Prestack Gaussian-beam depth migration in anisotropic media

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Summary

Gaussian-beam depth migration (Hill, 1990, 2001) is a useful alternative to Kirchhoff and wave-equation migrations: It overcomes the inabililty of Kirchhoff migration in imaging multiple arrivals while retaining its efficiency and its capability of imaging steep dips with turning waves. By directionally downward extrapolating local plane waves instead of single-trace scattered waves, Gaussian-beam migration also avoids the migration swinging noises inherent in Kirchhoff migration, resulting in clean subsurface images comparable to those from wave-equation migration.

In this study, we extend prestack Gaussian-beam depth migration from isotropic to general anisotropic media based on the raytracing systems developed recently by Zhu et al. (2005). Formulated in terms of phase velocity, the newly developed raytracing systems for anisotropic media are much simpler and much more efficient than the traditional elastic-parameter based raytracing systems, especially for the dynamic raytracing. The new formulation also avoids potential ambiguity in specifying elastic parameters for a given medium. Test results with synthetic and field data show that our anisotropic prestack Gaussian-beam migration is accurate and efficient. It produces images superior to those generated by anisotropic prestack Kirchhoff migration.

Introduction

Kirchhoff migration has been a major tool over the past decade for prestack seismic imaging. It is efficient and can image steep dips with turning waves. Moreover, the methods are flexible with input geometry, a feature especially important to land seismic data processing where irregular data-acquisition geometries and large topographic differences are common. The Kirchhoff methods, however, have difficulties in handling multiple arrivals, making them inaccurate in structurally complex areas. Wave-equation migration, on the other hand, can image multiple arrivals properly, but suffers limitations in imaging steep dips. It is also difficult and expensive to extend wave-equation migration to general anisotropic media. Such extension is essential for accurate seismic imaging since many sedimentary rocks are anisotropic. Gaussian-beam migration (GBM) has been developed recently as an alternative to prestack Kirchhoff and wave-equation migrations (Hill, 1990, 2001). It retains many advantages of the Kirchhoff methods such as its efficiency and its ability of imaging steep dips with turning waves. Moreover, similar to wave-equation migration, GBM is capable of imaging multiple arrivals and producing accurate images in geologically complex areas.

Prestack GBM was originally developed for isotropic media (Hill, 2001). Extension of this method to general anisotropic media has, however, been hampered in the past by the difficulties of kinematic and dynamic raytracing in inhomogeneous, anisotropic media. Formulated in terms of elastic parameters, the traditional anisotropic raytracing systems, especially the dynamic raytracing system, are complicated to implement and inefficient for computation. It may also result in ambiguity in specifying elastic parameters for a given medium. These difficulties have been overcome recently by Zhu et al. (2005) who reformulated the raytracing systems in terms of phase velocity. The purpose of this study is to extend prestack GBM to general anisotropic media based on these newly developed raytracing systems and to show that the resulting anisotropic prestack GBM is efficient and accurate, producing seismic images superior to those generated by Kirchhoff migration in anisotropic media.

Common-shot prestack Gaussian-beam migration

Prestack GBM has been developed in both common-offset (Hill, 2001; Albertin et al., 2001) and common-shot (Nowak et al., 2003; Gray, 2005) domains. The former is more efficient, but less flexible in dealing with land data acquisition geometries and large topographic differences. We concentrate in this study on common-shot prestack GBM because our aim is to develop an algorithm for prestack depth imaging of both land and marine data. Hill (2001) has formulated prestack GBM in common-offset domain; we adapt his formulation here for common-shot GBM. The image at subsurface point $x$ from a shot record can thus be written as

$$I(x) = \sum_L \int_{p_i} \int_{\partial L} \int_{p_i} \int_{\partial L} D(L, x, p^d, \omega) \\delta \omega \\delta p^d \\delta p_i$$

