Promises and superiority of reverse time migration

Yu Zhang, CGGVeritas
Outline

- **Introduction**
  - Review: From Kirchhoff to RTM
  - Why do we prefer RTM?
  - Advantages and issues
- RTM: issues and solutions
- Some real data examples
- Conclusion and Discussion
Subsalt Imaging Technology Life Cycle

- **Birth**: Kirchhoff 1999
Ray approximation: Kirchhoff migration

**Wave equation**

\[
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) G_0 = 0
\]

**High frequency approximation**

\[ G_0 = A(x)e^{i\omega \tau(x)} \]

**Eikonal equation for traveltime**

\[ \left| \nabla \tau \right|^2 = \frac{1}{v^2} \]

**Transport equation for amplitude**

\[ 2\nabla A \cdot \nabla \tau + A \Delta \tau = 0 \]
Kirchhoff to Gaussian Beam migration

Kirchhoff migration (single arrival)

Gaussian beam migration (multiple arrivals)
High frequency approximation

Kirchhoff

One-way WEM
One way decomposition: OWEM

Wave equation

\[
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) G_0 = 0
\]

Dowgoing wave:

\[
\left( \frac{\partial}{\partial z} + \frac{i \omega}{v} \sqrt{1 + \frac{v^2}{\omega^2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)} \right) D = 0
\]

Upgoing wave:

\[
\left( \frac{\partial}{\partial z} - \frac{i \omega}{v} \sqrt{1 + \frac{v^2}{\omega^2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)} \right) U = 0
\]
From OWEM to Turning Wave OWEM

Conventional OWEM

Turning wave OWEM
(Zhang et al. 2006)
OWEM vs TW-OWEM

Conventional OWEM

Turning wave OWEM
OWEM algorithm comparison

20 velocity PSPI  

90° true amplitude FD
Two-velocity test: PSPI

\[ v = 4500 \text{ m/s} \]

\[ v = 2000 \text{ m/s} \]
Two-velocity test: true amplitude FD

v=4500 m/s

v=2000 m/s
Two-velocity test: NSPS

$v=4500\text{m/s}$

$v=2000\text{m/s}$
PSPI vs NSPS
## One-way wave equation migration algorithms

<table>
<thead>
<tr>
<th>Domain</th>
<th>algorithms</th>
<th>variations</th>
<th>comments</th>
</tr>
</thead>
</table>
| X      | Implicit FD (Claerbout, 1972) | 1. 15º (Claerbout and Doherty, 1972), 45º (Berkhout, 1979), high order (Ma, 1981; Lee and Suh, 1985; Zhang, 1988)  
2. Li’s correction (Li, 1991)  
3. Multiple-way splitting  
   1. square bin (Collino and Joly, 1995; Ristow and Ruhl, 1997)  
   2. non-square bin (Zhang et al., 2004)  
|        | Explicit (Holberg, 1988) | 1. Squared bin (Hale, 1991)  
| X-K    | SSF (Stoffa et al, 1990) |  | 5º accuracy |
|        | FFD (Ristow and Ruhl, 1994) | 1. High order  
2. Multiple-way splitting | Dispersion, dip limitation and 3D solver |
|        | PSPI (Gazdag, 1984) | NSPS (Margrave and Ferguson, 1997) | Many reference velocities Strong velocity contrast |
|        | GS (Le Rousseau and De Hoop, 2001) |  | Unstable |
|        | Stable x-k (Zhang, 2003) |  | Stable? |
|        | OSA (Chen and Liu, 2006) |  | Strong velocity contrast? |
One way decomposition: OWEM

Wave equation
\[
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) G_0 = 0
\]

Downgoing wave:
\[
\left( \frac{\partial}{\partial z} + \frac{i \omega}{v} \sqrt{1 + \frac{v^2}{\omega^2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)} \right) D = 0
\]

Upgoing wave:
\[
\left( \frac{\partial}{\partial z} - \frac{i \omega}{v} \sqrt{1 + \frac{v^2}{\omega^2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)} \right) U = 0
\]
Rigorous square-root operator definition


\[ \sqrt{1 + \frac{v^2}{\omega^2} \left( \partial_x^2 + \partial_y^2 \right)} = 1 + \sum_{l=1}^{N} \frac{\alpha_{N,l} \left[ (v \partial_x)^2 + (v \partial_y)^2 \right]}{\omega^2 + \beta_{N,l} \left[ (v \partial_x)^2 + (v \partial_y)^2 \right]} \]

