Summary

Migrated common-image-gathers (CIG’s) are of central importance to prestack depth migration. They allow velocity estimation by tomography and other methods, and they allow the analysis of seismic amplitudes in moderately complex structures. Historically, CIG’s have usually been indexed by source-receiver offset, measured along the Earth’s surface. The recent popularity of migration by wavefield extrapolation has given rise to other types of CIG’s. In particular, CIG’s indexed by opening angle at subsurface image locations have advantages over offset-indexed CIG’s. They allow more natural applications of tomography as well as wave-equation migration velocity analysis, and they allow amplitude analysis as a function of opening angle rather than offset (AVA vs. AVO). Another type of CIG, occurring naturally in migrations that use a synthesized linear source, is indexed by the incidence angle of the linear source at the Earth’s surface.

Gaussian beam migration (GBM) can easily produce offset-indexed CIG’s. Also, because common-shot GBM performs local slant stacks on shot records, the slant parameter can be used to identify data with a common emergence angle. This knowledge can be used to produce emergence-angle CIG’s. Finally, knowledge of propagation direction in the subsurface allows GBM to produce CIG’s indexed by subsurface opening angle. I present various ways of producing these types of CIG’s, and I discuss their relative advantages.
Different types of CIG’s

The most popular domain for prestack depth migrated CIG’s has been source-receiver offset 2h: individual migrations are performed, each with a particular small range of offsets, and the traces for the migrations are aligned at each surface location, ranging from near to far offset. For time migration, velocity analysis on these gathers is analogous to velocity analysis on unmigrated CMP gathers. Depth migration velocity estimation based on moveout on CIG’s that are indexed by offset can be either simple (vertical updating) or complicated (tomography). In either of these, a set of flat gathers allows subsequent AVO analysis.

The popularity of offset-indexed CIG’s has been due partly to the prevalence of Kirchhoff migration, which is flexible enough to produce such gathers very easily. CIG’s indexed by some other quantity, however, might be preferable. For example, CIG’s indexed by opening angle at each image location would provide amplitude-vs-angle (AVA) information more readily than offset-indexed CIG’s. They might also allow tomography programs to shoot specular ray pairs more conveniently than they currently can. Also, the rising popularity of migration by wavefield extrapolation (WEM) has come about in spite of its inability to produce offset-indexed CIG’s. Instead, a certain amount of extra work allows WEM to produce CIG’s indexed by *subsurface* offset, which can then be transformed into CIG’s indexed by subsurface opening angle $\theta$.

We might also consider CIG’s indexed by surface incidence or emergence angle $\alpha$, i.e., the angle at the source or receiver location that corresponds by Snell’s law to the subsurface incidence angle at a reflector. While this domain carries no obvious benefit over the others, it is a natural one for WEM methods that synthesize line or planar sources from individual shot records (Zhang et al., 2005). The chief benefit of this approach is, in some situations, to produce complete WEM images more efficiently than common-shot migration.

Figure 1 illustrates the geometric quantities involved in the three types of CIG’s.

![Figure 1: Geometric quantities involved in the three types of CIG.](image)

Migrated offset and angle gathers from Gaussian beam migration (GBM)

Kirchhoff migration can easily produce offset-indexed CIG’s. By considering the surface offset of source and receiver beam centers (Hill, 2001), GBM can produce such CIG’s as well. In doing this, one must take care to account for the small number of migrated beam centers compared with the large number of Kirchhoff migrated traces; failure to do so can result in serious sampling artifacts (amplitude striping) within the CIG’s.

Producing CIG’s indexed by opening angle can be a major challenge for Kirchhoff migration. In cases of moderate structural complexity, the travelt ime tables used by Kirchhoff migration can be well-behaved, with few discontinuities due to jumping from one branch of a multivalued travelt ime field to another. In these cases, it is possible to compute the components of the travelt ime gradient, and to manipulate those derivatives to determine the propagation angles from an image location to both the source and receiver locations. Where the subsurface velocity varies rapidly, this is no longer generally possible. The single-valued travelt ime tables used by standard Kirchhoff migration will become discontinuous, and reliably computing their derivatives to produce ray angles will be difficult or impossible. Then, using these angles in assigning migrated data to different traces in CIG’s will also be difficult. Not so for GBM, whose travelt ime tables are smoothly varying by their nature.
Since it is easy to compute the propagation angle within the Gaussian beams, it is easy to compute the opening angle. Knowing the opening angle at image locations allows one to accumulate the energy into CIG’s indexed by opening angle.

WEM uses a completely different strategy for assigning migrated data to opening-angle CIG’s. After downward continuation of the source and receiver wavefields, WEM images them, not only with the standard (zero subsurface offset) imaging condition, but also at a range of nonzero subsurface offsets. A subsequent transformation from offset to angle produces the desired CIG’s. This is accomplished without ray information, at the extra expense of applying the imaging condition multiple times.

Recently, Sava and Fomel (2006) proposed a different way to perform the same task. Instead of imaging the downward-continued wavefields at a range of nonzero offsets at zero time, they image them at a range of nonzero times at zero offset. This can be accomplished by applying a phase shift to the downward-continued wavefields, which can be more efficient than explicitly applying the imaging condition multiple times.

