Direct downward continuation from topography using explicit wavefield extrapolation

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ABSTRACT

Downward-continuation migration algorithms are powerful tools for imaging complicated subsurface structures. However, they usually assume that extrapolation proceeds from a flat surface, whereas most land surveys are acquired over irregular surfaces. Our method downward continues data directly from topography using a recursive space-frequency explicit wavefield-extrapolation method. The algorithm typically handles strong lateral velocity variations by using the velocity value at each spatial position to build the wavefield extrapolator in which the depth step usually is kept fixed. To accommodate topographic variations, we build space-frequency wavefield extrapolators with laterally variable depth steps (LVDS). At each spatial location, the difference between topography and extrapolation depth is used to determine the depth step. We use the velocity and topographic values at each spatial lateral position to build extrapolators. The LVDS approach does not add more data nor does it require preprocessing prior to extrapolation. We implemented the LVDS method and applied it to a source profile poststack migration technique. We also implemented the previously developed zero-velocity layer approach to use for comparison. For both algorithms, we modeled the acoustic source as an approximate free-space Green’s function, not as a simple extrapolated spatial impulse. Tests on a synthetic data set confirmed the method’s effectiveness in imaging shallow and deep structures beneath rugged topography.

INTRODUCTION

Many recursive explicit space-frequency wavefield extrapolation methods are approximations to the generalized phase shift plus interpolation (GPSPI) algorithm (Margrave and Ferguson, 1999), the limiting form of the phase-shift plus interpolation (PSPI) method (Gazdag and Squazzero, 1984). Also, GPSPI reduces to the phase-shift algorithm (Gazdag, 1978; Claerbout 1985) when the velocity is constant. At each output point, the wavefield is computed explicitly using an operator specified by the velocity at the output location.

Unlike ray-based algorithms, e.g., Kirchhoff methods, these downward-continuation approaches usually assume that extrapolation proceeds from one horizontal surface to the next, so they cannot directly handle data recorded from irregular topographic surfaces over which most land surveys are acquired. Further, they are often computationally more expensive than ray-based methods, and current approaches that deal with topography typically increase their computational cost.

One of the oldest topographic approaches is to static shift data (i.e., apply constant time shifts to individual traces) to a horizontal datum followed by downward continuation. This approach is inaccurate for nonvertically traveling energy and can produce artifacts in the shallow section after migration (Gray, 1997). Ji and Claerbout (1992) and Bevc (1997) use a more accurate approach by upward-continuing the data to the highest elevation using wave-equation datuming (Berryhill, 1979) prior to migration, which needs more computational effort than merely static shifting the data.

The zero-velocity layer approach is more efficient than wave-equation datuming (Beasley and Lynn, 1992; Gray, 1997) but still requires some processing before migration. In that approach, data are static shifted to a datum above or equal to the highest elevation before migration. The wavefield extrapolation then is carried out by assuming a zero velocity (for diffraction effects only) between the datum and recording surface. The velocity used to calculate the static shifts is also used in the thin-lens term of the extrapolator.

In a similar approach, Reshef (1991) proposes a variable depth-step approach, and Margrave and Yao (2000) use a method in the nonstationary phase-shift algorithm (Margrave and Ferguson, 1999) to extrapolate zero-offset data directly from topography. In these references, the zero-velocity layer method and the variable depth step...
approach are demonstrated only for stacked data. Shragge (2008) presents a novel approach in which downward continuation was implemented in Riemannian coordinates in the presence of topography.

In this paper, we show how space-frequency explicit wavefield extrapolation can be used to downward continue source record data directly from topography by building wavefield extrapolators with laterally variable depth steps. We call our approach the laterally variable depth step (LVDS) method. We first derive source profile migration using space-frequency explicit wavefield extrapolation. This formulation helps understand the derivation of the LVDS approach. Then we compare LVDS and the zero-velocity layer approaches using a synthetic data set modeled from a rough topography. Because the zero-velocity layer approach is derived only for zero-offset data, we describe its extension to prestack data in Appendix B.

**SPACE-FREQUENCY EXPLICIT WAVEFIELD EXTRAPOLATION**

In this section, we derive the theory of source profile migration using space-frequency explicit wavefield extrapolation. This algorithm is then modified to extrapolate data directly from topography.

In source-profile migration, two wavefields are needed — upgoing and downgoing — and the reflectivity at any depth is estimated in a recursive sense. Subscript \( z \) denotes the depth below some reference plane where receivers are placed. If the recording surface has topographic variations, then \( z \) depends on \( x \).