**Pseudo-PDE operator**: need many PDEs to approximate

**Singularity**: The convergence cannot be fast
Directly solve wave equation: RTM

Wave equation

\[
\frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) G_0 = 0
\]

Turning wave OWEM

RTM
Imaging complex structures – turning wave

One-way migration

Reverse time migration
Imaging complex structures – prismatic wave

One-way migration

Reverse time migration
Imaging complex structures – interbed multiple
Imaging complex structures – a shot RTM

Single shot record

RTM image
Imaging complex structures – refraction

Single shot record

RTM image
VTI RTM: VSP Image

Image of transmission waves
Marine data – Better subsalt illumination

OWEM

RTM
RTM-Sed Flood Migration – turning wave

1-way WEM

RTM

WAZ: Auger, Garden Banks
Garden Banks WAZ
Wave propagation
Wave propagation

T=4.000s
Why we prefer reverse time migration?

**Reverse Time Migration**
- is straightforward to implement
- can image turning wave, prismatic wave, multiple, refraction, etc.
- has no dip limitations in imaging
- handles strong velocity variation and complex structures
- is the most accurate propagation operator
- carries correct propagation amplitude
- is applicable for land, marine, OBC/OBS, VSP, WAZ data and anisotropy
- has no additional cost for WAZ imaging
Outline

- Introduction
- **RTM: issues and solutions**
  - True amplitude RTM
  - Finite difference implementation
  - Noise removal
  - TTI RTM
- Some real data examples
- Conclusion and Discussion
Conventional RTM formulation

Source (t increases)

\[ \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \right) p_F(x; t) = \delta(x - x_s) f(t) \]

Receiver (t decreases)

\[ \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \right) p_B(x; t) = 0 \]

\[ p_B(z = 0) = Q(x, t) \]

Imaging condition

\[ R(x) = \int p_B(x; t)p_F(x; t)dt \quad \text{or} \quad R(x) = \int p_B(x; t)p_F^{-1}(x; t)dt \]
True-amplitude common-shot RTM

Source (t increases)
\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \right) p_F(\bar{x}; t) = \delta(\bar{x} - \bar{x}_s) f(t)
\]

Receiver (t decreases)
\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \right) p_B(\bar{x}, t) = 0
\]
\[
p_B(z = 0) = Q(\bar{x}, t)
\]

Imaging condition (Zhang et al., 2004)
\[
R(\bar{x}; x_s, y_s) = \int p_B(\bar{x}; t) p_F^{-1}(\bar{x}; t) dt
\]
True-amplitude common-angle RTM

Source (t increases)

\[
\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta\right) p_F(\vec{x}; t) = 0
\]

\[
p_F(z = 0) = \delta(\vec{x} - \vec{x}_s) \int_0^t f(s) ds
\]

Receiver (t decreases)

\[
\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta\right) p_B(\vec{x}; t) = 0
\]

\[
p_B(z = 0) = Q(\vec{x}, t)
\]

Imaging condition (Zhang et al., 2007)

\[
R(\vec{x}; \theta) = \int p_B(\vec{x}; t) p_F(\vec{x}; t) dt
\]
Input, offset=15,000m, v=2,000+0.3z
True-amplitude Common-angle Gather Output

Subsurface angle

AVA Curves

Reflection angle (degree)

Amplitude

Depth:
- 1 km
- 2 km
- 3 km
- 4 km
- 5 km
3D True-amplitude angle/azimuth RTM

Source (t increases)

\[
\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta\right) p_F(\vec{x}; t) = 0
\]

\[
p_F(z = 0) = \delta(\vec{x} - \vec{x}_s) \int_0^t f(s) ds
\]

Receiver (t decreases)

\[
\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta\right) p_B(\vec{x}; t) = 0
\]

\[
p_B(z = 0) = Q(\vec{x}, t)
\]

Imaging condition (Xu and Zhang, 2009)

\[
R(\vec{x}; \theta, \phi) = \int \frac{v(\vec{x})}{\sin \theta} p_B(\vec{x}; t) p_F(\vec{x}; t) dt
\]
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Temporal derivatives: Explicit or implicit FD

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) p(\bar{x}; t) = 0
\]

