Least-squares migration

The nature of seismic wave propagation is well known, even if a suitable velocity is not. The wave equation is used to describe this process and the mathematics of both wave propagation and reflection are well established. Given a source and receiver location, as well as a velocity model and subsurface reflectivity, the acquired seismic data can be modelled.

Conversely, given recorded seismic data, the acquisition geometry and the velocity model, how is the reflectivity recovered? Conceptually, the entire set of recorded data can be expressed as a convolution of a diffraction operator and the reflectivity. A full inversion for the reflectivity is not viable, due to the scale of the problem. The inversion reduces to a much smaller problem if regular sampling and a simple velocity is assumed. With this assumption, the inversion reduces to a migration operator, applied to each trace individually. Various flavours of migration have been developed – such as Kirchhoff time and depth, one-way wave-equation, reverse time, etc. All of these migration algorithms implicitly assume a regular sampling of the reflected wavefield. Where this assumption is not valid, the migrated image will be degraded, irrespective of the migration algorithm used. This is commonly seen when migrating land seismic data – acquiring onshore seismic data efficiently will rarely produce a regular sampling. Consequently the image from migrated land seismic data will often exhibit artefacts, commonly known as acquisition footprints. It should be emphasised that these artefacts are not present in the data itself; they exist only because migration is not suitable for use with irregular sampled seismic data.

Any acquisition footprint must be removed from the image before AVO studies or further analysis is performed. Conventionally the footprint is often removed with post-migration filtering of the image. Such methods are cosmetically suitable, but they do not preserve the information stored in the acquired data.

Least-squares migration is an alternative to standard migration for imaging seismic data. Any image that can be diffracted onto a given acquisition geometry such that the recorded data is reproduced is fully consistent with the acquired data. This is not generally true of an image derived from standard migration. However, starting from a standard migration result, an image can be iteratively optimised in a least-squares sense to reproduce the recorded data: this is the concept behind least-squares migration and is described in more detail by Nemeth *et al* (1999). In practice, least-squares migration needs to be constrained beyond the best fit with the recorded data. The diffraction of very steeply dipping reflections will often produce little recorded energy. This is effectively a null space in the inversion, potentially allowing noise in the system to be amplified. Weak constraints can be imposed to prevent instability, such as requiring the images from adjacent offsets/azimuths to be similar, or requiring that the image at adjacent locations changes slowly.

Least-squares migration can generate an improved image, when compared with standard migration. For example, Kuhl & Sacchi (2003) demonstrate that least-squares migration can provide high-quality amplitude information, even when the velocity model has minor errors. This is true even when the surface wavefield is irregularly sampled and a standard migration would yield a significant acquisition footprint. Clearly this is a huge benefit for AVO analysis as well as structural interpretation of a seismic image. However the cost of these algorithms has been prohibitive, which has prevented their widespread use.

The method of conjugate gradients is a well-known algorithm that is suitable for solving least-squares problems involving a large number of parameters. It has been found (eg. Nemeth *et al* 1999) that using conjugate gradients to solve least-squares migration is a viable algorithm, although it may require many tens of iterations to achieve a reasonable convergence. Each iteration of conjugate gradients will require the computational equivalent of several standard migrations. Given this, any algorithm requiring more than about five iterations to converge upon the solution will not be used frequently. Additionally, methods to reduce the cost of each iteration are needed.

The least-squares migration demonstrated here is practical for imaging general 3D seismic data. The optimisation algorithm used is found to converge significantly faster than conjugate gradients: a solution can be expected within five iterations or less. Moreover, to reduce the cost of each iteration, a Kirchhoff diffraction operator is used with time migration assumptions, although many other diffraction methods could be used.

The least-squares migration algorithm developed is illustrated here by a synthetic dataset and also using seismic data acquired from onshore Texas.

