Removing S-wave noise in TTI reverse time migration

Houzhu (James) Zhang, CGGVeritas, Guanquan Zhang, LSEC, Academy of Mathematics & System Sciences, CAS, Yu Zhang, CGGVeritas

Summary

Anisotropy is intrinsically an elastic phenomenon. The widely-used acoustic anisotropic wave equation results in significant shear wave presence in both modeling data and reverse time migration images. In this abstract, we discuss two approaches to reduce the S-wave noise in migration. The first is based on an elastic anisotropic wave equation with non-zero S-wave velocity to attenuate S-wave noise. The second approach uses a supplementary routine to remove the S-waves after waveform modeling in each time step. Impulse responses and a simple 2D TTI example show that both methods reduce S-wave energy significantly and lead to a cleaner image than those based on the conventional acoustic anisotropic wave equation.

Introduction

Reverse Time Migration (RTM) has attracted much attention in the seismic imaging community in recent years. It provides a general imaging framework which images arbitrary dips, automatically handles turning and prismatic waves and naturally deals with anisotropy. When sedimentary layers are not horizontally oriented, such as near salt boundaries, imaging anisotropic structures under the VTI (Vertical Transverse Isotropy) assumption can introduce errors in both traveltime and amplitude. In these cases, TTI RTM can be a better choice.

Conventional TTI wave equation used for reverse time migration is based on the acoustic assumption (Alkhalifah, 2000). Although the resulting equations can be solved efficiently either by a pseudo-spectral (Zhang and Zhang, 2008) or a finite-difference method, there are significant S-waves presence in the images. For surface marine data, since S-waves are not propagated in the water layer where the sources and receivers are located, S-wave noise is not so apparent in images below water bottom. For land and VSP data, there may be strong anisotropy at the surface near the sources or receivers. So S-waves may appear as strong noise in the images.

One way to reduce S-wave noise is to set Thomsen’s parameter, $\varepsilon$ and $\delta$, equal in certain shallow areas (Alkhalifah, 2000; Duveneck et al, 2008). By doing so, the shallow area become elliptical anisotropic media and will not generate S-waves. S-waves become trapped in that thin layer. In spite of the simplicity of this method, it does not provide a general solution for S-wave noise removal and it is difficult to apply to VSP data and RTM for land data.

In this paper, we first discuss reverse time migration using the elastic wave equation. We demonstrate that the strong S-waves in acoustic anisotropy are the natural results of the elastic anisotropic wave equation when the S-wave velocity is very small. By introducing non-zero S-wave velocity during wavefield extrapolation, S-waves can be effectively attenuated (Tsvankin, 2001). Then we propose a set of new equations, based on the eigenvalue analysis of the original acoustic wave equation, to reduce the S-waves in TTI RTM. We describe a numerical procedure to remove the S-waves in 3D heterogeneous media, which is simple and computationally inexpensive.

At the end, we use a simple 2D TTI example to demonstrate that both approaches produce cleaner TTI RTM images.

The acoustic anisotropic wave equation and S-wave noise

The acoustic VTI wave equation was firstly derived by Alkhalifah (2000) by setting the S-wave velocity $v_{s0}$ to zero in the phase-velocity expression

$$\frac{v_s^2(\theta)}{v_{s0}^2} = 1 + \varepsilon \sin^2 \theta - \frac{f}{2} \pm \frac{f}{2} \sqrt{1 + \frac{4\varepsilon^2 \sin^2 \theta}{f^2}(2\delta \cos^2 \theta - \varepsilon \cos 2\theta) + \frac{4\varepsilon^2 \sin^4 \theta}{f^2}},$$

(1)

Where $f = 1 - \frac{\partial}{\partial \theta} v_{s0}^2$ and $v_{s0}$ is the S-wave velocity in the vertical direction.

It was found that even for zero $v_{s0}$, the diamond-shaped S-waves are still present in the modeled wavefield (Zhang et al, 2003; Grechka et al, 2004) and therefore, show up in RTM images as noise. The reason is that although the S-wave velocity is zero in the vertical direction, the S-wave phase velocity in other directions may still be non-zero. More importantly, the S-wave group velocity is not zero. Figure 1 shows the S-wave (inner red curves) and P-wave group velocities (outer red curves) for different $v_{s0}$’s. The corresponding black curves are the phase velocity. They are computed from the Christoffel equation of the elastodynamics (Cerveny, 2001). Compared with P-
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wavefront, the S-wavefront is much more complicated, especially when \( v_s \) is small. Figure 3(a) shows the reverse time migration of an impulse. The amplitude of the S-wave noise becomes stronger as anisotropy factor (the difference between \( \varepsilon \) and \( \delta \)) increases. To verify this, Figure 2 shows the corresponding wavefield by elastic anisotropic modeling, whose theoretical aspects will be discussed in detail below.

