Summary

The combination of AVO/AVA analysis of P(PP)-wave and converted C(PS)-wave data with seismic inversion offers new possibilities for fluid detection and lithology prediction. Unlike most conventional inversion schemes, which use single-mode waves in the inversion process, our inversion method simultaneously reconciles amplitude and time information from both PP-wave and PS-wave data. The inversion method used is a 1-D globally optimized seismic inversion technique using a variant of simulated annealing in which the parameter step length is automatically adjusted during the cooling schedule, ensuring that half of the function evaluations are accepted in one direction. This allows the algorithm to focus on the most promising area and hence increases the accuracy of the optimization result. Tests of this global optimization method on synthetic and real case studies were successful.

Introduction

With the advent of multicomponent sea-floor seismic technology, the study of mode-converted shear-waves has become increasingly common in the industry. However, due to the asymmetrical ray path, conversion point dispersal and polarity reversal, mode-converted waves require a non-standard processing flow and more interpretation effort than single-mode waves. Mode-converted waves retain the benefit of both P- and S-wave surveys and offer the potential for more reservoir characterization and monitoring when they are used simultaneously with PP-wave data in the inversion process. The dual inversion of PP and PS-wave AVO/AVA data provides a powerful means to extract elastic parameters by matching the amplitude and time information of both modes. Over the last years, both local and global optimization methods have been used, with some degrees of success, to invert seismic data in order to extract elastic properties of subsurface rocks. In our case, the objective function is multimodal and the parameter space is very large. It is well known that in such a situation, local optimization methods are prone to be trapped in local minima when the starting model is “far” from the global minimum. The use of simulated annealing, as a global optimization technique, avoids this limitation. This method uses information about the error surface more globally to compute an update to the current model.

Simulated Annealing (SA)

SA is a robust statistical technique, which attempts to solve the problem of finding global extrema to complex optimization problems. The idea comes from the cooling processes of metals, and the way in which liquids freeze and crystallize. Several methods of SA have been used to effectively solve different problems and in different scientific domains. The basic concept of SA used in this work is as follows: each value of the model parameter is sequentially visited and randomly perturbed, while the values of all other parameters remain fixed. At each step, the change in the energy function ($\Delta E$) is calculated. The new model is accepted unconditionally if $\Delta E \leq 0$ (downhill moves). However, if $\Delta E > 0$ (uphill moves), then the new model is accepted according to the Boltzmann probability distribution $P(\Delta E) \approx \text{EXP}(\Delta E / T)$, where $T$ is a control parameter equivalent to the temperature in annealing of crystal-forming material. The entire procedure is repeated for all model parameters. Then the temperature is lowered and the procedure is repeated until “crystallization” occurs, i.e. a low energy state is attained (see figure 2/a). Thus, in SA it is still possible for a worse model to be accepted. In this way, the solution can escape from local minima. A modification made by Goffe et al. (1994) to standard SA methods includes the adjustment of step length during the cooling schedule, in such a way that half of the function...
evaluations are accepted in one direction (see figure 2/b). The decrease in step length with falling temperature allows the algorithm to focus on the most promising area and hence increases the accuracy of optimization results. As the temperature falls, uphill moves are less likely to be accepted, and the percentage of rejections rises. When a set of cost function values for successive stages are less than the error tolerance for termination, the iteration process is stopped. For SA the theoretical convergence to a global minimum has been extensively proven. The system should find the global minimum if the initial temperature, the cooling rates and the number of tries, are set appropriately. Ideally, starting at a high temperature and cooling very slowly guarantees convergence but it takes enormous computing time. To avoid local minima and reduce computing time, SA parameters should be defined by a trial run at the outset. Roughly, the value of $T_0$ should be of the order of the average $\Delta E$ found during the first cycle, ensuring a high accepted/rejected ratio at the start.