Synthetic example

A synthetic image, shown in Figure 1a), was generated within a simple velocity environment. This image was diffracted using a Kirchhoff algorithm onto a defined acquisition geometry with a nominal square bin size of 25m, typical of onshore seismic data with a "brick" pattern. The following analysis uses the near-offset traces, some of which are shown in Figure 1b), where the fold of coverage is around half that required to be fully sampled. Moreover the near-offset traces had compounded periodic sampling.

Figure 1c) shows a true-amplitude Kirchhoff migration of the diffracted data. The general structure of the synthetic image can be seen, but there is a considerable amount of noise present; much of the noise follows the same periodicity as seen in the diffracted data. This noise comes from the poor sampling of the acquisition geometry (a regularly sampled acquisition would produce a much cleaner image after migration). If the Kirchhoff migrated image is diffracted, the original diffracted data is not recovered. In other words, the Kirchhoff image is inconsistent with the modelled data – due entirely to the sampling.

The least-squares migration algorithm was used upon this dataset, using the weak constraint that adjacent locations should display a similar image. If desired, this constraint could be modified to account for known structural dip. The result is shown in Figure 1d). Clearly the residual noise has been reduced significantly, when compared to the standard

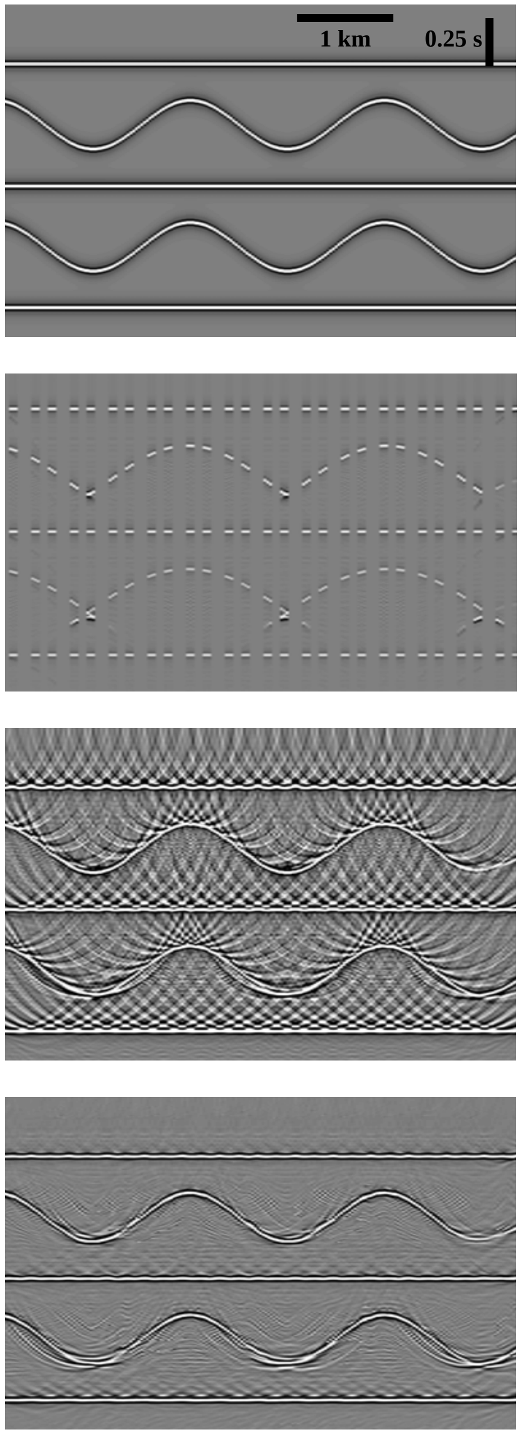

Figure 1.
a) Synthetic image.
b) Diffraction of image onto onshore acquisition geometry. Live data is shown at the mid-point locations.
c) Kirchhoff migrated image of the data in Figure 1b).
d) Least-squares Kirchhoff migrated image of the data in Figure 1b).