Assume, for the moment, that the sources and receivers are placed on the horizontal surface \( z = 0 \). The Fourier transform of the upgoing wavefield \( u_0(x,t) \) over the temporal coordinate \( t \) is

\[
U_0(x,\omega) = \int_0^\infty u_0(x,t) \exp(-i\omega t) dt, \tag{1}
\]

where \( \omega \) is the temporal angular frequency. Then the 2D Green’s function for a homogeneous unbounded medium can be used to calculate the downgoing wavefield or the simulated source wavefield at the first depth level \( D_{n\Delta z}(x,\omega) \). (We do not merely extrapolate a spatial impulse at the source location; see Appendix A for details.)

Downward continuing the upgoing wavefield and forward modeling the downgoing wavefield to depth \( z = N\Delta z \) in a recursive sequence (recursive means the output of one step is the input to the next step) of \( N \) steps can be described as

\[
U_{N\Delta z}(x,\omega) = \left( \prod_{n=1}^N W_{n\Delta z} \right) U_0(x,\omega) \tag{2}
\]

and

\[
D_{N\Delta z}(x,\omega) = \left( \prod_{n=2}^N W_{n\Delta z}^* \right) D_{\Delta z}(x,\omega), \tag{3}
\]

where \( W_{n\Delta z} \) is an operator that extrapolates a wavefield from depth level \( (n-1)\Delta z \) to depth level \( n\Delta z \) and \( ^* \) is the complex conjugate.

The cascade of operators \( \prod_{n=1}^N W_{n\Delta z} \) can be decomposed into

\[
\prod_{n=1}^N W_{n\Delta z} = W_{N\Delta z} \circ W_{(N-1)\Delta z} \circ \cdots \circ W_{\Delta z}. \tag{4}
\]

In expression 4, \( W_{n\Delta z} \) indicates an operation on the wavefield, and \( \circ \) denotes operator composition. In a one-way wavefield extrapolation, the \( n \)th operator depends only on the depth-averaged value \( V_n(x) \) over the interval \( z \in [(n-1)\Delta z, n\Delta z] \) of velocity \( V_f(z) \) in the lateral aperture of the operator. The explicit operation of \( W_{n\Delta z} \) is as a spatial convolution over the lateral coordinates \( [x \in \text{two dimensions}, (x,y) \text{in three}] \), where the convolution becomes nonstationary if velocity depends upon the lateral coordinates in any step. In a 2D setting, downward continuing the upgoing wavefield from \( z = (n-1)\Delta z \) to \( z = n\Delta z \) is expressed as

\[
U_{n\Delta z}(x,\omega) = W_{n\Delta z} U_{(n-1)\Delta z} = \frac{1}{2\pi} \int_{|x-x'|\leq \xi} U_{(n-1)\Delta z}(x',\omega) \times W_{n\Delta z}(x-x',k_n(x))dx', \tag{5}
\]

where \( \xi \) is the length of the aperture (i.e., spatial extent or support) of the operator and \( |x-x'| \leq \xi \) denotes that the integration domain can be limited if the operator has compact support (i.e., finite aperture). Similarly, forward modeling the downgoing wavefield is expressed as

\[
D_{n\Delta z}(x,\omega) = W_{n\Delta z}^* D_{(n-1)\Delta z} = \frac{1}{2\pi} \int_{|x-x'|\leq \xi} D_{(n-1)\Delta z}(x',\omega) \times W_{n\Delta z}^*(x-x',k_n(x))dx'. \tag{6}
\]

Theoretically, \( W_{n\Delta z} \) does not have compact support, but any practical approximation of it does (e.g., Thorbecke et al., 2004; Margrave et al., 2006). The expression \( W_{n\Delta z} \) is given explicitly by

\[
W_{n\Delta z}(x,k_n(x)) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \hat{W}_{n\Delta z}(k_x,k_n(x)) \exp(-ik_x x) dk_x, \tag{7}
\]

where \( k_s \) is the lateral wavenumber. The kernel of equation 7 is expressed as

\[
\hat{W}_{n\Delta z}(k_x,k_n(x)) = \exp(i k_z k_n(x) \Delta z), \tag{8}
\]

with the vertical wavenumber \( k_z \) defined by

\[
k_z(k_x,k_n(x)) = \begin{cases} \sqrt{k_n^2(x) - k_{x}^2}, & |k_n(x)| > |k_x| \\ i \sqrt{k_n^2(x) - k_{x}^2}, & |k_n(x)| < |k_x|. \end{cases} \tag{9}
\]

