**Surface multiple attenuation in shallow water and the construction of primaries from multiples**

*Neil Hargreaves, Veritas DGC Limited*

**Summary**

The central insight that comes from the work of Berkhout, Verschuur and others in the development of their method for surface-related multiple elimination (SRME) is that multiples, normally considered as unwanted noise in seismic data, are intimately related to, and can be constructed from, primary reflections. It follows that multiples contain equivalent information about the subsurface to that which is given by the primaries. There are circumstances, therefore, in which the multiples can add to signal, i.e. they can be an aid to seismic processing and interpretation rather than being viewed simply as unwelcome noise.

In this paper I first present an approach to multiple removal in shallow water that is a wavefield-consistent form of multi-channel prediction-error filtering. This is an alternative to inversion-based or iterative approaches to SRME in shallow-water applications where these methods encounter difficulties due to missing near-offsets. The multi-channel prediction filters are themselves estimates of the near-offset water-bottom reflection, derived from the primaries and multiples that are present at the available offsets.

I use these prediction operators (equivalently, the estimated water-bottom reflection) to derive an inverse operator that, in the spirit of the focal transform operator described by Berkhout and Verschuur, can construct primary reflections from multiples. As with SRME, the focal transform also encounters difficulties in shallow water due to missing near offsets. The prediction operators provide a way of overcoming this limitation and supply the additional information needed to successfully reconstruct primaries from multiples in shallow water.

**Introduction**

The perspective that surface multiples in marine seismic data are themselves combinations of primaries is the basis of a variety of powerful approaches to surface-related multiple elimination (SRME). An advantage of these approaches compared to other forms of multiple removal is that they do not rely on restrictive assumptions concerning the nature of the geology or the moveout behaviour of the multiples, and also do not need subsurface information in the form of a detailed earth model. They do however place considerable demands on data acquisition, as has been discussed by Dragoset and Jericevic (1998) in the context of 2D SRME, and more recently by a number of authors in the context of 3D SRME (e.g. Jakubowicz et al, 2004). These acquisition constraints become particularly severe in shallow water when the main generator of the multiples is the water-bottom reflection. The angular dependence of this key event often is such that it is not present on the offsets that are recorded during conventional marine seismic acquisition.

Although not explicit in its common implementation, SRME is at base an inversion process in which primaries are constructed from primaries and multiples. A full SRME inversion can, however, be a difficult and costly process. A more limited inversion, akin to multi-channel prediction-error filtering, is possible in shallow water when the water-bottom is the dominant multiple generator. The inversion is equivalent to estimation of the water-bottom primary at offsets close to zero, using the primaries and the water-bottom multiples that are present at larger offsets.

The prediction operator, or equivalently the water-bottom reflection, predicts first-order multiples from primaries, second-order multiples from first-order, and so on. The inverse of the operator predicts primaries from first-order multiples, first-order from second-order, and so on again. The inverse operators therefore provide a way of creating primaries and multiples solely from the multiple content of the original data – i.e. of constructing signal (primaries) from noise (multiples).

**SRME and multi-channel prediction**

The feedback model in Figure 1 (after Berkhout and Verschuur, 2006, Geophysics, in press) describes surface multiple generation as an iterative process of downward reflection at the surface followed by propagation down and back to the surface. (A full description is given in the work referenced above, and in earlier work from these authors – e.g. Berkhout and Verschuur, 2005, and references therein.)

In their recent work Berkhout and Verschuur have advocated a “detail hiding” description of their feedback model in the form:

\[ P = \Delta P + \Delta P A P . \]  

(1)

Here matrix \( P \) is a frequency-domain representation of the

![Figure 1: Feedback model for surface multiple generation (after Berkhout and Verschuur, 2006). The wavefield from source \( S \) propagates down to each reflector and then back to the surface according to the transfer operator \( \Delta X \), and is then filtered by detector \( D \). Multiples are generated by downward reflection at the surface with reflection coefficient \( R' \) and further propagation down and back to the surface. \( P \) is the total recorded wavefield consisting of both primaries and multiples.](Image 326x265 to 527x375)
acquired data, with element \((i, j)\) of the matrix representing the frequency component detected by a receiver at location \(i\) and a source at location \(j\). Matrix \(\Delta P\) similarly represents the primary response and matrix \(A\) represents the freesurface reflectivity plus a compensation for the source and detector properties.

This model of multiple generation is the basis of most of the common forms of surface-related multiple elimination (SRME). These are usually described as a process of multiple subtraction – first model the multiples then (adaptively) subtract. In a more general sense, formulations of SRME that are based on Equation (1) are an inverse process in which primaries are constructed from multiples. Taking expression (1) and writing it as

\[
\Delta P = P(A + \Delta A)P^{-1}
\]

explicitly describes the inversion of primaries plus multiples to obtain primaries. A common way of implementing this inversion in practice, however, is by the iteration

\[
\Delta P_i = P - \Delta P_{i-1}A P; \quad \Delta P_{i0} = 0
\]

which restores a multiple subtraction perspective - successive approximations to the primaries are obtained by subtracting successive approximations to the multiples.