Implementation

The “observed” data for the inversion are multicomponent seismic AVO/AVA attributes (intercept $R_{pp}$, gradient $G_{pp}$ and gradient $G_{ps}$), derived from the linearized approximations of PP and PS wave reflection coefficients (see Castagna et al., 1993):

$$R_{pp} = \frac{1}{2} \left( \frac{\delta R_{pp} + \delta \rho}{V_p} \right)$$

$$G_{pp} = \left( \frac{\delta V_p}{2V_p} - \frac{4}{\gamma^2} \frac{\delta V_p}{V_s} - \frac{2}{\gamma^2} \frac{\delta \rho}{\rho} \right)$$

$$G_{ps} = \left( -\frac{2}{\gamma} \frac{\delta V_p}{V_s} - \frac{2 + \gamma}{2\gamma} \frac{\delta \rho}{\rho} \right)$$

By combining these equations, it is possible to perform a robust multicomponent AVO inversion, valid for 1-D elastic isotropic earth models and limited angles of incidence. As shown by Garotta et al. (2002), the first step of this inversion scheme involves finding a finely sampled $\gamma$ model that minimizes the error energy (misfit) between $(\delta \gamma_\gamma)$ derived from transit times and $(\delta \gamma_\gamma)$ derived from seismic amplitudes:

$$\left( \delta \gamma \right)_\gamma = 2 \left( \gamma_i^{\text{obs}} - \gamma_i^\text{a} \right)$$

and

$$\left( \delta \gamma \right)_\gamma = \frac{1}{2} \left[ (4 + \gamma) R_{pp} (t_p^\text{a}) - \gamma G_{pp} (t_p^\text{a}) + (2 + \gamma) G_{ps} (t_p^\text{a}) \right]$$

Optimum $\gamma$ values obtained in the first step are then used to invert equations (1), (2) and (3) for the estimation of $V_p$, $V_s$ and $\rho$. The forward inversion of these equations is not stable and it fails for $\gamma = 2$. To solve the problem, we randomly perturb $V_p$ and $\rho$ parameters (whereas $V_s = V_p / \gamma$), from which we calculate synthetic $R_{pp}$, $G_{pp}$ and $G_{ps}$. The “observed” and “synthetic” AVO attributes (convolved with the seismic wavelet computed from input data) are then compared in a global optimization scheme using SA. The objective function (or error energy) $\Delta F$ that we use contains a least-squares deviation and $a priori$ parameter information constraints. Lower and upper bounds of parameter search region are derived directly from the respective low-frequency trends. In order to ensure the smoothness of the solution, lateral continuation constraints are also imposed during the inversion process. The implementation requires the seismic data to be processed in a true amplitude manner and the log data is available for the calibration. Using the full Knott-Zoeppritz equations does not provide direct insight into the separate contributions made by the different contrast parameters to be estimated, and that a stable inversion of the nonlinear problem is not an easy task. A relationship, involving Thomsen’s (2002) parameters may be used, but this increases the number of unknowns and hence the computing time.

Inversion results

The aim of our inversion scheme is: (1) the estimation of the high-frequency $V_p/V_s$ ratio ($\gamma$) of layers of 1-D Earth models and the generation of PS squeezed traces, (2) the derivation of $V_p$, $V_s$ and density parameters for each layer. The inversion is performed on a trace-by-trace basis. The inversion results may therefore be 2D or 3D, and can also be used to calculate additional lithological volumes, such as Poisson’s ratio, Lamé constants, and fluid factor. All inversion results are in PP-wave time scale and sampled as finely as input seismic data.

Synthetic data example

To illustrate the SA process, we show the inversion of noise-free band-limited synthetic AVO data calculated for a true 1-D Earth model of 150 layers sampled with a layer thickness of 4 ms two-way normal time. This is also the number of $\gamma$ parameters to be inverted. Figure 1 shows the “observed” seismic data (Rpp, Gpp and Gps) obtained by convolving the corresponding synthetic reflectivities (given by formulae (1), (2) and (3)) with a zero phase Ricker wavelet of 30 Hz. The SA technique described above was used to optimize $\gamma$ parameters in the working window from 1.2 to 1.8 s (two-way PP-time). The corresponding initial tie points in the PS time window are at 2.3 and 3.5 s. The starting point of the PS time window may be automatically adjusted during the inversion (by scanning around its initial value). The initial temperature used is 5 and the total number
of cost function evaluations is about 800000 (~ 450000 are accepted moves). Inverted results for $\gamma$ are shown in Figure 1 (right). The evolution of the cost function and the number of accepted and rejected moves for each temperature cycle are shown respectively in Figure 2 (a) and (b). It can be seen that even though the objective function consists of a single component (the squared error energy $\Delta F$), the match between inverted and well $\gamma$ is very good. Adding noise in “observed” data increases the residual error, but does not lead to instabilities during the inversion.

