Elastic Impedance Revisited

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Introduction

Elastic impedance is a relatively simple concept, but is somewhat elusive in practice. Just as contrasts in an acoustic impedance profile, convolved with a wavelet, generate normal incidence seismic data, there should be an "elastic impedance" profile whose contrasts would generate wide-angle reflections. The problem is that there is no simple closed form expression for this quantity. There are various ways to approximate it. One is to integrate Aki and Richards' (1980) approximation for the Zoeppritz equation (Connolly, 1999, and Sena, 1997). The result is usable but unsettling, with angle-dependent (or even depth-dependent) fractional units. Later work introduced normalizations that only partially alleviated the problems (Whitcombe, 2002). Subsequent to Connelly's original paper, there have been similar formulations of a PS elastic impedance (Landrø, et al, 1999) with the same drawbacks as Connelly's original PP elastic impedance, i.e. fractional units. An alternative formulation of elastic impedance has also been proposed (VerWest, 1998; and VerWest et al, 2000) which avoids the complications of the Connelly formulation. In this paper, a new derivation of elastic impedance will be presented which yields an even simpler form than previous formulations. This will then be extended to a PS elastic impedance.

PP Elastic Impedance

The definition of elastic impedance presented by VerWest was

\[ EI = \frac{\alpha \rho}{\sqrt{1 - \alpha^2 \rho^2}} \frac{(1 - \beta^2 \rho^2)^3}{(1 + \beta^2 \rho^2)} \left( \frac{\rho}{\rho_0} \right)^{-4 \beta^2 \rho^2} \]

where \( \rho, \alpha, \) and \( \beta \) are the density, compressional (p-wave) velocity, and shear velocity respectively, \( p \) is the ray parameter, \( p = \sin \theta / \alpha = \sin \phi / \beta \), \( \theta \) is incident angle, \( \phi \) is the s-wave emergent angle, and \( \rho_0 \) is a constant used to normalize the density. The first factor of this expression is the angle dependent impedance for a fluid-fluid interface and the remaining factors are from the shear rigidity. The reflection coefficient between two layers is given by

\[ r(\theta) = \frac{E_{i} - E_{i-1}}{E_{i} + E_{i-1}} \]

where the average angle is defined as \( \sin \theta = p(\alpha_i + \alpha_j) / 2 \). This expression was originally derived by postulating a form for EI which had this leading fluid factor and an unknown rigidity factor which was approximated by a rational approximation. The form of the rigidity factor was arrived at by trial and error while demanding that the resulting reflection coefficient was as good an approximation to the exact Zoeppritz’ equation as the Aki and Richards’ approximation.

A simpler way to achieve a similar but better result starts with Bortfeld’s (1961) approximation to Zoeppritz’ equation. Bortfeld derived the expression
\[ r(\theta) = \frac{1}{2} \Delta \ln EI_{pp} = \frac{A1_2 - A1_1}{A1_2 + A1_1} - 2p^2 \left[ (\beta_2^2 - \beta_1^2) + \frac{\beta_2^2 - \beta_1^2}{\rho} \right] \]

\[ = \frac{1}{2} \Delta \ln A1 - 2p^2 \Delta\beta^2 - 2p^2 \beta^2 \frac{1}{2} \Delta \ln \rho \]

where \( \rho = (\rho_2 + \rho_1)/2 \) and \( \beta = (\beta_2 + \beta_1)/2 \) and \( A1 = \frac{\alpha \rho}{\sqrt{1 - \alpha^2 \beta^2}} \). Integrating this expression gives

\[ EI_{pp} = \frac{\alpha \rho}{\sqrt{1 - \alpha^2 \beta^2}} \exp[-4p^2 \beta^2 (1 + \ln \rho / \rho_0)] \approx \frac{\alpha \rho}{\sqrt{1 - \alpha^2 \beta^2}} \exp[-4p^2 \beta^2 \rho / \rho_0] \]

The reflection coefficient that results from this form of the elastic impedance agrees with the Zoeppritz result to the same level of approximation as the Aki and Richards approximation, 4th order in angle and first order in the medium parameter changes as long as the difference between the reference density and the actual density is of the same magnitude as the medium parameter changes.

The two factors in the elastic impedance are the fluid factor and the rigidity factor following from the fluid and rigidity terms in the reflectivity expression of Bortfeld.