In the following sections, we first discuss modeling and migration using an elastic anisotropic wave equation in which S-wave noise is attenuated automatically during imaging computation. Then we propose a method based on the eigenvalue analysis of the original acoustic anisotropic wave equation to remove the S-waves. It computes the characteristic waveform from the already-propagated wavefield during each imaging-time step.

S-wave noises suppression using elastic anisotropic wave equation

The first order dynamic wave equations (Carcione, 1995) are widely used for modeling wave propagation in any physical media. In 2D, they are

\[
\begin{align*}
\frac{\partial^2 \sigma_{11}}{\partial t^2} & = \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{13} + f_1}{\partial x_3}, \\
\frac{\partial^2 \sigma_{13}}{\partial t^2} & = \frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{33} + f_3}{\partial x_3},
\end{align*}
\]

(2)

and

\[
\begin{align*}
\frac{\partial}{\partial t} \begin{bmatrix} \sigma_{11} \\ \sigma_{13} \end{bmatrix} & = \begin{bmatrix} C_{11} & C_{13} \\ C_{13} & C_{33} \end{bmatrix} \begin{bmatrix} \partial v_1/\partial x_1 \\ \partial v_3/\partial x_3 \end{bmatrix}, \\
\frac{\partial}{\partial t} \begin{bmatrix} \sigma_{13} \\ \sigma_{33} \end{bmatrix} & = \begin{bmatrix} C_{15} & C_{15} \\ C_{35} & C_{35} \end{bmatrix} \begin{bmatrix} \partial v_1/\partial x_1 + \partial v_3/\partial x_3 \end{bmatrix},
\end{align*}
\]

(3)

where \( C_{ij} \) are the elastic constants, \( \sigma_{ij} \) are the stress components, \( v_i \) are the particle velocity components and \( f_i \) are the components of body forces. Equation (2) and (3) can be solved by finite-difference on a staggered-grid (Ramos-Martínez et al, 2000). For VTI and TTI media, \( C_{ij} \) can be derived from Thomsen’s parameters (Thomsen, 1986).

System (2) and (3) are frequently used for multi-component modeling and imaging since they include all types of waves. Here, instead of using (2) and (3) for multi-component imaging, we use the same system for acoustic pressure imaging, where the acoustic pressure is defined as

\[
p = (\sigma_{11} + \sigma_{33})/2.
\]

(4)

Obviously, \( p \) contains both \( P \) and \( S \) waves. For TTI media, the strength of S-waves depends on the value of \( \varepsilon = (v_{po}/v_{so})^2(\varepsilon - \delta) \): smaller \( v_{so} \) gives bigger S-wave amplitude (Tsvankin, 2001). By decreasing \( v_{so} \), less distortion happens to the S-wave and the amplitude anomaly decreases. Figure 2 shows the elastic wavefield for different values of \( v_{so} \). It shows that the amplitude of the S-waves decreases when \( v_{so} \) increases and \( v_{so} \) does not significantly impact the kinematics and dynamics of P-waves. Therefore, the S-wave artifacts can be attenuated by adding non-zero \( v_{so} \) to the elastic equation system.

Removal of S-waves based on Eigenvalue-analysis

We start from the following 2D acoustic VTI wave equation (Zhou and Bloor, 2006)

\[
\begin{align*}
\frac{1}{V_0^2} \frac{\partial^2 p}{\partial t^2} & = (1 + 2\delta)\nabla^2_1 (p + q) + \nabla^2_2 p, \\
\frac{1}{V_0^2} \frac{\partial^2 q}{\partial t^2} & = 2(\varepsilon - \delta)\nabla^2_1 (p + q).
\end{align*}
\]

(5)

For a constant velocity, equation (5) can be rewritten in the following matrix form in the \((\alpha k)\) domain

\[
\begin{bmatrix}
V_0^2 \alpha^2 & 0 \\
0 & V_0^2 \beta^2
\end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} \alpha k_1^2 + k_2^2 & \alpha k_1^2 \\ k_1^2 \beta & k_1^2 \beta \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \Lambda \begin{bmatrix} p \\ q \end{bmatrix}
\]

(6)

where

\[
\alpha = 1 + 2\delta, \beta = 2(\varepsilon - \delta).
\]