![Synthetic Attributes](image)

**Figure 1.** Left = input AVO attributes: Rpp, Gpp and Gps used for the inversion (in their natural time scale), Gps-squeezed trace with inverted $\gamma$ (PP-time scale), and their power spectra. Right = inversion results: well log $\gamma$, low-frequency trend (reference) $\gamma$, inverted or best $\gamma$, and lower and upper bounds of the parameter search region. Note the remarkable match between well log $\gamma$ (red) and the inverted $\gamma$ (gray filled).

![Objective Function Evolution](image)

**Figure 2.** (a) The objective (cost) function evolution and (b) the number of accepted and rejected moves, for each temperature cycle.

It should be noted that in this case the optimization process for $\gamma$ does not depend on the starting model. One only needs to provide a very rough low-frequency trend used to calculate the lower and upper bounds, restricting the parameters to be perturbed in these regions. However, the inversion results do depend upon the initial temperature and the seed for the random number generator. If nearly the same optimum is found for different seed values, there is a high degree of confidence in the global optimum found. In practice, the model parameterization as finely as the input data sample rate is time intensive. Therefore, alternative means need to be explored such as: sample at larger interval, optimize SA parameters (initial temperature, cooling schedule, etc).

### Real data example

This procedure was successfully applied to invert a 2D line extracted from a set of OBC data recorded in the North Sea. Norsk Hydro conducted the true amplitude processing through the prestack time migration as described by Riste and Fjellanger (2000). PP and PS-wave seismic AVO attributes: intercept Rpp and gradients Gpp, Gps (figure 3) were calculated from the processed data and then inverted for $\gamma$, Vp, Vs and density ($\rho$). Well log data used to calibrate AVO attributes were available from one well close to the line. The reservoir is about 80 m thick. It consists of a series of prograding sand lobes with different porosity and disconnected by thin layers of shale. Figure 4 shows the generated attribute time sections. The initial temperature for the SA is 20 and the number of forward calculations is about 400000. The spatial variation of wavelet frequency is taken into account by estimating it (from input Rpp data) for each processed trace. The low-frequency trend of $\gamma$ is estimated from PP and PS window tie points. Initial models for Vp and $\rho$ are respectively estimated using inverted optimum $\gamma$ values and Gardner’s relationship between velocity and density. The spatial continuity and the good match between the inverted results and well log data confirm the consistency of our inversion.
procedure. The interpretation of the generated inversion results can be used to estimate lithologies. This is done either by interpretation of the inversion results directly or by comparing the relationship of the inverted properties in the well logs.

Figure 3. Input AVO attribute sections (Rpp, Gpp, Gps) used for the inversion in their natural time scale and Gps squeezed section (reverse polarity) in PP-time scale. Their normalized power spectra are shown at the bottom.

Figure 4. Color displays of inverted results for $\gamma$, $V_p$, $V_s$ and $\rho$ with corresponding well logs.

Conclusion

Combined inversion of PP and PS-wave AVO attributes increases the robustness and resolution of $V_p/V_s$ ratio definition. It also discriminates between $V_p$, $V_s$ and density information, hence offering new possibilities for fluid detection and lithology prediction. Matching time and amplitude information from geological events allows the tying of PP and PS-wave data sample by sample. The global optimisation using a form of simulated annealing, in which the step length is automatically adjusted during the cooling schedule, is a robust technique allowing escaping local minima and increasing the accuracy of inversion results. The approach is capable of revealing the high-resolution detail needed for reservoir characterization and lithology prediction.

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References

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