**PS Elastic Impedance**

The reflection coefficient for PS converted wave reflections is (Bortfeld, 1961)

\[ r(\theta) = \frac{1}{2} \Delta \ln EI = -p \left[ 2(\beta_2 - \beta_1) + \frac{\rho_2 - \rho_1}{\rho} (\alpha / 2 + \beta) \right] \]

Integrating this expression for the PS elastic impedance yields

\[ EI_{ps} = \exp[-4p \beta - p(\alpha + 2\beta) \ln \rho / \rho_0] \approx \exp[-4p \beta - p(\alpha + 2\beta) (\rho / \rho_0 - 1)] \]

This form only has an exponential factor since the PS reflection will only have a rigidity contribution and not an acoustic factor. As was true in the PP case, there is no need to normalize the velocities. The only additional constant is the reference density. The reflection coefficient that results from this form of the elastic impedance agrees with the Zoeppritz result to the same level of approximation as the Bortfeld approximation, first order in angle and first order in the medium parameter changes.

**Examples**

Figures 1 and 2 below show comparisons of the computed reflection coefficients for PP and PS reflections using various approximations. These are for a shale over a wet sand or a gas sand and the shale parameters are \( \alpha = 2.77 \text{km/s}, \beta = 1.52 \text{km/s} \) and \( \rho = 2.29 \text{g/cc} \), the wet sand parameters are \( \alpha = 3.85 \text{km/s}, \beta = 2.24 \text{km/s} \) and \( \rho = 2.34 \text{g/cc} \), and the gas sand parameters are \( \alpha = 3.08 \text{km/s}, \beta = 1.98 \text{km/s} \) and \( \rho = 2.14 \text{g/cc} \). The reference density used was 2.3 g/cc. This new formulation of elastic impedance produces reflection coefficients in good agreement with Bortfeld and Aki and Richards. In the case of the gas sand there is good agreement with the Zoeppritz results. In the case of the wet sand all the approximations deviate somewhat from Zoeppritz due the neglect of higher order terms since some of the medium parameter changes are relatively large. This is true for both the PP and PS result. The PS results deviate significantly from the Zoeppritz result above 20 degrees because there are \( p^3 \) contributions that are first order in the medium parameter changes that are not included. In the case of PP reflections, there are no \( p^4 \) contributions that are first order in the medium parameter changes in the rigidity term.

Figure 3 shows the PP and PS elastic impedance for various angles computed from log curves. Also shown is the “elastic density” formed by dividing the elastic impedance by the p wave velocity. The p wave velocity and the elastic density can be used to calibrate seismic inversion of angle stacks. Also plotted are the ratios of the elastic impedance to acoustic impedance for the PP case. This ratio is
highly correlated to the Poisson’s ratio for larger angles such as 30 degrees. The PS elastic impedance is also highly correlated to the Poisson’s ratio.

Figure 1. Comparison of PP reflection coefficients as a function of average angle calculated from Zoeppritz (bold, solid curve), Bortfeld (bold, dashed curve), Aki and Richards (fine, dashed curve), and new elastic impedance (red curve). The upper set of curves is for a shale over a wet sand and the lower set of curves is for a shale over a gas sand. The medium parameters are given in the text.

Figure 2. Comparison of PS reflection coefficients as a function of average angle calculated from Zoeppritz (bold, solid curve), Bortfeld (bold, dashed curve), and new elastic impedance (red curve). The upper set of curves is for a shale over a gas sand and the lower set of curves is for a shale over a wet sand. The medium parameters are given in the text.

Conclusions

Defining elastic impedance by integrating the Bortfeld approximation to Zoeppritz equations has the advantage to producing a form that avoids having to normalize the velocity factors and honors the correct acoustic limit. This form yields reflection coefficients that are as accurate as any of the approximations commonly in use for seismic modeling and inversion. The inversion results for the PP elastic impedance ratio and the PS elastic impedance are predictors of Poisson’s ratio. Determining Poisson’s ratio from PP elastic impedance inversion requires doing two inversions and taking their
ratio while determining Poisson’s ratio from PS elastic impedance inversion only requires doing a single elastic impedance inversion.

Figure 3. Example from well curves showing input velocities and density, computed PP elastic impedance for average angles of 15 and 30 degrees, “PP elastic density”, PP elastic impedance ratios to acoustic impedance, and PS elastic impedance for 15 degrees.

References