Summary

A good migrated image depends strongly on the accuracy of the velocity model. For tomographic velocity analysis, pick errors, null space, velocity ambiguity, and intrinsic non-linearity all have a strong effect on the accuracy of the velocity model. Our post-migration tomography algorithm can work on either common image gathers (CIG's) or common angle gathers (CAG's). A large number of depth residuals and the structural dip field are automatically picked and high-graded. In the conjugate-gradient solver, we apply a left preconditioning operator in data space in order to reduce the influence of extreme picks and apply a right preconditioning operator in model space with a variable-length controlled direction smoother in order to reduce the null space, speed up the convergence and stabilize the solutions. Our smoothing algorithm uses a nonlinear diffusion equation, which can automatically detect velocity boundaries and perform velocity smoothing without crossing the boundaries. To prevent the solution from going wild, we apply hard constraints that limit the velocity updates within a certain range. Both numerical and real data tests show that our algorithm works well and produces encouraging results.

Despite many successful applications of tomographic velocity analysis, challenges still remain for this ill-posed problem. The presence of a null space and pick errors guarantee some level of velocity ambiguity. In this abstract, we discuss some of the challenges facing tomographic velocity analysis, and we introduce some ways to cope with them.

Introduction

For estimating velocities near complex geologic structures, vertical updating methods such as the Deregonowski (1990) have both theoretical and practical limitations and usually do not give entirely satisfactory results. Tomographic velocity analysis offers a powerful alternative for velocity analysis in complex areas where conventional velocity analysis methods fail. However, velocity analysis is often a nonlinear, under-determined problem. The quality of the picks and adequate filling of the null space will have a major effect on the final solution.

In this abstract, we discuss post-migration CIG and CAG tomography. These work by projecting the residual depth errors picked on CIG’s or CAG’s back along the traced ray paths. They compute the velocity updates by minimizing the depth discrepancies in CIG’s or CAG’s (Stork et al., 1992; Wang et al., 1995; Woodward et al., 1999; Zhou et al., 2001; Guo et al., 2002). Our tomographic implementation (1) automatically picks events on the CIG’s or CAG’s, (2) estimates the dip fields on the migrated stacked volume, high-grading the picks based on the semblance of the event on CIG’s or CAG’s and the spatial coherency on the migrated stacked volume, and (3) solves the resulting system of equations with both left and right preconditioning. A left preconditioning operator using the $L_p$ norm reduces the effect of the extreme picks, and a right preconditioning operator with variable smoothing length performs smoothing along the structural dip to reduce the null space. Our smoothing algorithm uses a nonlinear diffusion equation, which can automatically detect velocity boundaries and perform velocity smoothing without crossing the boundaries. In addition, we apply hard constraints, depth and illumination scales and quality factors of the picks to weight the equations. Synthetic and real data tests show that our method can efficiently and accurately pick the events and the structural dips, and handle the null space problems adequately.

However, seismic tomography remains an ill-posed problem, with problems of non-uniqueness and dependence on pick errors. We will discuss some challenges with the tomographic velocity analysis and introduce some ways to cope with them.

Method

- **Model representation**
  Tomographic velocity analysis begins with the parameterization of the velocity model. Among different possible model parameterizations, blocky (or layered), gridded, tessellated, and B-splined models are among the most widely used. In our implementation we use gridded models. While not as compact as other parametrizations, they provide a natural, easy-to-control, and well resolved representation of velocity.

- **Automatic event and structural dip field picking**
  Picking is crucial in tomography, since the best tomographic results can perform only as well as the picks permit. We use an automatic picker that can pick a large number of events without any horizon consistency restriction. It can also be constrained to avoid picking multiples. Event picking is based on a robust single parameter scheme, and dip estimation is based on LSQR. We high grade the picks using the event semblance on CIG’s or CAG’s and spatial coherency on the migrated stacked volume. The net effects of the high-graded, primaries-only picks are to make the inversion problem...
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better determined and to yield a highly resolved inverted velocity field.

- **Iterative Linear Solver**

The high-graded picks are combined with information obtained from raytracing through the initial velocity model to provide a set of tomographic equations. We allow two kinds of picks: from CIG’s and from CAG’s. For CIG’s, the surface offset, depth and structure dips are known, and we need to find the ray pair that honors all above information. Finding the ray pair is an optimization problem. For CAG’s, the depth, subsurface open angle and structure dips are known, and it is much easier to find the ray pair. Since the inverse problem is under-determined, there may be many solutions that minimize the objective function. In order to find the most reasonable solution, we need to provide constraints, leading to the following equations:

\[
\begin{align*}
L x' &= d, \\
x &= R x', \\
\min x &\leq x' \leq \max x,
\end{align*}
\]

where \( F \) is the Frechet derivative matrix, \( x \) is the matrix of slowness perturbations, and \( d \) is the matrix of residual depths. \( L \) is the left preconditioning operator in the data space, which is used to reduce the effects of extreme picks. \( R \) is the right preconditioning operator in the model space, which is used to fill in the null space. In our implementation, \( R \) is applied as a smoother, and we progressively shrink the smoothing diameter with increasing iteration number. To make the velocity model more geologically reasonable, we constrain the tomographic inversion with a controlled direction smoother that performs along predefined dip directions. This can both accelerate the convergence and stabilize the solution. It uses a nonlinear diffusion equation, which can automatically detect velocity boundaries and perform velocity smoothing without crossing the boundaries. \( \min x \) and \( \max x \) are lower and upper bounds of the permitted slowness perturbation; these are applied to stop the solution from straying too far.