Biersteker (2001) describes an alternative approach to the inversion when he writes Equation (1) as:

\[
\Delta P = P - P F
\]

Here \(F\) is a multi-channel prediction filter which can be obtained by minimising the prediction error on the righthand side of expression (4). Comparing this expression with the inversion in expression (2) above we can see that in general it is possible to equate \((I - F)\) with \((I + A P)^{-1}\) – i.e. the inversion in (2) can be considered as equivalent to multichannel prediction-error filtering. With some constraints on the behaviour of matrix \(A\), corresponding physically to the assumption of angle-independent reflectivity and point sources and receivers, it is also possible to associate the filter \(F\) with \(\Delta PA\).

Estimation of the multi-channel prediction filter is equivalent to the estimation of a (scaled) version of the primaries.

Whilst the estimation of primaries by the iteration of expression (3) is usually preferable to estimation either by explicit inversion or by multi-channel prediction filtering, there is at least one set of circumstances where the iteration cannot be used. In shallow water, and where the water-bottom reflection is the main generator of the multiples, this key event is often not present in the acquired data. As an approximate rule of thumb, the water-bottom reflection has a significant amplitude only for offsets that are less than the water depth – e.g. for a water-bottom two-way traveltime of 200 ms the water-bottom reflection is significant only at offsets less than 150 m. These are offsets that are rarely recorded with typical acquisition configurations. Thus, unless additional information is available, the iteration of Equation (3) will fail to estimate the short-period multiples.

Biersteker shows, however, that it is possible to estimate an offset and time limited prediction filter via Equation (4) and that this can be used for short-period multiple removal. This in essence amounts to estimation of the water-bottom primary reflection from the short-period multiples themselves; application of the prediction-error filter then models and subtracts those multiples.

**Multi-channel prediction and focal transformation**

One “round-trip” of the feedback loop in Figure 1 adds a further order of multiples to the wavefield; expansion of expression (1) shows that this is equivalent to multiplication of the previously-generated set of multiples by \(\Delta PA\). On the other hand, multiplication of expression (1) by \(\Delta P^{-1}\) (that is, multiplication by the inverse of the primary matrix) represents the removal of one round trip, converting higher-order multiples into the next lower order and converting the first-order multiple into primaries:

\[
\Delta P^{-1}P = I + A P
\]

This is the “focal transform” described by Berkhout and Verschuur (2006). In that paper they point out that this leads to a new version of the primaries and multiples (a new version of matrix \(P\) that has been constructed from multiples and which, at least in theory, can be used to derive an image of the subsurface from the multiples.

The focal transform operator as described by Berkhout and Verschuur is the inverse of an initially derived estimate of the primary content of the data. Two obvious difficulties that can arise in this approach are (a) obtaining the initial estimate of the primaries, and (b) inversion of the resulting primary matrix. In shallow water an alternative is to construct the focal transform operator from the offset and time limited prediction operator described above. When a few dominant primaries are responsible for the generation of the multiples we can write \(\Delta P\) as \(\Delta P_0\) (the dominant multiple generators) plus \(\Delta P_1\) (other primaries) and write expression (1) as:

\[
P = \Delta P_0 + \Delta P_1 + \Delta P_0 A P
\]

Multiplication of this expression by the inverse of the offset and time limited prediction filter \(F\) can be equated with multiplication by an estimate of \(\Delta P_0 A\):

\[
F^{-1}P = (\Delta P_0 A)^{-1}P = I + (\Delta P_0 A)^{-1} \Delta P_1 + P
\]

Since \(\Delta P_1\) is also available (from the result of prediction-error filtering of \(P\)) a combination of space or time muting plus adaptive subtraction can then be applied to obtain a new estimate of \(P\) that has been constructed from multiples. An alternative approach would be to use the available estimates of \(\Delta P_0 A\) and \(\Delta P_1\) to construct the full primary matrix \(\Delta P\), and to then derive the focal transform operator.
Surface multiple attenuation in shallow water

from $\Delta P^1$. There are some possible practical advantages, however, to using $(\Delta PA_0)^{-1}$ for the focal transform compared to using the inverse of the full primary matrix. The full matrix has $(2N + 1) \times (2N + 1)$ elements, where $N$ is the number of traces per shot. $\Delta P_0$ has the same number of elements but only $(2M + 1)$ near-offset traces are non-zero, and for shallow water $M$ is very much less than $N$. Computation of the focal transform using the inverse prediction filter is likely to be cheaper than using the full primary matrix; estimation error and stability issues may also be less problematic.

