Migration in the angle domain.

Migration in the angle domain comes as a recent development, Xu et al. (1998), in the long-running story of true amplitude Kirchhoff migration. The classic approach, in the offset domain, for true amplitude migration uses the Beylkin determinant (Beylkin, 1985), which is the Jacobian of the change of coordinates from the surface acquisition (source-receiver couple) to the illuminations vector at the depth point. The Beylkin Jacobian handles the irregularity of illumination due to the propagation in the subsurface, assuming a reasonably regular acquisition. To meet this requirement, some Voronoi binning can be applied beforehand. Other true amplitude approaches perform an explicit regularization of illumination at the depth point: the micro-local inversion in the angle domain of Brandsberg-dahl et al. (1999), the regularization of dip in the offset domain of Albertin et al. (1999), and two different implementations of the regularization of illumination in the dip angle and scattering angle domains, Rousseau et al. (2000) and Audebert et al. (2000). We detail here this latter approach.

A depth point sees migration as the coincidence of source wavefronts coming from a variety of incidence directions, with receiver wavefronts “reflected” at the depth point towards a variety of emergence directions. In the high-frequency limit, these wavefronts can be parameterized by a source isochron and a receiver isochron, each with its associated Green’s function attributes: traveltime, geometrical spreading and traveltime gradient (i.e. slowness vector). Each of the two slowness vectors is characterized by a norm (the local phase slowness) and a direction. Together they define a total of four angles in 3D (two in 2D). The two scattering angles are the scattering opening angle (in single mode, the half-angle between source and receiver rays) and the scattering azimuth angle (azimuth of the plane containing both source and receiver rays). The scattering angles play the role, in the local depth point references, of the acquisition offset and azimuth in the surface-related references. The two illumination dip angles are the angular coordinates (for instance, dip and azimuth) of the illumination slowness vector, the sum of the source and receiver (incidence and reflection) slowness vectors. This vector has the direction of the normal to the reflection isochron.

Restored amplitude imaging by hitcount compensation.

It is natural to apply the explicit regularization of the illumination at the depth point, in angular coordinates, i.e. in both the scattering and illumination dip angle domains. We will thus take naturally into account all regular or irregular illumination effects, originating from either an irregular acquisition or a complex propagation in the subsurface. Rather than compensating for the fold on the acquisition side (in the offset-CMP domain), we will compensate the illumination fold directly in the angle domain at the depth point. And rather than triangulating the illumination fold as in Rousseau et al. (2000), we will restore a constant fold among equal-area bins of illumination dip angles. First, we set the bin area to a constant unit, and then we restore the fold to 1 in each angle bin, since ideally the
imaged point should receive one and only one contribution per unit bin (scattering angles, illumination dip angles). For this, we store the number of contributions per unit bin into a hit-count table. This hit-count will appear in the denominator of the Kirchhoff migration kernel, in lieu of the Beylkin determinant. Fig. 1 summarizes the building of a common scattering angle trace pertaining to a CIG in the angle domain. For the given scattering azimuth \( \phi \) and opening angle \( \theta \), we produce the corresponding common scattering angle gather, in both seismic contribution (Fig. 1, left) and hit-count (Fig. 1, center). We divide the seismic contribution by the hit-count; thus producing the fold-corrected common scattering angle gather (Fig. 1, right). Stacking of this gather produces the \( \theta \) scattering angle trace that will stand in the final AVA gather (Fig. 2). After integration along dip angles, only the portions of the gather of Fig. 1, right, where the seismic response has horizontal tangency, will sum in constructively and constitute the final common scattering angle trace of Fig. 2.

**The Kirchhoff integral in the illumination dip domain.**

After regularization, the common scattering angle gather of Fig. 1, right, represents the best discrete approximation of the continuous Kirchhoff integral in its canonical form: an ideally well sampled and continuous integration upon all illumination dip angles. Nevertheless, this integration is conducted for a constant depth: it is a horizontal summation on the gather of Fig. 1, right. But, within the common scattering angle gather, the imprints or the trajectory of the reflectors appear as smiles in the dip angle direction, not as straight lines. Nevertheless, these smiles remind us of the familiar shape of the migration impulse response.

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**Fig. 1:** Hit-count correction in a common scattering angle gather, \( \Theta = 10^\circ \).

**Fig. 2:** AVA-gather, hit-count corrected.

**Fig. 3:** Horizontal reflector impulse responses.

**Fig. 4:** Angle domain imprint of a flat horizontal reflector. From pink to blue (fanning inwards), scattering opening angle: \( \Theta = \{0^\circ, 60^\circ, 80^\circ\} \).

