Global solution to water column statics: a new approach to an old problem
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Summary
We develop a novel procedure to solve the long-standing problem of water column statics. Our procedure consists of automatic picking of the relative static shift between the sail lines, and solving this problem as a global nonlinear inversion problem. Like most inversion problems, the solutions are not unique. So, we introduce a priori constraints to limit the final solution. The constraints require the least movement of traces when statics are derived. Furthermore, we implement a hybrid $\ell/1/\ell^2$ norm in our algorithm for a robust inversion; this norm is relatively insensitive to large residuals, which may occur as the result of incorrect picks in the automatic picking. Results on synthetic and real data sets show that our method is robust and powerful.

Introduction
Water column statics correction is the first step of 3D deep-water marine seismic data processing. As described in (Barley, 1999), water column statics may exceed 10 ms, and this is enough to degrade a stack. The correction is a difficult problem, and even today it is generally done manually. Thus, the process is time consuming and very costly. For a large marine project with hundreds of offshore blocks, it might take more than a month to pick the statics, and the results are not always reliable. This is because the process depends on many factors (e.g. the amount of data, the allocated time, and the experience of processors). Finally, the picked water column statics function at one offset cannot be guaranteed to be consistent with the function at other offsets.

Water column statics can be caused by the change in the state of the water with the different times of the acquisition (e.g. the primary shoot, the re-shoot, and the in-fills). Due to seasonal changes or ocean currents, the state of water can vary as a function of the temperature and salinity, resulting in a change in the seismic velocity (MacKay et al., 2003; Bertrand and MacBeth, 2003). Different shooting times will experience different water velocities, so the observed travel time of the seismic wave at the receivers can be different for the same configuration. This effect is easily observed on a near offset cube along a cross line section where some of traces seem to be shifted up or down relative to their neighbors.

Normally, one sail line acquisition takes a relatively short time. One can therefore consider that during that short period, the water velocity does not change; approximately, then, one constant static function per sail line should work for most cases. Some small variations along the sail line may exist because of other factors like tidal variations, which can cause the water level to vary in a short time, but these variations are normally small.

To solve the statics problem, we develop an automatic method for water column statics correction. Our algorithm picks relative static shifts between two adjacent traces that belong to different sail lines along a cross line section. With these picks, we then formulate the problem as a nonlinear inversion problem whose solutions are the desired static corrections.

After the inversion of statics function for all sail lines in a near-offset cube, the correction for the offset effect can be done based on the work of Wombell (1997).

Automatic Picking
Automatic picking is a crucial step in our method. To make the picking robust, we make two basic assumptions:

- Diffractors are far from the receiver lines.
- The seismic propagation velocity is constant in the vicinity of receiver lines.

Figure 1 illustrates the physics that underlie these assumptions. The reflection surface can be rugose but the recorded seismic events are smooth, so the observed travel time of the seismic wave at the receivers can be different for the same configuration. This effect is easily observed on a near offset cube along a cross line section where some of traces seem to be shifted up or down relative to their neighbors.

Figure 1: Assumption of automatic picking: that seismic events are smooth, the diffraction wave front is also a smooth event when recorded along a receiver line.

This physical fact allows us to devise a stable strategy to pick the statics. Figure 2 shows a seismic shot gather, with...
the red line showing the picks along the water bottom reflection event. When there are diffraction points at the water bottom, the red line, while continuous, might not be a smooth function. The “kink” is the place where the two seismic events (green lines) cross each other. Thus, picks for the reflection events are smooth, but the water bottom picks (red line) are not.

Traditional methods of determining statics shifts from water bottom picks are problematic because the discontinuities in the derivatives of these picks with respect to the cross line direction can be due to statics shifts or rugosity. The reflection events, on the other hand, are typically smooth. Thus, we can compute the derivative function from their traveltime picks (i.e., seismic event slope) and guarantee that any discontinuities observed will be due to pure water column statics. To select the correct water statics, we use a small cross correlation window to reduce the chance of picking crossing events. We also compute multiple cross-correlation windows along the trace. Among all computed shifts for a given pair of traces, the water statics are picked based on the following criteria:

- The seismic events in the cross correlation window have good reflection energy; the RMS in the window is compared with the RMS of the trace.
- The cross correlation coefficients in the vicinity (each side 3-5 traces) of the events satisfy a user-specified threshold.

Figure 3 illustrates how our picking works. As mentioned above, we use cross correlation to pick the time shifts from trace to trace. In this example, the picked shift between two adjacent traces is due to the static shift plus the slope of the seismic event. To extract the water static shift, it is necessary to get rid of the effects of the general geology (seismic slope). Equation (1) describes the seismic event slope, which should be a continuous function along a given seismic event:

\[
D_{(y)} = \frac{dT}{dy} \tag{1}
\]

Where the T is the travel time of the event, y denotes cross line coordinate of the trace’s midpoint. Static shifts occur when the function (1) becomes discontinuous, with the very high frequency part (a spike) in function (1) indicating a static shift (see Fig. 3). Detecting the “spikes” of this function and properly remove the slope of the seismic events yield the desired water statics.

