TTI Wave-Equation Migration for Canadian Foothills Imaging

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In order to achieve good imaging in areas such as the Canadian Foothills, there are several key issues which must be addressed: the presence of very rough surface topography; structural complexity; and anisotropy with a tilted symmetry axis (TTI). To image structural complexity with greater accuracy than we can typically accomplish at present one must move beyond ray-based Kirchhoff methods to techniques employing a full wavefield description, such as wave-equation migration (WEM), and these techniques must be extended to handle propagation through TTI media and migration from topography.

This paper reports on the development of such technology and its application to Foothills data from the Canadian Rocky Mountains region. A methodological advance of this nature is of considerable interest in exploration, as it permits improved imaging of the complex targets typical of a Foothills play and should therefore lead to improved interpretation and better drilling results.

Background

Wave equations are essential to all migration methods. Sometimes they are solved very approximately, as in the ray-tracing of Kirchhoff methods, while other times they are solved to produce full wavefields, as in wave-equation migration. In either case, the complexity of the wave equation depends upon the earth model employed, which may range from a simple acoustic isotropic model to one incorporating varying degrees of anisotropy, anelasticity, etc. Ideally one will use the simplest possible model which still incorporates the essential features necessary to properly image the subsurface. For the Foothills environment, an acoustic TTI model is currently perceived as producing a good balance of accuracy and computational tractability. Here we review the concepts of TTI media and of wave-equation migration that underlie our development of TTI WEM.

Tilted Transverse Isotropy (TTI): Figure 1 shows a photograph taken along the Trans-Canada highway in Alberta which reveals a set of dipping shales, tilted as a result of tectonic forces during formation of the Rocky Mountains. This gives rise to a tilted transversely isotropic system. Transversely isotropic implies that velocities depend only on the angle relative to a symmetry axis. Vertical transversely isotropic (VTI) is a special case for a vertical symmetry axis. Tilted transversely isotropic (TTI) means that the symmetry axis is non-vertical. The angle of the symmetry axis is shown in Figure 1 as $\theta_s$, and is perpendicular to bedding. Velocities are lowest along the axis and highest parallel to bedding. As in the case of VTI, the actual variation of velocity can be described by two Thomsen parameters $\varepsilon$ and $\delta$, except that for TTI they are measured relative to the axis instead of relative to vertical. Parameter $\varepsilon$ essentially measures the fractional difference between velocities parallel and perpendicular to bedding. The effect of parameter $\delta$ is more complicated, but characterizes variation at intermediate angles. It has the most impact on the phase velocity at approximately 45°.
Figure 1. Photograph of dipping shales in Canadian Foothills, taken from Trans-Canada highway

The importance of properly handling anisotropy is illustrated in Figure 2. As is well known, neglect of anisotropy is one source of mis-ties between well logs and seismic data. For a VTI medium, as illustrated in the second picture, this failure tends to mean that data are imaged with a vertical mislocation. This typically means the driller may reach the target shallower or deeper than expected. However, in the presence of TTI, the error is more serious. The image is laterally (as well as vertically) shifted, meaning that the well may completely miss the target, or require a sidetrack. This issue is now commonly addressed in the Canadian Foothills using a TTI form of Kirchhoff migration.
Wave equation migration (WEM): Kirchhoff migration in other structural areas has been largely superseded by wave-equation migration (WEM) or even reverse time migration (RTM). These methods are based upon a more direct application of the wave equation to the data, without using the high-frequency approximation of Kirchhoff migration. This approximation is inherent in the ray-tracing basis of Kirchhoff migration, which also limits its ability to handle multi-pathing. In contrast, WEM (or RTM) handles multi-pathing of energy naturally, as illustrated in Figure 3. For these reasons, WEM generally delivers a superior image, albeit usually for somewhat greater computational effort.
The term wave-equation migration (WEM) is typically used (some would say misused) to describe algorithms which operate by recursive wavefield extrapolation. The wavefield at depth $z$ is extrapolated downwards by a small step $\Delta z$, based upon a homogeneous solution to the wave equation. This is done recursively until all steps are completed. The extrapolation is applied to both shot and receiver wavefields, which are then combined to form the image at each depth step.