Guo et al. (2002) have described a left preconditioner and its least-squares conjugate gradient algorithm. A right preconditioning algorithm can be written as

\[
\begin{align*}
FR x' &= d, \\
x &= Rx'
\end{align*}
\]

with its least-squares conjugate gradient algorithm

\[
r = d - Fx, \\
\beta = 0
\]

iterate \{
\[
\Delta x = F' r, \\
z = RR' \Delta x, \\
if not first iteration \quad \beta = \frac{(\Delta x, z)}{\gamma} \\
\gamma = (\Delta x, z) \\
s = z + \beta s, \\
\Delta r = Fs \\
\alpha = \gamma / (\Delta r, \Delta r) \\
x = x + \alpha s \\
r = r - \alpha \Delta r
\]
\}

Here, \( z \) is a preconditioner parameter. This algorithm can be easily extended to one with both left and right preconditioning operators. If an appropriate preconditioning operator is applied, the condition number of the inverse problem can be reduced significantly, and the solution can converge much faster.

- **Velocity ambiguity**

Even when the algebraic inverse problem is solved correctly, the estimated (inverted) velocity model differs from the true velocity model: this is velocity ambiguity. In general, there are two reasons for this. The first reason is the non-uniqueness of the inverse problem, which is usually related to the null space. Under-determination, shadow zones, limited angle coverage, and decreasing resolution with depth all contribute to the size and shape of the null space (Ross, 1994; Prucha et al., 1999; Guo et al., 2002). The proper handling of the null space is one of the most important issues and the main challenges for seismic tomography. The effect of the null space can be reduced by re-gridding the velocity model with a larger grid size and by adding a right preconditioning operator to the model space. The second reason for velocity ambiguity is pick errors, which introduce error to the estimated velocity model. A major challenge for tomography is to reduce velocity ambiguity by realistically constraining the effects of the picks in the inversion.

**Examples**

Here, we illustrate the performance of the algorithm outlined in the preceding section. The chosen dataset for our first test is from an area with very strong velocity contrast. Fig. 1 is the result of a prestack depth migration using a vertically updated velocity model. Fig. 2 shows the migrated image after updating the model using tomography. Fig. 2 shows greater detail and continuity than Fig. 1. Fig. 3 and Fig. 4 show selected CIG’s from the two migrations. The large residual curvatures in Fig. 3 have been almost completely flattened in Fig. 4.
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Our next example is meant to illustrate a common problem with estimating velocity from migrated seismic data. Fig. 5 shows selected common angle gathers from the Sigsbee2A synthetic dataset (SMAART) migrated with 90% of the correct velocities in the subsalt areas. The events in the subsalt areas are nearly flat, which is due to the small reflection angles in the subsalt areas, with almost vertical incidence. The event flatness is not sensitive to velocity changes, and this insensitivity will pose a severe challenge for the tomography. Fig. 6 is the migrated section with picks overlaid, the picks match the events very well. Fig. 7 shows the illumination section, which counts the number of rays passing each cell. From this section we see that some of the subsalt areas are un-illuminated or poorly illuminated. Fig. 8 shows the velocity updates calculated by tomography. In the poorly illuminated areas, the velocity updates are very small. To overcome the problems caused by the poor ray coverage and the decreasing angular coverage with increasing depth, we need to incorporate other constraints, such as interpreter’s knowledge of velocity, well information, etc.

Conclusions

We have implemented and tested a version of post-migration tomography. Our implementation uses automatically picked events on CIG’s or CAG’s and the structure dips from the migrated stacked volume. To handle the extreme picks and null space, we add a left preconditioning operator (an Lp norm diagonal weighting operator) and a right preconditioning operator (a controlled direction smoother that performs the smoothing along predefined dip directions to the inversion). Our directional smoother is based on the diffusion equation; it can automatically detect the velocity boundaries and perform smoothing without crossing the velocity boundaries. Our inversion strategy uses variable smoothing scale lengths and re-parameterization of the velocity model with variable grid sizes as the iteration increases. Tests using numerical and field datasets show that our algorithm has produced satisfactory results. In addition, we have also discussed some challenges with tomographic velocity analysis.

References

Deregowski, S. M., 1990, Common offset migrations and velocity analysis: First Break, 8, 224-234.
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Fig. 3, Selected common image gathers from Fig. 1.

Fig. 4, Selected common image gathers from Fig. 2

Fig. 5, Selected common angle gathers of Sigsbee2A dataset

Fig. 6, Picks overlaid on the migrated stacked section

Fig. 7, Illumination (the number of rays passing each cell)

Fig. 8, Velocity perturbation calculated by tomography