TTI wave-equation migration
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Summary

We describe a phase-shift plus interpolation (PSPI) method for wave-equation migration in TTI media. To apply the PSPI methodology for anisotropy, we generate reference operators based upon phase error criteria with respect to the symmetry axis direction, and exploit correlations between parameters. The method is demonstrated on an elastic synthetic dataset generated over a thrust-belt setting, as found in the Canadian Foothills.

Introduction

Many hydrocarbon reservoirs, such as those in the Rocky Mountain Foothills of western Canada, lie below dipping clastic sequences characterized by tilted transverse isotropy (TTI) (Isaac and Lawton, 2004). Several authors (e.g. Vestrum et al., 1999) have shown the importance of accounting for the tilt of the symmetry axis when imaging such reservoirs using anisotropic migration, in order to correctly locate structures laterally. To realize this goal, typically Kirchhoff algorithms have been upgraded to handle TTI. Improved results can be obtained using Gaussian beam TTI migration (Zhu et al., 2005).

As for isotropic migration, superior results for significantly greater effort are expected from the use of wave-equation migration methods on TTI data. In contrast to ray-tracing based methods, wave-equation migration is able to handle multi-pathing in a natural way, and is not based upon a high-frequency approximation to the wave equation.

Shan and Biondi (2005) have demonstrated both 2-D and 3-D implementations of TTI wave-equation migration, using an implicit operator with explicit correction, applied in the space-frequency (x-y-f) domain. For isotropic migration, the Hale-McLellan transform (Hale, 1991) offers an efficient method to produce accurate 3-D responses (without numerical anisotropy) using an x-y-f domain operator. However, since Hale-McLellan is based on circular symmetry, this luxury is not obviously available for TTI, which lacks such symmetry.

An alternative approach to wave-equation migration is based upon applying phase-shift operators in the wavenumber-frequency (k-f) domain. This choice has advantages of operator stability and accurate steep dip behavior. The main drawback, compared to x-f migration, is that lateral variations in the medium are not naturally accommodated by k-f domain operators. For isotropic migration, a number of methods have been proposed to address this issue, including: phase-shift plus interpolation (PSPI), split-step and Fourier finite difference. Generally, all of these are based upon the idea of migrating with a number of reference velocities, and then applying some form of correction to improve the fidelity for lateral velocity variations. We first outline the basic phase-shift operator, and then describe how PSPI and split-step methods can be adapted for the TTI algorithm.

Phase-shift operator

The acoustic VTI approximation (Alkhalifah, 1998) is given by setting the shear-wave velocity parallel to the symmetry axis equal to zero, to obtain the dispersion relation between the horizontal wavenumbers \( k_x \) and \( k_y \), and the vertical wavenumber \( k_z \)

\[
k_z^2 = \frac{\omega^2}{V_{p0}^2} \left( \frac{\omega^2 - V_{m0}^2}{\omega^2 - 2V_{m0}^2} \right)^2 + k_x^2 + k_y^2 - k_0^2,
\]

(1)

where \( V_{p0} \) is the velocity parallel to the symmetry axis (vertical for VTI), \( \eta = \epsilon - 2\delta \) for Thomsen (1986) parameters \( \epsilon \) and \( \delta \), and \( V_{m0} = V_{p0} \sqrt{1 + 2\delta} \).

An acoustic TTI approximation can be obtained by rotating the coordinates of (1). The resulting dispersion is given by a quartic equation (Shan and Biondi, 2005)

\[
A k_x^4 + B k_x^2 k_y^2 + C k_y^4 + D k_x^2 k_y^2 + E k_y^4 = 0,
\]

(2)

where \( A, B, C, D \) and \( E \) are polynomials in \( k_x \) and \( k_y \) of degrees 0, 1, 2, 3 and 4, respectively. There are four solutions to this equation, of which two correspond to the desired P-waves (propagating in opposite directions).

The appropriate P-wave solution is selected from these, and is used recursively to drive the extrapolation of the wavefield from depth \( z \) to \( z + \Delta z \), via

\[
P k_x, k_y, z + \Delta z, \omega = e^{ik_z \Delta z} P k_x, k_y, z, \omega.
\]

(3)

Equation (3) can be used to apply wavefield extrapolation for a medium that varies only with depth, approximating the variation by a series of homogeneous slabs, one for each depth step. In order to address lateral medium
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variation, equation (3) must be incorporated within a PSPI or split-step type of algorithm.

**PSPI for TTI**

The obvious and challenging problem in generalizing PSPI for use with anisotropic media is the so-called “curse of dimensionality”, defined as “the problem caused by the rapid increase in volume associated with adding extra dimensions to a (mathematical) space” (Bellman, 1961). Instead of a single parameter, namely velocity, which can be sampled to define reference operators, we have 3, 4 or 5 parameters depending on whether we are dealing with VTI, TTI in 2-D or TTI in 3-D. We will consider the case of TTI in 2-D, characterized by $V_{P0}$, the velocity parallel to the symmetry axis, $\varepsilon$ and $\delta$, the Thomsen parameters referenced from the symmetry axis, and $\delta_{\theta}$, the tilt of the symmetry axis in the plane of propagation.