Finally, the magnitude of the wavenumber vector for the \( n \)th step is \( k_n(x) = \omega / V_n(x) \). The convolution in equations 5 and 6 becomes nonstationary when \( V_n(x) \) is not constant and is stationary otherwise. Lateral velocity variations are accommodated by allowing the operator to depend explicitly on the velocity at each output point (Berkhout, 1981; Holberg 1988; Hale, 1991), as stated in equation 9. As mentioned, for
practical implementation of equations 5 and 6, the infinitely long operator W needs to be approximated with a compactly supported operator that is numerically stable.

There are different methods for designing practical stable operators, e.g., Hale (1991), Soubaras (1996), Thorbecke et al. (2004), and Margrave et al. (2006). In this paper, the optimized forward operator and conjugate inverse (FOCI) algorithm (Al-Saleh and Margrave, 2006; Margrave et al., 2006) is used to approximate the infinitely long theoretical operators. The prestack deep migrated (PSDM) image is then obtained by invoking the crosscorrelation imaging condition (Claerbout, 1971):

\[
I(x, z) = 2 \text{ Re} \left[ \int_{0}^{\omega_{\text{max}}} U_r(x, \omega) D^s \left( x, \omega \right) d\omega \right],
\]

where \( \omega_{\text{max}} \) is the maximum signal frequency. Equation 10 estimates the migrated image for a single source record. A common approach is to average similar estimates from all sources in a survey.

LATERALLY VARYING DEPTH STEP (LVDS) APPROACH

In a 2D setting, the upcoming wavefield for a particular source location is the surface recorded data \( U_{hs}(x, \omega) \), where \( h(x) \) is a positive function describing the topography as a curve below the datum, which is \( z = 0 \) (Figure 1). We assume that the highest point on the topography corresponds to \( z = 0 \).

Consider a source at \( x = x_s \), and \( z = h(x_s) \) (Figure 2). Generally, \( h(x_s) \) will fall between two depth levels on the extrapolation grid (i.e., the set of depths \( \{0, \Delta z, 2\Delta z, \ldots, N\Delta z\} \)). We call these levels \( s \Delta z \) and \( (s + 1) \Delta z \) for some integer \( s \), so that \( s \Delta z \leq h(x_s) < (s + 1) \Delta z \). Because \( h(x) \) presumably varies smoothly, there will be an area around \( x_s \), say, \( \sigma \), for which \( h(x) < (s + 1) \Delta z \) for \( x \in \sigma \). For points in this area, we approximate the source wavefield using the 2D Green’s function for a homogeneous unbounded medium:

\[
D_{(s+1)\Delta z}(x, \omega) = \begin{cases} 
G(x_s, h(x_s), x, (s + 1) \Delta z, V(x), \omega), & x \in \sigma \\
0, & \text{otherwise}
\end{cases}
\]

where \( G(x_s, h(x_s), x, (s + 1) \Delta z, V(x), \omega) \) denotes the 2D free-space Green’s function for a source at \( (x_s, z_s) \) evaluated at \( (x, z) \) with the local velocity \( V(x) \) and angular frequency \( \omega \). Equation 11 implicitly allows moderate lateral velocity variation in a way consistent with GPSPI (Margrave and Ferguson, 1999).

The explicit analytic form of \( G(x_s, h(x_s), x, (s + 1) \Delta z, V(x), \omega) \) for the Helmholz equation can be found in many texts (e.g., Morse and Feshbach, 1953; Zauderer, 1989) and is

\[
G(x_s, h(x_s), x, (s + 1) \Delta z, V(x), \omega) = \frac{i}{4} H^{(0)}_0 \left( \frac{\omega r}{V(x)} \right),
\]

where \( H^{(0)}_0 \) symbolizes the zero-order Hankel function of the first kind and \( r = \sqrt{(x - x_s)^2 + (h(x_s) - (s + 1) \Delta z)^2} \). In equation 11, the source wavefield is evaluated as nonzero only at those lateral coordinates for which the source depth is less than \( (s + 1) \Delta z \) (see Figure 2). We discuss the use of a Green’s function as a source in Appendix A.