Examples

Figure 2(a) shows a near-offset section and a shot record from a synthetic dataset containing primaries and shallow water-bottom multiples; the shot record is from a location slightly to the right of centre of the near-offset display. The upper event in these displays is a primary reflection from a syncline whilst the deeper events are successive orders of multiples from a water-bottom reflection at a zero-offset time of 200 ms. The water-bottom reflection amplitude decreases smoothly with offset and is non-zero only on offsets from 0 to 125 m. The synthetic dataset, however, lacks those offsets and contains only traces with offsets greater than 125 m.

Figure 2(b) shows the result of applying multi-channel prediction-error filtering to the data in Figure 2(a) using a 360 ms by 125 m (six trace) operator derived from a least-squares solution to Equation (4). Most of the multiple energy has been successfully predicted and removed from the data. (The small amount of residual multiple in the display is believed to be related to the white noise used to stabilise the operator computation.)

Figure 2(c) is the result of focal transformation of the synthetic data according to the formulation in Equation (7). The inverse of the multi-channel prediction operators was applied to the multiples-removed data of Figure 2(b), and this was then adaptively subtracted from the result of applying the inverse filters to the original data. The figure shows that there is a reasonable degree of similarity between this reconstructed version of the data and the original synthetic. Note, however, the relative weakness of one limb of the triplication in the shot record of Figure 2(c) compared to 2(a). The conversion from multiples to primaries has moved energy from high to low offsets and this event has been weakened as a consequence of the finite offset range of the data.

The synthetic water-bottom reflection that generated the multiples is shown in Figure 2(d) at selected shot locations and for offsets from 0 to 125 m, together with the derived prediction operators. The operators closely resemble the water-bottom reflection at near offsets, apart from a slight loss of high frequencies, despite being designed from data in which those offsets were missing.

Figure 2. Multi-channel prediction-error filtering and data reconstruction for a portion of synthetic data. (a) On the left a near-offset section and on the right a shot record for a synthetic dataset containing a primary reflection from a syncline (topmost feature) plus water-bottom multiples. The water-bottom reflection responsible for multiple generation is non-zero only on offsets from 0 to 125 m and these offsets are missing from the synthetic data. (b) The result of multi-channel prediction-error filtering for multiple removal. (c) Data reconstruction from multiples via the focal transform. Focal transform operators were constructed from the inverse of the prediction operators and applied according to Equation (7). (d) Selected shot records showing the water-bottom reflection at offsets from 0 to 125 m and the derived prediction operators. The operators are an estimate of the water-bottom reflection at near offsets, derived from data not containing those offsets.
Surface multiple attenuation in shallow water

Figure 3(a) shows a portion of unprocessed North Sea stack data from an area with a water-bottom zero-offset time of approximately 120 ms; the nearest offset in the acquired data was 200 m. Figure 3(b) is the stack after multi-channel prediction-error filtering using a 280 ms by 175 m (8 trace) operator. Again, despite the missing offsets in the input data, it has still been possible to achieve a good level of multiple removal.

Figure 3(c) is the result of data reconstruction via focal transformation and adaptive subtraction. The main features of the original section appear to have been recreated by this process; the section does show, however, a slight loss of high frequencies and some loss of lateral detail compared to the original stack.

Conclusions

Although the algebraic formulation of the feedback model in Equation (1) is relatively simple we have seen that it can be expressed in a variety of different ways, each with its own physical interpretation and potential application to primary and multiple estimation. Of these, the explicit inversion, or equivalently multi-channel prediction, is a useful alternative to the more common iterative approach for surface-related multiple elimination. The synthetic and North Sea examples presented here show that this approach is capable of compensating for the missing near offsets that are critical for successful multiple removal in shallow water.

Focal transformation via the inverse of the prediction operators is an attractive option for the generation of primary information from multiples in shallow water. It leads to the intriguing possibility of improving the signal to noise ratio of the data by adding imaged noise to signal rather than by simply subtracting the noise. It remains to be shown in practice, however, what the limitations and benefits of this approach might be. In commenting on the synthetic results, for example, I pointed out the aperture limitations of the focal transform. Other issues such as estimation error in the construction of the focal transform operators remain to be investigated, as does the closely related question of how to best combine the information from the different derived multiples and primaries images.

References

Berkhout, A.J., & Verschuur, D.J., 2005, Inverse data processing, a paradigm shift? 75th SEG Meeting, 2099-2102.


Acknowledgements

The author would like to thank AW for providing the thought that underlies this work – that necessity is expressed through chance, and that noise is a form of signal.

He would also like to thank Helmut Jakubowicz and other colleagues at Veritas DGC for helpful discussions and advice during the course of this project.

The author must also thank the management of Veritas DGC for permission to publish this paper.