The simplest method for doing this extrapolation is by phase shift. A spatial Fourier transform is applied to the wavefield at depth $z$ to get it into the wavenumber domain. Then a phase shift is applied to the wavefield $P$, using

$$P \mathbf{u}_x, k, z + \Delta z, \omega \approx e^{ik \Delta z} P \mathbf{u}_x, k, z, \omega,$$

which makes use of a dispersion relation between the horizontal wavenumbers, $k_x$ and $k_y$, and the vertical wavenumber, $k_z$. The temporal frequency is given by $\omega$.

The approach of using phase-shift operators, as in equation (1), has advantages of operator stability and accurate steep-dip behavior. The main drawback, compared to space-frequency migration, is that lateral variations in the medium are not naturally accommodated by wavenumber domain operators.
For isotropic migration, a number of methods have been proposed to address this issue, including: phase-shift plus interpolation (PSPI), split-step and Fourier finite difference. Generally, all of these are based upon the idea of migrating with a number of reference velocities, and then applying some form of correction to improve the fidelity for lateral velocity variations.

Why has WEM not been used extensively for Foothills imaging? There are a number of reasons.

First, the extension of WEM to properly handle TTI has only recently been investigated. There are significant challenges to each of the possible WEM methods for TTI. Second, Kirchhoff migration naturally handles migration from topography, whereas this is more complex for WEM. Third, there are often noise issues for land data which are conveniently addressed by muting angle gathers after Kirchhoff migration, but less readily so with WEM. Fourth, Foothills data are not usually acquired on the very dense, regular spatial grid required for WEM. Finally, and related to the sampling issue and the TTI issue, WEM is more computationally-intensive than Kirchhoff. In spite of these obstacles, the potential advantages in the use of WEM for imaging in Foothills environments have motivated the development which is described in the remainder of this article.

A 2-D Foothills model: As an introduction to the promise of TTI WEM for Foothills data, consider Figure 4. The model (Figure 4a) is based on a real geological profile, and broadly consists of lower velocity clastic sediments which have
Figure 4. (a) Foothills 2D model for generation of synthetic datasets using an elastic finite-difference algorithm. Comparison of (b) TTI Kirchhoff migration and (c) TTI WEM migration. We observe an improvement in the WEM imaging over the Kirchhoff near the popup and the carbonate sheet below it.

been deformed to create a TTI system, overlying an isotropic region dominated by high-velocity carbonates. The black line shows the boundary between them. The direction of the symmetry axis is indicated by the black arrows. They are approximately normal to bedding, though they do not vary within the geologic units. Synthetic data were generated using a 2D elastic finite-difference algorithm.

The main imaging goal in this model is to delineate structures which lie within the carbonates. In the Kirchhoff image, which is generally good, there are some areas of ambiguity; under the low-velocity sheet embedded in the carbonates there appears a possible second horizon which does not exist in the model. The area around the popup is also ill-defined. Both are better resolved by the WEM. The basement reflections are also more continuous.

Method
We describe here a form of PSPI migration adapted for TTI media (Bale et al., 2007; Du, 2007). Other methods for implementing TTI WEM have been explored and these have been reviewed elsewhere (Bale et al., 2007). The basic phase shift operator is given by equation (1). This must be supplemented by a relation of the form $k_z = k_z(k_x, k_y, \omega)$, known as a dispersion relation. For isotropic media, this is the familiar expression $k_z = \sqrt{\left(\frac{\omega}{V}\right)^2 - k_x^2 - k_y^2}$, where $V$ is the velocity. For transverse isotropy, the dispersion relation is far more complicated. An important and accurate approximation to it is the “acoustic” TI approximation, introduced for VTI by Alkhalifah (1998), and given as

$$k_z^2 = \frac{\omega^2}{V_{p0}^2} - \frac{\omega^2 - V_{nmo}^2}{\omega^2 - 2V_{nmo}^2} + 2\eta \frac{\hat{\mathbf{\eta}} + k_z^2}{\hat{\mathbf{\eta}} + k_z^2},$$