where $x_s$ is source location, and $p_i$ and $p^d$ are slowness vectors at source and receiver, respectively. Position vector $L$ represents the center of the local slant stack of Gaussian-windowed common-shot traces $D(L, x_s, p^d, \omega)$; $L$ is also referred to as the beam center. The local slant stack $D(L, x_s, p^d, \omega)$ is calculated from data in a manner similar to that described by Hill (2001) except that our slant...
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stack is performed in common-shot instead of common-offset domain. Coefficient $C$ is a constant related to the geometry of the lattice for beam centers and summation is carried out over all beam centers for the shot record. Function $U(x, x', L, p^L, \omega)$ in (1) is a propagation integral given by

$$U = \frac{1}{2\pi} \int dp^L_1 dp^L_2 u^*_G(x, x', p^L, \omega) u_G(x, L, p^L, \omega), \quad (2)$$

where $u^*_G$ is the complex conjugate of the normalized Gaussian-beam solution to wave equation (Hill, 2001). Other computational formulae used in our prestack GBM, such as the formulae for beam and beam-center sampling, are similar to those given by Hill (1990, 2001).

Anisotropic kinematic and dynamic raytracing

Calculation of the Gaussian-beam solutions $u_G$ in (2) requires both kinematic and dynamic raytracing (e.g. Cerveny, 2001): the former is required for computing the path and traveltime along the central ray of a given beam solution while the latter is for computing complex amplitude and local wavefront along the beam. Thus extension of GBM to anisotropic media requires anisotropic kinematic and dynamic raytracing. Kinematic and dynamic raytracing in general inhomogeneous, anisotropic media has been traditionally formulated in terms of elastic parameters. Cerveny (1972), for example, has derived the kinematic raytracing system from the equation of motion:

$$\frac{dx_i}{d\tau} = a_{ijkl} p_i g_j g_k \quad (3a)$$

$$\frac{dp_i}{d\tau} = -\frac{1}{2} \frac{\partial}{\partial x_i} \left( \right. p_m p_i g_j g_k \left. \right), \quad (3b)$$

where $\tau$ is traveltime along the ray, $a_{ijkl}$ are the density normalized elastic parameters, and $g_k$ is the polarization vector. The polarization vector is calculated by solving an eigenvalue problem of a 3x3 matrix equation. The functions on the right-hand sides of equations (3) are complicated. Evaluation of these functions is time consuming and requires solving an eigenvalue problem at each ray step.

The raytracing system (3) also requires the medium to be specified in terms of elastic parameters. This is inconsistent with the common practice in seismic data processing where anisotropy is usually described with Thomsen (1986) parameters. This inconsistency may result in ambiguity in specifying the elastic parameters. For P-wave imaging in the widely used weak transversely isotropic (TI) media, for example, usually only P-wave velocity along the symmetry axis $\alpha$ and Thomsen parameters $\delta$ and $\varepsilon$ are estimated from field data. Determination of the elastic parameters of such a medium from parameters $\alpha$, $\delta$ and $\varepsilon$, on the other hand, requires explicit knowledge of S-wave velocity along the symmetry axis.

Elastic-parameter based dynamic raytracing system is even more complicated as the system is derived by differentiating both sides of ray equations (3) with respect to ray parameters $\gamma_1$ and $\gamma_2$ of the ray coordinate system $(\gamma_1, \gamma_2, \tau)$. This leads to very complicated right-hand-side functions for the derived dynamic raytracing system (e.g., Cerveny, 1972, 2001). Thus, kinematic and dynamic raytracing for constructing a Gaussian-beam solution in an anisotropic medium becomes a very complicated and expensive process, especially for prestack GBM in 3D media.

To overcome these difficulties, Zhu et al. (2005) have reformulated the kinematic raytracing system in general anisotropic media in terms of phase velocity in a manner similar to how the ray equations in isotropic media are formulated. This gives

$$\frac{dx_i}{d\tau} = V_i \quad (4a)$$

$$\frac{dp_i}{d\tau} = -\frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} v, \quad (4b)$$

where $v$ is phase velocity and $V_i$ group velocity. The latter can be calculated from the former with the formulae given, for example, by Tsvankin (2001). Similar to its counterpart in isotropic media, raytracing system (4) now takes a very simple form with its right-hand sides given simply by group and phase velocities rather than the complicated functions in (3). System (4) is thus much more efficient than (3), especially for TI and orthorhombic media where the phase and group velocities can be calculated quickly with the simple analytical expressions given by Thomsen (1986) and Tsvankin (2001). Solution of eigenvalue problem at each ray step is no longer needed. Since the medium for raytracing is now specified with phase velocity, the ambiguity problem in specifying elastic parameters for weak TI media is also eliminated.