Kirchhoff result. On closer inspection, the three flat events are much more continuous, comparing well to the original image. The two oscillating events are much clearer, but their correct structure is not fully recovered. Figure 1b) shows that the troughs of the oscillations are focussed into a small data space by diffraction. Conversely, the peaks of the oscillating events are broadened by diffraction. Since the data space occupied by the troughs is smaller than that for the peaks, detail of the troughs is more easily lost when data is missing. Hence the structure of the troughs is poor, whereas the least-squares migration recovers the peaks accurately.

How accurate is the image? An objective measure of this is the least-squares fit between the acquired data and the diffracted image. By this metric, least-squares migration is always better than a standard migration, provided that the constraints are realistic. Given that the starting point for the least-squares inversion is a standard migration, any number of iterations will improve upon the image.

Coastal plain from onshore Texas

The second example uses seismic data acquired from the coastal plain of onshore Texas. For these tests the full dataset was used, along with a copy of the data where every second source was removed. Figure 2 shows a fold map for the full data and the decimated data. Clearly neither dataset provides a regular sampling of the subsurface.

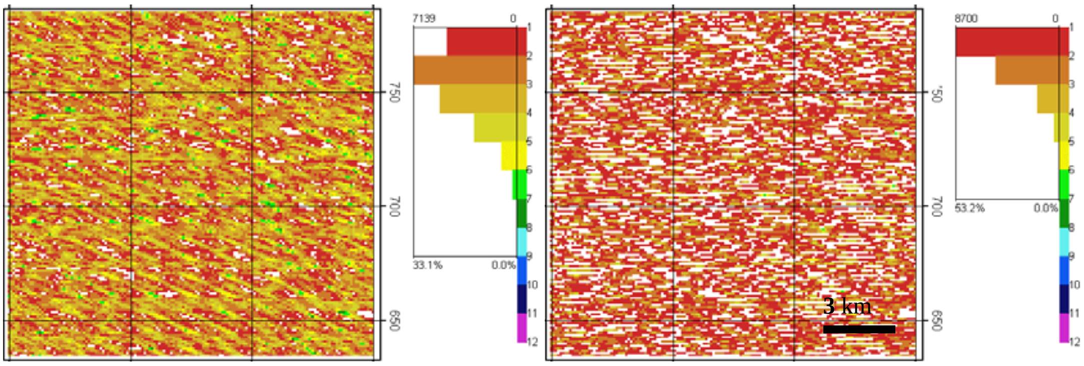

Figure 2. Fold maps and histograms for offset 2172m for the two datasets used:
a) all sources.
b) alternate sources.

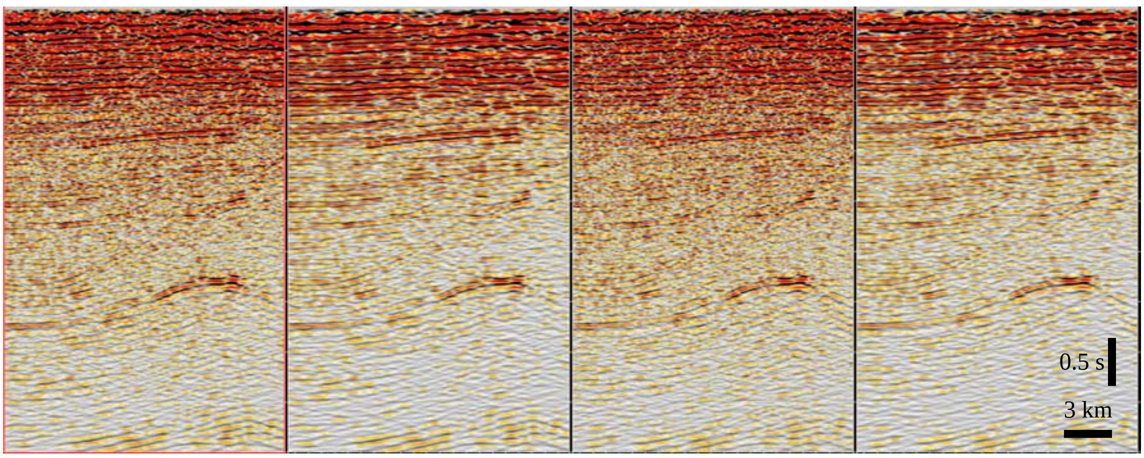

Figure 3. Sections through the images generated by:
a) Standard migration using all the seismic sources.
b) Least-squares migration using all the seismic sources.
c) Standard migration using alternate sources.
d) Least-squares migration using alternate sources.