(2)

where $V_{p0}$ is the velocity parallel to the symmetry axis (vertical for VTI), $\eta = \hat{\mathbf{\eta}} - \delta \hat{\mathbf{\eta}} + 2\eta \hat{\mathbf{\eta}}$ for Thomsen parameters $\varepsilon$ and $\delta$, and $V_{nmo} = V_{p0}\sqrt{1+2\delta}$. Rotating the coordinates of the system gives the corresponding relation for TTI which is a quartic equation in $k_z$. Two of the solutions are spurious S-wave solutions and the remaining two are the up- and down-going P-wave solutions of interest (Bale, 2007).

The main challenge of TTI WEM using PSPI is the so-called “curse of dimensionality”. TTI migration in 3D involves five parameters: $V_{p0}$, the velocity parallel to the symmetry axis, $\varepsilon$ and $\delta$, the Thomsen parameters referenced from the symmetry axis, and $\theta_s$ and $\phi_s$, the tilt and azimuth respectively of the TTI symmetry axis. As shown in Figure 5, this dramatically increases the number of reference states, and hence the computational burden. For example, isotropic migration using six reference velocities would be quite efficient. For TTI, if six values are required to characterize velocity, and each of the other parameters, then we have $6^5 = 7776$ reference operators to apply! Thus TTI WEM would be orders of magnitude slower than isotropic WEM. In practice, we have used various strategies to significantly reduce the actual requirements to a manageable level. Such strategies in fact constitute the fundamental design framework for the development of this technology.

(a)

| V=2000 m/s | V=3000 m/s | V=4000 m/s | V=5000 m/s | V=6000 m/s |

(b)
Figure 5: (a) Five possible reference states are illustrated for an isotropic WEM calculation, representing the range of velocities in the velocity model. Propagation from $z$ to $z+\Delta z$ is carried out at each constant velocity, and these results are then interpolated in the spatial domain as guided by the true velocity model. (b) In a TTI WEM calculation, the reference state is defined, not by a single velocity value, but by a unique combination of velocity parameters. Thus, for example, five vertical velocities and three tilt angles would give rise initially to 15 reference states as shown. However, each of these would in turn give rise to many more states by including all possible combinations of azimuth, $\alpha$, and $\delta$. This results in an intractable calculation for current computing resources.

While extension to TTI is the principal technical challenge in this method, it is also nontrivial to allow migration from topography, and this is necessary for it to play a role in imaging Foothills geology. Our approach to topography is a modification of methods advocated by Reshef (1991) and Ng (2008). We have elsewhere described how these methods may be implemented using the concept of a “zero slowness” layer (Ursenbach et al., 2009).

Data example

The Copton Foothills 3D seismic survey is a component of the CGGVeritas Data Library and covers complex Alberta Foothills geology between the Smoky and Narraway rivers, approximately 15 kilometers northwest of Grande Cache. We have chosen a subset of the survey in the south consisting of 34 shot lines, and 32 receiver lines covering an area of approximately 400 sq. km to run our migration
tests. The input data were first regularized with respect to receivers using the proprietary 5D interpolation technique described in Trad (2007), so that each shot record has a properly sampled receiver domain. The original receiver locations and the regularized receiver locations prepared for migration are shown in Figure 6. The shot locations were not regularized.