Based on the ray equations in (4), Zhu et al. (2005) have also reformulated the dynamic raytracing system in terms of phase velocity. The resulting new dynamic raytracing system is much simpler and computationally much more efficient than that formulated in terms of elastic parameters since, instead of differentiating the complicated functions on the right-hand sides of equations (3), dynamic raytracing now requires only simple calculation of derivatives of phase and
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Examples

We have implemented prestack Gaussian-beam depth migration in general anisotropic media based on the formulae described in the previous sections. The algorithm has been tested with both synthetic and field data. The results show that our anisotropic prestack GBM is efficient and accurate, producing images superior to those from Kirchhoff prestack migration.

Figure 1 shows a structural model used in our testing where an anisotropic thrust sheet is embedded in an otherwise homogenous isotropic medium. This model is designed to illustrate the problems that arise when one produces seismic images beneath a dipping thrust sheet without considering the effects of its anisotropy. The finite-difference data generated from the model were first migrated with isotropic prestack GBM by ignoring the anisotropy of the thrust sheet. The resulting image (Figure 2) shows a spurious pull up of the flat basal reflector beneath the dipping thrust sheet. Some defocusing effects along the thrust sheet are also visible. The image obtained with our prestack anisotropic GBM (Figure 3) shows, on the other hand, the basal reflector has been properly flattened and the image of the thrust sheet are better focused, indicating that inclusion of anisotropy is essential for accurate imaging of this structural model. Compared to the cost for the isotropic prestack GBM, the additional cost for the anisotropic migration is negligibly small (about 1%), showing that our anisotropic GBM is considerably more efficient than that of Alkhalifah (1995) who extended the poststack GBM of Hill (1990) to anisotropic media using the elastic-parameter based raytracing systems and reported an increased computational effort of 40% for his anisotropic GBM.

As a comparison, we have also migrated the dataset with an anisotropic prestack Kirchhoff algorithm and the resulting image is displayed in Figure 4. The anisotropic Kirchhoff migration has also imaged the thrust sheet and basal reflector well; this is expected since few multiple arrivals are observed with this structural model. Even in this case, however, the image from prestack GBM appears to be cleaner and better focused than that from the Kirchhoff migration (compare, for example, the image of the thrust sheet near the surface).

Conclusions

We have implemented a prestack anisotropic depth GBM in common-shot domain. This algorithm is more flexible than common-offset GBM in dealing with irregular data-acquisition geometries and large topographic differences. Such flexibility is essential for accurate land seismic imaging. Employing the newly developed phase-velocity based kinematic and dynamic raytracing systems, our anisotropic prestack GBM is efficient, requiring only small amount of additional computational cost compared to isotropic prestack GBM. As the medium for our anisotropic raytracing is now specified with phase velocity, the possible ambiguity problem in specifying elastic parameters for a given medium is also eliminated. Test results with synthetic and field data show that our prestack anisotropic GBM is accurate in imaging structures in anisotropic media, producing images superior to those generated by Kirchhoff migration. The results also show that our anisotropic GBM is more efficient than the anisotropic GBM implemented with elastic-parameter based raytracing systems.

References


Zhu, T., Gray, S., and Wang, D., 2005, Kinematic and dynamic ray tracing in anisotropic media: theory and
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Figure 1: Velocity model with a dipping anisotropic thrust sheet. The angles of the bedding vary from 0 to 61 degree. The velocities along and perpendicular to the bedding are 3364 and 2925 m/s, respectively.

Figure 2: Depth image obtained with isotropic prestack GBM by ignoring the anisotropy of the thrust sheet.

Figure 3: Depth image obtained with anisotropic prestack GBM.

Figure 4: Depth image obtained with anisotropic prestack Kirchhoff migration.