offsets on the R-T record. This could be the result of a polarization rotation, perhaps near a singularity or, since we have not displayed true trace-to-trace relative amplitudes, the effect could be one of amplitude scaling. At first glance it appears as if the \( S_2 \) phase on the T-T record is scarcely visible on the T-R record (Fig. 8) and that the same \( S_1 \) arrival dominates both records. However, the apparent onset times of these phases on the two records are not the same at far offsets: the \( S_1 \) seems to arrive later on the T-T record. In fact, the earlier far-trace onset on the T-R (and R-T) record may include some \( SP \) head-wave energy beyond the shear-wave window, although one might have expected to see this also on the R-R record.

**CONCLUSIONS AND FUTURE DIRECTIONS**

We have shown that observed stacking or NMO velocities for \( qP \)- and \( SH \)-wave propagation in symmetry planes of our orthorhombic medium conform closely, that is within about 1 per cent, to the theoretical group-velocity variation. The observed \( SH \) velocities do, however, exhibit some anomalous variation (departure from the expected monotonic variation).
We have suggested as possible reasons for this: effects of large transducer size (geometrical or effective path length effects, array attenuation effects, and free-surface contamination), sample inhomogeneity and/or anelasticity, and experimental error. We are pursuing the investigation of these possible explanations.

It has become clear to us in examining the full vector-wavefield records, even for this orthorhombic medium that is only moderately anisotropic, that it is extremely difficult to identify all wave phases and determine intuitively what sorts of propagation effects one is seeing when the sagittal plane is an off-symmetry plane. What we feel is needed then is to
Scaled physical modeling of anisotropic wave propagation

compare physical modelling results with synthetic seismograms and other theoretical results (e.g. positions of singularities, wave-surface mapping, etc.) computed by means of seismic anisotropic numerical modelling. Only then, in our opinion, will it be possible to identify many of the complicated and rapidly (spatially) fluctuating effects that occur; for example, around singularities and cusps. Moreover, by making such comparisons with a number of numerical modelling algorithms based on different theoretical schemes, it may well be possible to determine which of those techniques are superior for the tasks at hand. We are now, therefore, pursuing collaborations with a number of groups who have developed these capabilities.

Some of our next laboratory experiments will entail the generation and recording of waves on ray paths close to singular or conical directions in order to observe what sorts of polarization and amplitude fluctuations actually occur and to compare these, and the different patterns of arrivals in different sagittal planes, with those predicted numerically.

ACKNOWLEDGMENTS

We are most grateful to the sponsors of the CREWES Project (Consortium for Research in Elastic Wave Exploration Seismology) at the University of Calgary for their crucial financial support, and to its director Rob Stewart for his scientific and moral support of this work. Eric Gallant is acknowledged for his stalwart technical assistance as is David Eaton for pointing out the negative reciprocity in the nine-component record set (Fig. 8). We thank Stuart Crampin and Dan Ebrom, as well as an anonymous reviewer, for providing salient and constructive suggestions which have contributed to an improved paper. Finally, John Lovell displayed a blend of saintly patience and friendly persuasion in helping us to meet the publication deadlines.

REFERENCES