![Figure 6: Receiver locations and receiver fold (colour) for (a) original receivers, and (b) regularized receivers. The vertical black line in (b) shows the approximate location of Crossline 720 displayed in Figure 7, while the horizontal line shows the location of Inline 400 displayed in Figure 8.](image)

The data were migrated using the TTI WEM algorithm and a TTI Kirchhoff algorithm for comparison. The results on one crossline (parallel to receiver lines) are shown in Figure 7. The TTI model which was used for both migrations contains significant tilt of the symmetry axis - up to 67°. It has constant Thomsen parameters $\varepsilon=0.12$ and $\delta=0.06$ in the dipping, clastic overburden, above the Nordegg. Below the Nordegg, the model is isotropic for the high-velocity carbonates. Output bin sizes were 70m x 35m.

The migrations included frequencies up to 80 Hz. The cost of WEM increases at least quadratically as the maximum frequency increases, whereas the cost of Kirchhoff increases linearly with maximum frequency, and, in the past, this has limited the useable frequencies to be obtained from WEM. However, with increases in computational performance, this difference is less significant today.

The WEM result at and below the Nordegg shows improvements over the Kirchhoff image, as indicated by the solid ellipses. Generally it is expected that WEM illuminates structurally complicated areas better than Kirchhoff, since it uses the full wavefield, whereas single-arrival Kirchhoff migration relies upon raypaths which may have “shadows” i.e. areas the rays do not reach due to the complexity of the velocity model. This expectation seems to be borne out by the example shown here. The WEM image is also less noisy below the complex Nordegg horizon. Overall, the application of TTI WEM in a Foothills environment should thus serve to increase confidence in trap identification.
Figure 7: A cross-line (#720) image from the 3D Copton survey, showing a comparison of (a) TTI Kirchhoff migration with (b) TTI wave-equation migration. Both migrations included frequencies up to 80 Hz. Solid yellow ellipses indicate areas where WEM appears superior to Kirchhoff. The dashed ellipse shows an example where Kirchhoff is superior. Note that the black arrow at the bottom of (a) shows the location of the inline displayed in Figure 8.

Large-angle noise in the near surface

In contrast to the results at depth, the Kirchhoff migration produces a slightly better imaged result for the shallow regions, as shown by the dashed ellipses in Figure 7. This is because events in the image near the surface are vulnerable to corruption by high-angle arms of the migration operator. These are conveniently dealt with in Kirchhoff migration by muting of common image gathers, which are straightforward to generate for Kirchhoff migration, whereas this is possible but more costly for WEM. In this example, the WEM result was directly imaged without any post-migration mutes applied, while image-gather mutes were applied to the Kirchhoff image.

While the routine generation of common image gathers is not currently practical for WEM, other approaches can be used to remove large-angle noise from the near surface. We have previously shown, for instance, that improvement of the near surface can be obtained very efficiently through slowness-filtering during the migration (Ursenbach et al., 2009). Here we present results obtained instead by muting migrated shot gathers prior to stacking. Applying such mutes significantly reduces these artefacts to show shallow events more clearly, as seen in the inline images in Figure 8.
Figure 8: An inline (#400) image from the 3D Copton survey. The (a) and (b) panels are taken from the same image volumes as Figure 7a and 7b. Panel (c) is the same as (b) except that muting has been applied to the migrated shots prior to stacking. The shallow events are much clearer after muting. Note that the black arrow at the bottom of (a) shows the location of the crossline displayed in Figure 7.

**Conclusions**

We have developed a TTI WEM algorithm which is suitable for imaging complex Foothills structures, and it has been tested on a structured, 3D dataset with topography. This new method can be employed at frequency ranges typical of Kirchhoff migration to yield improved images at depth with accurate depth control. The application of shallow mutes to migrated shots prior to stacking also produces acceptable near-surface images.

The overall improvement of TTI WEM migration relative to TTI Kirchhoff migration is to be expected based on a more accurate treatment of the wavefield, which is able to describe the multi-pathing effects so prevalent in seismic responses from complex structured media. That these theoretical expectations appear to be realized in practice is encouraging for exploration in the Foothills.
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Suggested Reading


Ng, M., [2008], A fast and accurate migration from topography via coarse step downward wavefield extrapolation. 78th Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, SPMI P2.4.