Source-profile migration can be implemented directly from topography by modifying the wavefield extrapolators to handle topographic and lateral velocity variations. For this discussion, we treat the recorded wavefield as if it exists at datum \( (z = 0) \), and we build an extrapolator that will downward continue that wavefield correctly at all depths below the actual recording surface. Downward continuing the recorded data \( U_{hs}(x, \omega) \) to the first depth level, with respect to the datum \( z = 0 \), is described by

\[
U_{\Delta z}(x, \omega) = \frac{1}{2\pi} \int_{|x - x'| \leq \xi} U_{hs}(x', \omega) \Theta_{\Delta z}(x - x', k_1(x)) dx',
\]

where \( \Theta_{\Delta z} \) is a modified wavefield extrapolator. Similarly, forward modeling the downgoing wavefield to the second depth level below the source is described as

\[
D_{(s+2)\Delta z}(x, \omega) = \frac{1}{2\pi} \int_{|x - x'| \leq \xi} D_{(s + 1)\Delta z}(x', \omega) \times \Theta^*_{(s + 2)\Delta z}(x - x', k_{s + 2}(x)) dx'.
\]

Figure 2. A representation of the topographic surface \( h(x) \) containing a source located at the coordinates \( x_s \) and \( h(x_s) \). The source at \( h(x_s) \) will fall between two depth levels such that \( s \Delta z \leq h(x_s) < (s + 1) \Delta z \), where \( \sigma \) is an area around \( x_s \) for which \( h(x) < (s + 1) \Delta z \) for \( x \in \sigma \).
The modified wavefield extrapolator $\Theta_{\Delta z}$ is constructed so that when the extrapolation depth is above the topography, $\Theta_{\Delta z}$ is an identity operator. It otherwise accomplishes a wavefield extrapolation with a depth step that might vary laterally. We define $\Theta_{\Delta z}$ as

$$
\Theta_{n\Delta z}(x - x', k_n(x)) = \begin{cases} 
\delta(x - x'); & \gamma(x) \leq 0 \\
W_{n\Delta z}(x - x', k_n(x)); & \text{otherwise}
\end{cases}
$$

(15)

where

$$
\gamma(x) = n\Delta z - h(x)
$$

(16)

is the negative of the depth of the topography relative to the extrapolation depth. $\gamma$ is negative if the topography is below the extrapolation depth (i.e., when $n\Delta z < h(x)$), and the laterally variable depth step is given by

$$
\Delta\tilde{z}(x) = \begin{cases} 
\gamma(x); & 0 < \gamma(x) < \Delta z \\
\Delta z; & \text{otherwise}
\end{cases}
$$

(17)

Equation 15 defines $\Theta_{\Delta z}$ to be a Dirac delta function, the identity operation under convolution, when the extrapolation depth is above the topography. This is equivalent to taking a depth step of zero. So from the analogy of a marching scheme, the data start marching downward from the highest elevation, and any particular trace marches in place until the extrapolation depth moves below the trace elevation (Al-Saleh et al., 2006); then it starts marching downward.

Thus, in equation 13, $U_{\Delta z}(x, \omega)$ equals the input data $U_{\delta}(x, \omega)$ for those $x$-coordinates where the extrapolation depth is equal to or above the topography. When the topography is above the extrapolation depth, $\Theta_{\Delta z}$ is equivalent to $W_{n\Delta z}(x, \omega)$ defined by equation 7. The depth step will vary laterally, as defined by equation 17, when the vertical distance from the topography to the extrapolation depth is less than $\Delta z$ but greater than zero. This laterally variable depth step causes no additional complications in equation 7, which is already adapted to handle the lateral phase variations caused by velocity variations. The variable depth step merely introduces additional lateral phase variation. When the extrapolation depth is entirely below the topography, then $\Theta_{\Delta z} = W_{\Delta z}$ and the LVDS method becomes identical to ordinary space-frequency wavefield extrapolation.

Using equations 13 and 14 simultaneously allows source-profile extrapolation to be implemented directly from topography. The LVDS approach can also be extended to the 3D case using wavefield extrapolators built with a lateral depth step $\Delta\tilde{z}(x, y)$. The full derivation involves no additional difficulties beyond those encountered in formulating explicit 3D wavefield extrapolation. The topography, the velocity model, and the depth step become functions of two lateral coordinates instead of one.

