Introduction

Kirchhoff migration has been the workhorse of prestack seismic imaging for over a decade. It allows time and depth migration methods to be incorporated within a single basic program, facilitates target-oriented migration, and enables straightforward migration velocity analysis. While the imaging accuracy of single-arrival Kirchhoff prestack depth migration has been sufficient for all but the most challenging structural imaging problems, accuracy comparisons with many wavefield extrapolation methods have often brought out its shortcomings. In particular, in complicated geology, where several arrivals are required to give a good image, we must choose one particular arrival, thereby degrading the image.

Recently, developments in algorithms and hardware, have allowed wavefield extrapolation methods that can image all events to become viable alternatives to Kirchhoff techniques. However, these methods can have problems imaging steep or overturned events, as well as accounting for anisotropy. Furthermore although wavefield extrapolation methods can be affordable, they are still much more expensive than Kirchhoff migration. As a result, they are typically reserved for situations that really require multi-arrival imaging, such as geology below complicated salt bodies.

Another, perhaps less well-known imaging technique is Gaussian beam migration (Hill, 1990, 2001). This method retains the strengths of Kirchhoff migration, but can also image multiple arrivals. Although Hill gives the theoretical basis for Gaussian beam migration in his two classic papers, the method involves many steps and is difficult to implement. Indeed this is, perhaps, the main reason it has not become more popular. Gaussian Beam migration solves many of the imaging accuracy problems of single-arrival Kirchhoff migration, while retaining many of the advantages of the Kirchhoff method including its ability to image steeply dipping or overturned events, as well as imaging in the presence of TTI anisotropy.

In this paper we describe the Gaussian beam migration method and show examples of its use on synthetic and field data.

Gaussian Beam Migration

Hill formulates Gaussian beam migration as a wavefield continuation method operating on common-offset, common-azimuth data volumes. The wavefield continuation itself provides a kinematically correct imaging condition, while the migration is similar to Kirchhoff migration, but applied to local slant stacks using complex-valued traveltimes and amplitudes. These latter complex quantities come from expressing the wavefield as a sum of Gaussian beams, which are finite-frequency, ray-theoretical approximate solutions to the wave equation. In Hill’s formulation, Gaussian beam migration is performed by imaging local slant stacks of traces from each common-offset data volume, and summing the contributions from all the local slant stacks. Any given local slant-stacked trace, centered at $x^m$ and with slant-stack vector $p^m = p^d + p^s$ (m for midpoint, d for detector, s for source), will then be imaged using the complex Gaussian beams shot from $x^m + h$ and $x^m - h$ ($h$ = half-offset vector) with takeoff angles determined by $p^d$ and $p^s$.

Figure 1. Schematic representation of the traveltime tables for five Gaussian beams shot from the same surface location with different takeoff angles. The left panel shows the real part of the traveltime, and the right panel shows the exponential of the imaginary part of the traveltime at a particular reference frequency. The real part of the traveltime has moderate curvature, while the imaginary part of the traveltime causes exponential decay of energy away from the raypath.
Gaussian beams provide complex values of time and amplitude for imaging. For example, the real part of time corresponds to the standard Kirchhoff imaging condition, while the imaginary part of time provides an exponential decay of wavefield strength away from a raypath (Figure 1). The real part of the time determines the mapping along the ray, while the imaginary part controls the tapering of the amplitudes away from the ray. In principle, this involves a double loop over \( p^m \) and \( p^h = p^d - p^s \), but Hill uses the locally planar geometry of the traveltimes to reduce the amount of work required for this while preserving most of the multi-arrival imaging. This results in an efficient, kinematically accurate migration. Figure 2 shows migrations of a pair of input spikes using single-arrival Kirchhoff migration and Gaussian beam migration, illustrating the multi-valued imaging capability of Gaussian beam approach.

Figure 2. Migrated impulses from single-arrival Kirchhoff migration (left) and Gaussian beam migration (right). The velocity model is the same for both migrations. The interpolation of traveltimes onto the migration grid has produced glitches and sharply truncated holes in the migrated image obtained with the Kirchhoff technique (left). By contrast, Gaussian beam migration has preserved the multi-arrival nature of the actual wave propagation.

Gaussian beam migration replaces a single real-valued Kirchhoff traveltime table (plus, perhaps, an amplitude table) from each source or detector location with many complex-valued traveltime and amplitude tables from a restricted set of upper-surface locations (Figure 1).

The multiplicity of tables used by Gaussian beam migration provides its multi-valued imaging capability. Wavefield sampling theory provides rules for determining the total number of tables from each point, and for the total number of points that act as beam center locations.

**Examples**

We first compare Gaussian beam with Kirchhoff and wavefield extrapolation migration on the SEG/EAGE 3D salt model shown in Figure 3. All three migration methods give similar results in the sedimentary area above the salt body. The top of salt and horizons to the left of the salt also look very similar; this is to be expected since a single arrival is adequate to image these events. However, the base of salt and horizons below the salt show differences between the three methods. The Kirchhoff result shows more migration swings below the salt than either the Gaussian beam or wavefield extrapolation results, and these swings partially obliterate the flat horizon. By contrast, the wavefield extrapolation and Gaussian beam results are in general quite similar, with the latter perhaps looking slightly cleaner. In all three cases, the left part of the bottom of the salt is not imaged well; this is probably because this part of the model is poorly illuminated, and, or, the recording time is not long enough to capture the possibly overturned energy illuminating it. Nevertheless, this example shows the power of the Gaussian beam method relative to Kirchhoff migration, and that it can be close to or equal to wavefield extrapolation methods in quality.
The second example is from UK-CS Quad 30 in the Central Graben of the North Sea. The inline shown is from a 3D dataset, and the seismic data corresponds to results obtained with Kirchhoff and Gaussian beam migrations. The subline shown in Figure 4 slices through the edge of what is believed to be an overhanging salt dome shown in the middle of the section. The tertiary data above the salt is again imaged equally well by the two methods because a single arrival is adequate. However, below the complex geology of the salt, the Gaussian beam result looks cleaner. In particular the “smiles” that can seen in “quiet” areas of Kirchhoff migration are much reduced. It is also easier to interpret the edge and the area below the overhanging salt on the Gaussian beam-migrated section.

Conclusions

Gaussian beam migration is an elegant, accurate, and efficient depth migration method. It has the ability to image complicated geologic structures with fidelity exceeding that of single-arrival Kirchhoff migration and approaching that of wave-equation migration. It retains the steep dip imaging and TTI imaging capability of Kirchhoff and has the multi-arrival imaging capability of wavefield extrapolation methods, yet retains the flexibility of Kirchhoff migration, and is less computationally demanding than wavefield extrapolation methods.

We believe Gaussian beam migration is a viable alternative to Kirchhoff depth migration, and is in a position to replace it.

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References


Figure 4. Line through the edge of an overhanging saltdome from a North-Sea, Central Graben, 3D data set. The top part shows the Kirchhoff while the bottom part shows the Gaussian beam migration.