GBM does not explicitly downward continue wavefields before imaging, so it would appear that it cannot apply a strategy similar to WEM’s. However, Sava and Fomel’s (2006) time-shift imaging condition, implemented as a phase shift, can be applied to the image, whether the image was obtained from WEM, GBM, or even Kirchhoff migration. So this strategy is applicable to both Kirchhoff migration and GBM. Given two different methods to produce subsurface opening-angle CIG’s for GBM, is one of them preferable? I address that question in the next section.

For completeness, the third type of CIG considered here is indexed by surface incidence or emergence angle. Since delayed-shot (i.e., linear source) WEM naturally produces these CIG’s, it is reasonable to consider them even if they do not appear to be as useful as the other two types. Marine-style common-offset, common-azimuth GBM (Hill, 2001) does not retain source or receiver surface angle information in its input gathers, but common-shot or common-receiver GBM does. For example, common-shot migration (Gray, 2005) migrates data from many different receiver beam center locations. At each of these locations, it accumulates data from many different (surface) emergence angles. Keeping track of these angles while imaging allows the migration to bin the output energy into CIG’s indexed by emergence angle. This procedure is slightly simpler than producing subsurface opening-angle CIG’s for GBM, since it relies only on the emergence angles of the receiver rays.

**Examples**

Figure 2 shows examples of the three types of CIG’s produced by GBM. These are taken from a 2-D Canadian Foothills line, and the sets of gathers are all extracted from the same set of locations. All three panels (six CIG’s each) contain the same information, and they all produce the same stacked traces when the traces in the CIG’s are stacked together. The left panel contains CIG’s indexed by surface offsets \(2h\), ranging from 0 m (the rightmost traces within a CIG) to 4500 m (the leftmost traces within the same CIG). The center panel contains CIG’s indexed by surface emergence angle \(\alpha\), ranging from -50° (right) to 50° (left). The right panel contains CIG’s indexed by subsurface opening angle \(\theta\), ranging from 0° (right) to 90° (left).

The left panel is what we normally use to analyze velocities and AVO from prestack depth migration. At shallow depths, the information is restricted to the nearest offsets, making velocity analysis difficult. At greater depths, the presence of nonzero data at medium and far offsets makes it easier to analyze residual moveout for velocity analysis. The situation seems to be reversed for the center and right panels, which both show more live traces at shallow depths than at greater depths. This difference is due to the fact that, at shallow depths, near-offset information contains a wide range of both emergence angles and opening angles while, at great depths, the opposite holds true.
Tomography programs based on shooting specular ray pairs from image locations at depth will perform similarly on the left and right sets of CIG’s, although CIG’s indexed by subsurface opening angle might provide a more natural set of traces to be analyzed when the ray angles match the opening angles. As mentioned above, however, the gathers indexed by surface emergence angle - the center panel - are of limited use. They provide direct input neither into tomographic velocity analysis, which requires either surface offset or subsurface opening angle, nor into AVO/AVA analysis. However, they do give limited information, in the sense that residual moveout of events on the CIG’s indicate velocity errors, and amplitude behavior will predict qualitatively the amplitude behavior on the opening angle gathers.

![Figure 2](image1.png)

Figure 2. Three types of CIG’s for prestack depth migration. CIG’s indexed by: (left) surface offset $2h$; (middle) surface emergence angle $\alpha$; (right) subsurface opening angle $\theta$.

Finally, Figure 3 shows, from the same locations used in Figure 2, the first step in producing subsurface opening angle gathers using the time-shift imaging condition. The center trace in each CIG in Figure 3 is equal to the stack of the corresponding CIG in Figure 2, and the other traces were produced by applying a time shift to the input traces before summing into the image. Because each trace in these CIG’s is equivalent in total effort to producing an entire CIG in any of the panels in Figure 2, using the time-shift imaging condition is less efficient for GBM than producing CIG’s by the other methods. Also, since more work (slant stack plus further velocity-dependent processing) is required to transform this set of CIG’s into a set of subsurface opening angle gathers, there appears to be no compelling reason to produce these CIG’s from GBM as an alternative to the more efficient gathers shown in the right panel of Figure 2.

![Figure 3](image2.png)

Figure 3. The first step in producing subsurface opening angle CIG’s using Sava and Fomel’s time-shift imaging condition. The traces in each CIG are indexed by the time shift applied in the migration. Slant stack plus further processing applied to each CIG will yield CIG’s equivalent to those in the right panel of Figure 2.
Conclusions
Prestack GBM can produce several different types of CIG’s with relatively little effort over that of simply migrating the data. Some of these are difficult or expensive to produce by other migration methods. In that sense, GBM is even more flexible than Kirchhoff migration. Of the different types of CIG’s, those indexed by surface offset or subsurface opening angle appear to be the most useful, the former in standard tomography and AVO analysis, and the latter in opening-angle tomography and AVA analysis, and also in comparisons with WEM.

Of the two approaches to computing opening-angle CIG’s, the straightforward approach based on tracking the propagation angles of the beams during migration appears to be more efficient than the one based on the time-shift imaging condition. It also appears to be more efficient than analogous approaches used by WEM, based on applying the imaging condition over a range of either time shifts or subsurface offsets.

References