**EXAMPLES**

The Aruma data set was generated at Saudi Aramco using an acoustic finite-difference modeling program. The data set consists of 626 sources, and the acquisition geometry has a roll on/roll off at the beginning and end of the line, where the number of receivers per source profile ranges from 188 to 375, depending on the lateral position of the source. The source and receiver intervals are 16 m, and after downward continuation but before invoking the imaging conditions, source spacing and receiver spacing were interpolated to 8 m. The spatial extent of the wavefield extrapolator is 25 points (400 m), designed using the optimized FOCI algorithm (Al-Saleh and Margrave, 2006; Margrave et al., 2006). The elevation profile of this data set is shown in Figure 3 where the vertical axis is exaggerated to show the rough topography. Figure 4 shows the velocity model. Figure 5a illustrates a raw source gather as recorded from the surface where reflection events are distorted by the rough topography, and Figure 5b is the zero-offset unmigrated section. Reflections are greatly distorted by the rough topography and do not suggest the geology of the subsurface.

![Figure 3. Elevation profile of the Aruma data set.](image)

![Figure 4. Velocity model of the Aruma data set. The velocity above the topography has been set to 6000 m/s to make the topography visible.](image)
Figures 6 and 7 show the source-profile PSDM images using the zero-velocity layer (Appendix B) and LVDS approaches where the two results are comparable. However, the reflectors at about 400 and 550 m have better continuity in the LVDS result than in the zero-velocity layer image (Figures 8a and 9a, see arrows). Also, the LVDS approach more effectively resolves the edges of the shallow channels at about 700 m than the zero-velocity layer approach. The deep structures are well resolved in both results (Figures 8b and 9b), where the reflectors suffer less from the elevation related statics. The LVDS result, however, has slightly better resolution of the reflectors at about 1900 m than the other approach (Figures 8b and 9b). These examples demonstrate the effectiveness of LVDS in extrapolating data directly from an irregular surface.

![Figure 5.](image1.png)  
(a) Source profile at a lateral location 5000 m. (b) Unmerged zero-offset section of the Aruma data set.

![Figure 6.](image2.png)  
PSDM image using source profile migration with the zero-velocity layer approach. Boxes indicate zoom view locations shown in Figure 8a and b.

![Figure 7.](image3.png)  
PSDM image using source profile migration with LVDS. Boxes indicate zoom view parts of Figure 9a and b.

![Figure 8.](image4.png)  
Close-up views of (a) shallow and (b) deep sections of Figure 6. Arrows show areas for comparison with the LVDS image in Figure 9.
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APPENDIX A

SOURCE REPRESENTATION BY THE GREEN’S FUNCTION

In equation 11, we introduce our use of the Green’s function for the scalar Helmholtz equation as a model for the seismic source. We avoid the common practice of extrapolating a discrete impulse because, as discussed by Wapenaar (1990) and Zhang et al. (2005), this is known to be correct only in its kinematics and not in amplitude. Our procedure is consistent with that advocated by those authors but perhaps simpler. We work in three dimensions because the mathematics is more transparent, but our conclusions hold when specialized to two dimensions.

First, we establish that the extrapolation of a unit impulse is not a proper source model by explicitly comparing the analytic form of such an extrapolation with the plane-wave decomposition of the free-space Green’s function. The latter can be found in many texts (e.g., Aki and Richards, 2002, p. 192) where it is called the Weyl integral. For a source at the origin, the Weyl integral is given by

\[ G(0, x) = (x, y, z), \nu, \omega) \]

\[ = \frac{1}{R} \exp(i \omega R / V) \]

\[ = \frac{1}{2 \pi} \int_{-\infty}^{\infty} \exp\left(\frac{i(k_z z - k_x x - k_y y)}{i k_z}\right) dk_x dk_y, \]

(A-1)

where \( R = |x| \) is the distance from the source and

\[ k_z = \begin{cases} \sqrt{\frac{\omega^2}{v^2} - k_x^2 - k_y^2}, & \omega^2 > k_x^2 + k_y^2 \\ \sqrt{k_x^2 + k_y^2 - \frac{\omega^2}{v^2}}, & \text{otherwise} \end{cases} \]

(A-2)

On the other hand, the extrapolation of a unit impulse is, by definition, just the impulse response of the wavefield extrapolator; this is given by

\[ W_{3D}(x, \nu, \omega) = \frac{1}{4 \pi^2} \int_{-\infty}^{\infty} \exp(ik_z (z - k_x x - k_y y)) dk_x dk_y. \]

(A-3)

comparison of equations A-1 and A-3 gives the fundamental result

\[ W_{3D}(x, \nu, \omega) = \frac{1}{2 \pi} \frac{\partial}{\partial z} G(x, 0, \nu, \omega). \]