Kirchhoff time migration and least-squares time migration were performed on the full and decimated datasets. Sections through the standard migrated images using the full and decimated seismic data are shown in Figure 3 a) and c), respectively. Decimating the data has reduced the signal-to-noise ratio, as should be expected. Moreover, as the fold maps suggest, neither migration is clean due to the irregular acquisition. Sections through the least-squares migrated images using the full and the decimated data are shown in Figure 3 b) and d), respectively. The least-squares migrated results are considerably cleaner than those of the standard migration. This is especially clear in the shallow region, where the acquisition footprint is most significant. Figure 4 shows timeslices through the images in Figure 3 at 2.2 seconds. Figure 4 reinforces the conclusions taken from Figure 3. The timeslices demonstrate that the least-squares migration will reduce the amount of noise in the system, but not at the expense at losing resolution of signal.

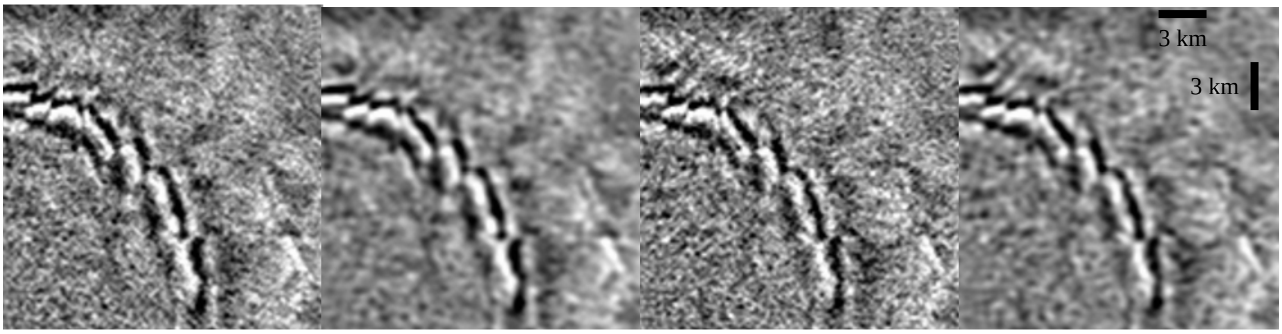

Figure 4. Timeslice at 2.2 seconds through the images generated by:
a) Standard migration using all the seismic sources.
b) Least-squares migration using all the seismic sources.
c) Standard migration using alternate sources.
d) Least-squares migration using alternate sources.

Conclusion

Migration of seismic data is a valuable technique in forming an image. All flavours of migration assume that the data is regularly sampled. When this assumption does not hold, the migrated image will be unsatisfactory. When this occurs, post-migration filtering of the image is often performed; a process that does not preserve the acquired data.

Least-squares migration is a general technique for imaging irregularly sampled seismic data. This class of algorithms form an image that, when diffracted, is consistent with the acquired data. Fewer assumptions are made, but additional constraints are required for stability.

It has been shown on synthetic and real land data that least-squares migration can form a good image with a reasonable computational cost. The resulting image is suitable for further analysis and interpretation, with less need for cosmetic manipulation. This means that the image is more accurate than an equivalent image using a standard migration.

References:

Nemeth T., Chengjun W. & Schuster G. T., 1999, Least-squares migration of incomplete reflection data, Geophysics, pp 208-221

Kuhl H. & Sacchi W.D., 2003, Least-squares wave-equation migration for AVP/AVA inversion, Geophysics, pp 262-273.