**Fig. 5:** Angle domain imprint of a flat horizontal reflector. Scattering opening angles \( \Theta = [0^\circ : 90^\circ] \).
Let us consider an isolated flat horizontal reflector, in a constant velocity medium: for the zero scattering angle, all isochrons are circles, but only the isochrons tangent to the horizon (the impulse responses) will carry energy. The circular impulse responses, tangent to the reflector at zero dip and depth $Z_0$, express a simple depth-dip relationship: $Z = Z_0 \cdot \cos(dip)$. The smiley reflector imprint is thus a cosinusoid. For increasing offsets, the circular impulse responses in the space domain become ellipses (Fig. 3). The cosinusoid in the dip angle direction, within a common scattering angle gather, fans in towards zero dip (i.e. the specular dip) when the scattering angle increases (Figs. 4 and 5), because the sum of the dip and scattering angles cannot exceed 90 degrees, in a constant velocity medium. Note that only the portion of the seismic responses with horizontal tangency in the vicinity of the specular dip will survive integration in the dip angle direction. Additional summation upon scattering angles (fig. 5) will further improve resolution.

Let us now consider a flat dipping reflector, for a scattering angle equal to zero. The impulse responses, Fig. 6, are rolling circles of increasing radius. While the circle rolls and passes through a given location, it appears first at zero depth for a dip of $-90$ degrees, then at increasing depth as the dip increases, until a maximum depth is reached, (the actual depth of the reflector) at the specular dip. Beyond the specular dip, the depth decreases with increasing dip. Fig. 7 shows the dipping reflector imprint in the illumination dip angle domain: when the reflector dip increases, the cosinusoid becomes skewed, with horizontal tangency at the specular dip (i.e. the reflector dip).

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Fig. 6: Dipping reflector impulse responses, $\Theta = 0$.

Fig. 7: Angle domain imprint of dipping reflectors. Opening angle $\Theta = 0^\circ$. From pink to blue (left to right) reflector dip $\alpha = \{0^\circ, 30^\circ, 60^\circ\}$.

Fig. 8: Point diffractor impulse responses, $\Theta = 0^\circ$.

Fig. 9: Dip angle domain imprint of a point diffractor. Opening angle $\Theta = 0^\circ$, increasing observation distance, from pink to blue (left to right): $X_{crp}/Z_0 = \{0., 0.1, 1., 2.\}$. 

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Let us consider now a single point diffractor in a constant velocity medium, at zero scattering angle. The impulse responses in the space domain, Fig. 8, are circles of decreasing then increasing radius that roll around the point diffractor. The minimum radius is equal to the diffractor depth, for the zero-offset isochron located right at its apex. We compute theoretical CIG gathers (all superimposed in Fig 9) at increasing distance from the point diffractor apex ($X_{crp} = 0, 0.1, 1, 2$ times the diffractor depth). For the CIG located right upon the point diffractor, all the impulse responses, for any dips, will meet at the point diffractor depth: the imprint in the dip angle domain is a straight line, and the Kirchhoff integration is perfectly constructive. This is the very definition of the Kirchhoff integral: a horizontal integration along the dip angle axis, for a constant scattering angle. Starting from a horizontal line at the apex of the point diffractor, the increasingly steeper curves in fig. 9 correspond to CIGs at increasing distances from the point diffractor (10, 100 and 200 % of its depth). Note that away from the point diffractor, there is no horizontal tangency (no stationary phase) at all.

**Synthetic examples.**

We tested the migration in the angle domain on a canonical synthetic case, with a regular acquisition: a set of horizontal reflectors, with constant, angle-independent reflectivity, in a constant velocity medium (2000m/s). We checked that in this simplistic case, migrations with 1) constant velocity true amplitude weights, 2) Beylkin Jacobian and 3) hitcount compensation, all gave similar results. We illustrate on this case the theoretical aspects examined earlier.

**Conclusion:**

We described a Kirchhoff-type migration in the scattering angle domain, which performs a hitcount compensation in the illumination dip angle. This Kirchhoff migration in the dual scattering angle – illumination dip angle represents the canonical form of the Kirchhoff integral, and as such, gives interesting insight into regularization issues. It offers a practical understanding of constructive and destructive interference, and a visualization of the otherwise abstract concept of stationary phase.

**References:**