<table>
<thead>
<tr>
<th>Method</th>
<th>Stability</th>
<th>Dispersion</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Explicit FD</td>
<td>Conditional</td>
<td>Yes</td>
<td>Straightforward, but with small time step</td>
</tr>
<tr>
<td>2. Implicit FD</td>
<td>Guaranteed</td>
<td>Yes</td>
<td>Mostly need iterations to solve the big matrix</td>
</tr>
<tr>
<td>3. One-step extrapolation</td>
<td>Guaranteed</td>
<td>No</td>
<td>Time step can be as big as possible</td>
</tr>
</tbody>
</table>
Two-step Explicit method

One step explicit formulation (Tal-Ezer, 1987; Etgen, 1989):

\[ p(t + \Delta t) + p(t - \Delta t) - 2 \cos(\Delta t \sqrt{-\Delta}) p(t) = 0 \]

Find an optimized approximation (Soubaras and Zhang, 2008)

\[ \cos(\Delta t \sqrt{-\Delta}) \approx \sum_{n=0}^{N} a_n \left( -v^2 (\Delta t)^2 k^2 \Delta \right)^n \]

For

\[ vk \leq 2\pi f_{\text{max}} \]

and

\[ \Delta t \leq \frac{1}{2f_{\text{max}}} \]
Spatial derivatives: FD or PS

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) \right) p(\tilde{x}; t) = 0
\]

Pseudo-spectrum (exact):

\[
\frac{\partial^2 p}{\partial x^2} = FFT^{-1}\left(-k^2 FFT\right) p
\]

Finite difference (approximate):

2nd order:

\[
\frac{\partial^2 p_m}{\partial x^2} \approx \frac{p_{m+1} - 2p_m + p_{m-1}}{\Delta x^2}
\]

4th order:

\[
\frac{\partial^2 p_m}{\partial x^2} \approx \frac{-p_{m+2} + 16p_{m+1} - 30p_m + 16p_{m-1} - p_{m-2}}{12\Delta x^2}
\]

6th order:

\[
\frac{\partial^2 p_m}{\partial x^2} \approx \frac{2p_{m+3} - 27p_{m+2} + 270p_{m+1} - 490p_m + 270p_{m-1} - 27p_{m-2} + 2p_{m-3}}{180\Delta x^2}
\]

8th order:

\[
\frac{\partial^2 p_m}{\partial x^2} \approx \frac{-9p_{m+4} + 128p_{m+3} - 1008p_{m+2} + 8064p_{m+1} - 14350p_m + 8064p_{m-1} - 1008p_{m-2} + 128p_{m-3} - 9p_{m-4}}{5040\Delta x^2}
\]
Finite difference schemes: 8th order

\[ \frac{d^2}{dx^2} \quad \frac{d}{dx} \]
Finite difference schemes

\[ \frac{d^2}{dx^2} \]

\[ \frac{d}{dx} \]
New finite difference scheme
Comparison: PS vs FD

- Pseudo-spectrum
- 8th order optimized FD
  - dz deduced by 30%
Comparison: PS vs FD

Pseudo-spectrum

New FD
dz deduced by 10%
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Raw output from RTM
Why migration artifacts?

\[ t = t_s + t_r \]
Solution 1: Angle mute

\[ \theta = 90^\circ \]

\[ \theta < 90^\circ \]
RTM Subsurface Angle Gathers

Courtesy of BP
Solution 1: angle mute (stack of 0°-60°)
Solution 2: Laplacian filter

Apply a Laplacian filter to the stacked image

\[- \Delta R(\bar{x})\]

Because

\[k_x^2 + k_y^2 + k_z^2 = 4 \frac{\omega^2}{v^2} \cos^2 \theta\]

Zhang and Sun, 2009
Solution 2: Laplacian filter
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  - **TTI RTM**
- **Some real data examples**
- **Conclusion and Discussion**
Back to elastic VTI equation

Elastic VTI equation:

\[
\begin{align*}
\frac{\partial^2 u_x}{\partial t^2} &= \frac{\partial}{\partial x} \left[ \nu_0^2 \left( (1 + 2\varepsilon) \frac{\partial u_x}{\partial x} + (1 + 2\varepsilon) \frac{\partial u_y}{\partial y} + \sqrt{1 + 2\delta} \frac{\partial u_z}{\partial z} \right) \right] \\
\frac{\partial^2 u_y}{\partial t^2} &= \frac{\partial}{\partial y} \left[ \nu_0^2 \left( (1 + 2\varepsilon) \frac{\partial u_x}{\partial x} + (1 + 2\varepsilon) \frac{\partial u_y}{\partial y} + \sqrt{1 + 2\delta} \frac{\partial u_z}{\partial z} \right) \right] \\
\frac{\partial^2 u_z}{\partial t^2} &= \frac{\partial}{\partial z} \left[ \nu_0^2 \left( \sqrt{1 + 2\delta} \frac{\partial u_x}{\partial x} + \sqrt{1 + 2\delta} \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) \right]
\end{align*}
\]