Equation (6) represents an eigenvalue problem of matrix \( \Lambda \). Both \( p \) and \( q \) are combinations of the P-wave and S-waves. To obtain a wavefield which contains P-wave or S-wave only, we compute the eigenvalues and eigenvectors of matrix \( \Lambda \) and define the characteristic waveform by

\[
(\bar{p}, \bar{q}) = X^{-1}(p, q)^T.
\]

(8)

where \( X \) is the matrix whose column vectors are the eigenvectors of \( \Lambda \). After some algebraic manipulations, we derive the following expression for the P-wave characteristic waveform

\[
\bar{p} = p + q \frac{\alpha k_1^2}{(\alpha k_1^2 + k_2^2)}.
\]

(9)

Assuming \(|q|\) is small, equation (9) can be implemented numerically based on the following scheme,

\[
\bar{p}(x) = FT_{k_1}^{-1}\left\{ \frac{1}{k^2} FT_k \tilde{p} \right\}.
\]

(10)

Where \( H \) is the right hand side term of the first equation in (5) and \( FT \) stands for spatial Fourier Transform.

For 3D TTI, we use

\[
\bar{p} = p + q \frac{\alpha k_1^2}{(\alpha k_1^2 + k_2^2)}.
\]

(11)

where \( k_1 \) is the projection of wavenumber vector in the symmetry direction and \( k_{so}^2 = k_1^2 - k_2^2 \). Equation (10) remains unchanged in 3D. In a TTI modeling or migration,
the projection filter (10) or (11) can be applied to the computed wavefields at each output step.

A 2D TTI example

In this section, we use a simple 2D example to verify the methods outlined above. The velocity model is shown in Figure 4(a) with the values of $\varepsilon$, $\delta$ and symmetry axis overlaid. Since $\varepsilon,\delta=0.249$, strong S-waves are generated due to the strong anisotropy. This can be clearly seen in Figure 4(b) which is based on the acoustic TTI wave equation and with no effort to remove the S-waves. The linear noise is due to the diamond-shaped S-waves which tilt 45° to the right (see also Figure 1 and 2).

Since the anisotropic layer is homogeneous, we cannot apply the thin-layer-isotropy trick here. We proceed with the two approaches discussed in the previous sections. The images from elastic anisotropic wave equation are shown in Figure 5(a), 5(b) and 5(c) and those from acoustic anisotropic wave equation but applied S-wave removal techniques are shown in Figure 5(d). Both methods give very clean images. The images for different S-wave velocities for elastic wave equation are very similar. This indicates RTM based on elastic wave equation is robust. The numerical procedure based on the acoustic anisotropic wave equation with S-wave removal is straightforward and can be quickly added to the existing TTI RTM routine.

Conclusions

We have presented two approaches to attenuate S-wave noise in anisotropic reverse time migration. The first method is based on the most general elastic anisotropic wave equation and has the capability to image the structures and automatically attenuate S-wave noise in more general media, such as orthorhombic media. The second approach provides an efficient and practical way to remove the S-wave noise in the current acoustic VTI and TTI reverse time migration.

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Figure 1: S-wave (Inner red curve) and P-wave (Outer red curve) group velocity for different values of $v_s$. (a): $v_s=1500$ m/s; (b): $v_s=500$ m/s; (c): $v_s=1$ m/s. $v_p=1500$ m/s, $\varepsilon=0.2$, and $\delta=-0.2$. The corresponding black curves are the phase velocity.

Figure 2: Wavefield snapshots using elastic anisotropic wave equation modeling. Here $v_s=1500$ m/s and $v_p$ varies. (a): $v_p=1000$ m/s; (b): $v_p=500$ m/s; (c): $v_p=1$ m/s. As $v_s$ increases, the S-wave distortion decreases.
Figure 3: 2D impulse responses of reverse time migration based on an acoustic anisotropic wave equation. (a): Without S-wave removal; (b): After S-wave removed based on Equation (10).

Figure 4: (a): 2D TTI model; (b) Reverse time migration images based on the acoustic anisotropic wave equation with no S-wave removal.

Figure 5: Reverse time migration images based on the elastic anisotropic wave equation: (a) \( v_{s0} = 1 \text{m/s} \); (b) \( v_{s0} = 500 \text{m/s} \); (c): \( v_{s0} \) is computed by assuming Poisson’s ration is 0.25. (d) Reverse time migration images based on the acoustic anisotropic wave equation after S-wave removed based on Equation (10).