**Water column statics global inversion**

Automatic picking gives only the relative static shifts between sail lines. For each picked data, we can build the following linear equation:

\[
S_{(x)}^i - S_{(x)}^j = \Delta T_{i,j} \tag{2}
\]

Where \( S_{(x)}^i \) denotes the static function of sail line i at cross line x, and \( \Delta T \) is the picked time shift. To solve the static problem, we construct the cost function

\[
\Phi = \sum_k \left\| S_{(x)}^i - S_{(x)}^j - \Delta T_{i,j}^k \right\|, \tag{3}
\]

where the index k is the summation over all the picks.

The distribution of the residuals in the picks is generally not a Gaussian distribution, so least-squares inversion (Claerbout and Muir, 1973; Taylor et al., 1979; Chapman and Barrodale, 1983; Scales et al., 1988) will not necessarily work well on this problem. Because least-squares inversion is sensitive to large residuals, a small number of bad picks will affect the global solution, leading to a very incorrect result. Fortunately, robust error measures such as the \( \ell^1 \) norm or the Huber function (Huber
Water column statics inversion

1973; Guitton and Symes 1999) have been implemented in a number of applications in geophysics. As measures of data misfits, these robust methods show considerably less sensitivity to large measurement errors than the least-squares ($l^2$ norm) measures. This insensitivity to large noise has a statistical interpretation: robust measures are related to long tailed density function, in the same way that $l^2$ norm is related to the short-tailed Gaussian distribution function (Tarantola, 1987).

Meanwhile, like most geophysical inverse problems, our problem is also ill posed; as a result, relatively noise-insensitive misfit measures can yield far more stable estimates of the model parameters than does the $l^2$ norm. In our application, because we use an automatic method for picking the static shifts between the sail lines, it is difficult to QC every pick, and a robust inversion can help make the solution insensitive to erroneous picks.

The norm $\| \cdot \|$ that we use here is a hyperbolic function:

$$\| x \| = (l^2 x^2 + \varepsilon^2/2 - \varepsilon)^{1/2}.$$  \hspace{1cm} (4)

Here $\varepsilon$ is a positive small constant, which controls the gradual transition from $l^1$ behavior (for large residuals) to $l^2$ behavior (for small residuals). Minimizing the cost function (3) will give a solution of equations (2). If we take (4) as the error measure, our solution method is the hybrid $l^1/l^2$ norm inversion (Bube and Langan, 1997).

**A priori constraints**

We may obtain an estimate of our unknown statics function $S^i_{(x)}$ by directly minimizing the cost function (3).

However, the solutions of statics functions $S^i_{(x)}$ in equations (2) are not unique: if a constant is introduced to all the $S^i_{(x)}$ functions, the equations (2) will still hold and the cost function (3) will have the same value. This means that our method potentially allows a DC shift between different solutions. To solve this DC floating problem, a priori information should be introduced to constrain the solution. In reality, the statics occur only for a few sail lines; normally these are infill lines. Thus, we can modify our cost function to the following:

$$\Phi = \sum_k \| S^i_{(x)} - S^i_{(x)} - \Delta T^i_{k,j}(x) \| + \sum \| \delta^i S^i_{(x)} \|. $$  \hspace{1cm} (5)

The second part of the cost function (5) will limit the solution to the following condition: the least movement of the sail line traces for water column statics correction to best fit the data. The parameter $\delta^i$ in equation (5) is the weight of the a priori information. The choice of this parameter depends on the acquire geometry: if it is too small, it can not control the DC floating, and if it is too large, it will constrain the solution to converge away from the true solution.

**Synthetic data test**

For a “synthetic” test, we take a production data set with manually corrected water column statics, and we introduce artificial shifts to several sail lines. We select the area where the sea bottom is rugose to test our method. This synthetic data test will have all the difficulties of a real data application.

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Table 1. Synthetic test, all the units are in ms.

Table 1 shows the result of our test. Our method has found the correct statics within the numerical accuracy. Our results are slightly different from the exact solutions because the data sample rate is 4 ms while the resolution of the cross correlation in the automatic picking is 2 ms.

**Real data applications**

We apply our method to a marine data set in a deep-water environment, where the water bottom is rugose. We can observe large water column statics in this data set. Figure 4 shows a near offset crossline section after binning; the original data have large water column static shifts in the

![Figure 4: Real data application, near offset cross line section before water column static correction.](image-url)
Water column statics inversion

center. The traces of four sail lines have up to 10 ms water column statics, while the statics of most other sail lines are less than 2ms.

Figure 5 shows the result of our automatic statics correction. After the correction, the seismic events have become smooth. Our method yields a better quality than the manual correction, because it corrects not only the large statics but also small ones, which are difficult to see.

Figure 6 shows how some segments of statics functions change along the sail lines; the shifts are almost constant with small linear changes.

Conclusion

We have developed an automatic water column static correction method, which consists of automatic picking and performing a nonlinear global inversion. The only assumptions in our approach are that the seismic events recorded in the near offset sections are smooth and that the water column statics function slowly changes in a given sail line. The applied a priori constraints make the statics solution unique and introduce the least movement of traces to best fit the data. Results on both synthetic and real data applications show that our method is robust and powerful. The described framework can be extended to use multi-offset data so that consistent statics solutions can be obtained.

References

Barley B. 1999, Deep-water problems around the world: The leading Edge, 18, 488-494.


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