Horizontal stress: \( p = \nu_0^2 (1 + 2\varepsilon) \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) + \nu_0^2 \sqrt{1 + 2\delta} \frac{\partial u_z}{\partial z}, \)

Vertical stress: \( r = \nu_0^2 \sqrt{1 + 2\delta} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) + \nu_0^2 \frac{\partial u_z}{\partial z}, \)
Stable acoustic VTI/TTI equations

Acoustic VTI equation (Duveneck et al., 2008):

\[
\frac{1}{v_0^2} \frac{\partial^2}{\partial t^2} \left( \frac{p}{r} \right) = \left( \frac{1}{\sqrt{1+2\varepsilon}} \begin{pmatrix} \frac{\partial^2}{\partial z^2} & 0 \\ 0 & \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \end{pmatrix} \right) \left( \begin{pmatrix} \sqrt{1+2\delta} \\ 1+2\varepsilon \end{pmatrix} \right) \left( \begin{pmatrix} \frac{\partial^2}{\partial x^2} \\ \frac{\partial^2}{\partial y^2} \end{pmatrix} \right) \left( \frac{p}{r} \right)
\]

A stable TTI formulation (Zhang and Zhang, 2009)

\[
\frac{1}{v_0^2} \frac{\partial^2}{\partial t^2} \left( \frac{p}{r} \right) = \left( \frac{1}{\sqrt{1+2\varepsilon}} \begin{pmatrix} \frac{\partial^2}{\partial x^2} \\ \frac{\partial^2}{\partial y^2} \end{pmatrix} \right) \left( \begin{pmatrix} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \\ \frac{\partial^2}{\partial z^2} \end{pmatrix} \right) \left( \begin{pmatrix} D_{z,z}^T & D_{z,z}' \\ D_{x,x}^T & D_{x,y}^T + D_{y,y}^T \end{pmatrix} \right) \left( \frac{p}{r} \right)
\]
Anisotropic Imaging: Green Canyon Huang et al. 2009

- Dual Azimuth Orthogonal shooting
  - East-West Survey
  - North-South Survey

Top of salt
Provide simpler velocity structures
Flat gathers for different azimuths

<table>
<thead>
<tr>
<th>90°</th>
<th>0°</th>
<th>Semblance</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO</td>
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<tbody>
<tr>
<td>TTI</td>
<td></td>
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</tbody>
</table>
Improve salt geometry and subsalt imaging
Outline

- Introduction
- RTM: issues and solutions
- **Some real data examples**
  - Kirchhoff/OWEM/RTM
  - Beam/RTM
  - ISO RTM/TTI RTM
  - VTI RTM/TTI RTM
- Conclusion and Discussion
Imaging improvement - Mississippi Canyon

Walker Ridge WAZ, Bigfoot Area
Walker Ridge: ISO vs TTI RTM
Walker Ridge WAZ: VTI vs TTI RTM
Summary

Development of depth imaging

- Single arrival Kirchhoff migration cannot provide satisfactory subsalt imaging
- Ray based beam migrations suffer from high frequency approximation, they assume uncomplicated velocity and wave propagation
- One-way wave equations are singular pseudo-PDEs, geophysicists have struggled more than 30 years for a fast, accurate and dip unlimited solution
- Reverse time migration images steep-dips and provides superior subsalt images
Summary

- Stable true-amplitude inversion formulations have been proposed for RTM.
- Angle gathers can be output for AVA and model building.
- Spatial and temporal dispersion can be overcome by pseudo-spectral or accurate FD methods.
- Migration artifacts can be removed by an angle gather mute or a Laplacian filter.
- Stable TTI RTM has been developed and provides better image with WAZ acquisition.
Subsalt Imaging Technology Life Cycle

- TTI RTM 2008
- RTM 2007
- CBM 2005
- 1-way WEM 2002
- Kirchhoff 1999

Full waveform inversion?
Wave-EQ tomo?
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