At first glance, this 4-D parameter space appears potentially intractable. For example, if 5 values are required to adequately represent variation of each parameter within a depth slice, then the total number of reference operators would be 625 within that slice. However, some simple observations imply that this number may be reduced by at least an order of magnitude. First, in many cases there is strong correlation between different parameters, so that the effective dimensionality of the parameter space is reduced. For example, if $\varepsilon$ and $\delta$ are not assumed to vary independently, it is possible to sample them using a single (possibly curvilinear) axis instead of two axes. In practice, limitations on our ability to estimate $\varepsilon$ often make this assumption necessary. A second helpful observation is that sampling of $\delta_{\theta}$ does not need to be equal or uniform for all values of $\varepsilon$ and $\delta$. Obviously, in the case of isotropy ($\varepsilon = \delta = 0$) one value is enough and the choice of $\delta_{\theta}$ is irrelevant since the operator is independent of $\theta$. More generally, the required sampling of $\delta_{\theta}$ can be computed, for a given phase error, $\Delta \xi$, by using the relationship

$$\Delta \theta_{\xi} = \left( \frac{\partial \xi}{\partial \theta_{\xi}} \right)^{-1} \Delta \xi,$$

where

$$\xi = \sqrt{p_{x}^{2} + p_{\xi}^{2}},$$

with $p_{x}$ and $p_{\xi}$ representing the horizontal and vertical slowness values respectively, such that $k_{x} = \alpha p_{x}$ and $k_{\xi} = \alpha p_{\xi}$. A suitable value for $\Delta \xi$ can be obtained by wavelength considerations. A final observation is that even after computing appropriate combinations of reference parameters using the above, there will still be unused parameter combinations, which may therefore be omitted from the set of reference operators. When combined, these reductions may allow PSPI extrapolation for TTI at an acceptable cost.

Figure 1 illustrates several of the above points. The figure shows the variation of the velocity and anisotropy parameters for a depth slice through an actual Canadian Foothills 2-D depth model. The black circles represent the actual values of the model, whereas the blue asterisks represent the reference operators selected for extrapolation over this step. The model contains a carbonate thrust sheet which causes uplift of the overlying clastic sediments. Both of these are present in this slice, with isotropic high velocities for the carbonates and variable TTI media for the clastics. The Thomsen parameter $\delta$ is not shown, but is highly correlated with $\varepsilon$, allowing representation by one variable only. The isotropic carbonates can be represented by a single choice of symmetry axis (namely vertical), whereas the anisotropic clastics require sampling of the axis according to equation 4. Also, the high velocity carbonates and lower velocity clastics lead naturally to areas of parameter space (e.g. high $V_{P0}$, high $\phi$), which are not populated.

**Split-step correction for TTI**

The accuracy of the operators may be enhanced by application of a split-step correction. The standard split-step correction (Stoffa et al., 1990) can be thought of as a vertical shift to account for the difference between the reference velocity and the actual (x-dependent) velocity. The result is that near-horizontal reflectors are accurately imaged, while dipping reflectors have residual errors. In the presence of TTI, it is often assumed that the symmetry axis is normal to bedding (in fact, this assumption may be needed to determine the axis direction). It is more natural, in this case, to apply the split-step correction parallel to the symmetry axis direction rather than vertically. Tests on model data indicate that this approach is superior for imaging the TTI layers, but not for targets beneath the TTI layers.

**Example**

An elastic finite difference code was used to generate synthetic shot records, for a model based on thrust belt geology found in the Foothills of the Canadian Rockies. The model contains high velocity Mississippian carbonates (pink colours in figure 2), which have caused displacement and tilt of the overlying Cretaceous clastic sediments (purple to green colours), resulting in a complex TTI overburden.

The P- and S-wave velocities and Thomsen parameters $\varepsilon$ and $\delta$ are constant within each geological unit of the model,
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with $\beta = 0.05$ and $\epsilon = 0.1$ for the whole clastic sequence. Both are equal to zero in the carbonates. The symmetry axis is constant within each unit, based on the dominant local structure, giving a variation between $-40^\circ$ and $50^\circ$.

The data were migrated using the TTI migration described above, an isotropic migration, and a PSPI VTI migration. The results are shown in figure 2, overlaid on the velocity model (in colour), with the velocities being those parallel to the symmetry axis (i.e. $V_{p0}$). The TTI migration was performed using the full set of model parameters. The isotropic migration used only the velocities along the symmetry axis direction, and the VTI migration used $V_{p0}$, $\epsilon$, and $\alpha$ but assumed the symmetry axis was vertical.

**Discussion and conclusions**

Figure 2 demonstrates the necessity to properly account for tilted anisotropy, and shows the high fidelity result that can be obtained using wave-equation migration. A Gaussian beam TTI migration on the same data (not shown) achieved similar results, with slightly less clear delineation of the faults. Both the isotropic and VTI migrations misplace events and suffer from poor focusing of the basement reflections. It is interesting to observe that the VTI result is in most places inferior to the isotropic migration. This may be attributed to the fact that the velocity is correct for the normal incidence propagation direction with the isotropic migration, whereas for VTI it is incorrect for all angles. In practice, it is unlikely that the true symmetry axis velocity would be used for either isotropic or VTI type migrations, as it does not necessarily lead to optimal focusing. Nonetheless, the results suggest that performing VTI migration in areas where TTI is clearly called for may be a wasted exercise.

In conclusion, we have described a PSPI type approach to TTI wave-equation migration which has a high fidelity response. The lateral variation of the medium parameters can be accounted for by a careful choice of reference operators to sample the parameter space optimally, using a phase error criterion with respect to the symmetry axis direction. This is combined with a modified split-step correction. Results on an elastic synthetic dataset for a complex thrust-belt model show superior imaging from the TTI algorithm.
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Figure 2: Results of wave-equation migration on elastic modeled data, using: (a) TTI PSPI algorithm described in text; (b) isotropic PSPI migration, and; (c) VTI PSPI migration. All results are overlaid on the velocity model for comparison, ranging from 3400m/s to 6500m/s.
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References