(A-4)
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The factor of $ik$, that appears in the denominator of equation A-1 and not in equation A-3 amounts to a division by $\cos \theta$ in the plane-wave domain ($\theta$ being propagation angle). Although a Green’s function is, by definition, a model of a source, the extrapolation of a unit impulse gives a source model that is effectively suppressed at large angles by a factor of $\cos \theta$. The Weyl integral representation is implied by equation 14b in Wapenaar (1990) and equation 15 in Zhang et al. (2005) but is not stated explicitly in either paper. The denominator in equation A-1 becomes zero at the evanescent boundary, implying difficulties for a direct numerical evaluation. We avoid this by direct evaluation of the Green’s function at the first depth level below the actual source level, also avoiding division by $R = 0$. Equation A-4 is found in Robinson and Silvia (1981, pp. 371–373).

In summary, a source model superior to the extrapolation of a unit impulse is obtained by direct numerical evaluation of the free-space Green’s function at the first depth level below the source.

APPENDIX B

THE ZERO-VELOCITY LAYER APPROACH

In Beasley and Lynn (1992), Lynn et al. (1993), and MacKay (1994), the zero-offset images using the zero-velocity layer approach show dramatic improvements in image quality over conventional static-shift processing. This method can be extended to nonzero offset data using source profile migration in which upgoing and downgoing wavefields are needed (Claerbout, 1971).

In the zero-velocity layer approach, the upgoing (recorded) and downgoing (source) wavefields are time shifted to the datum at $z = 0$. In the temporal frequency domain, the recorded data or upgoing wavefield at the datum is given by

$$ U_0(x, \omega) = U_{h(x)}(x, \omega) \exp(i \omega t_0(x)),$$

(B-1)

where the static time shift is $t_0(x) = h(x)/V_s$ and where $V_s$ is called the replacement velocity, i.e., a chosen value for the medium between the recording surface and the datum. The static shift in equation B-1 corrects only for topography and not for rapid lateral variation of near-surface properties that are addressed commonly by residual and refraction statics solutions. Therefore, we assume these procedures have been accomplished, if necessary, and that the data have been adjusted such that the topography is the floating datum resulting from statics procedures, before applying equation B-1.

The downgoing wavefield is the result of a simulated source at the actual source point and is referenced to the datum by a static time shift. However, the extrapolation of the downgoing wavefield will be in the opposite direction of the upgoing wavefield (i.e., forward in time), so we apply a negative time shift to shift the downgoing wavefield to $z = 0$. Using equation 11 as our source model at depth $(s + 1)\Delta z$ gives

$$ D_0(x, \omega) = D_{(s+1)\Delta z}(x, \omega) \exp(-i \omega (t_0(x) + t_1(x))),$$

(B-2)

where

$$ t_1(x) = \frac{\Delta z}{V(x,(s+1)\Delta z)},$$

(B-3)

where $V(x,(s+1)\Delta z)$ indicates the velocity from the migration velocity model just beneath the source location.

Applying this static shift to the downgoing wavefields causes it to start at a negative time. These static shifts extrapolate the upgoing and downgoing wavefields from the recording surface $h(x)$ to the datum $z = 0$, consistent with the use of a zero-velocity layer (Gray, 1997). Thus, time-shifting the data to the datum can be viewed as a wave-equation process if the focusing term of the phase-shift operator uses zero velocity so that no lateral propagation occurs.

Downward continuing the upgoing wavefield from $z = 0$ to $z = \Delta z$ is expressed as

$$ U_{\Delta z}(x, \omega) = (W_{\Delta z}U_0)(x, \omega) = \frac{1}{2\pi} \int_{|t' - t|\leq \xi} U_{h}(x, \omega) \Gamma_{\Delta z}(x - x', k_1(x)) dx',$$

(B-4)

where $\xi$ is the aperture of the operator. Similarly, forward modeling the downgoing wavefield to the first depth level is expressed as

$$ D_{\Delta z}(x, \omega) = (W_{\Delta z}D_0)(x, \omega) = \frac{1}{2\pi} \int_{|t' - t|\leq \xi} D_{h}(x, \omega) \Gamma_{\Delta z}^{+}(x - x', k_1(x)) dx',$$

(B-5)

The kernel of equations B-4 and B-5 is defined as

$$ \Gamma_{\Delta z}(x - x', k_1(x), \Delta z) = \begin{cases} \exp(i \omega \Delta z/V_s); & n \Delta z < h(x) \\ W_{\Delta z}(x - x', k_1(x)); & \text{otherwise} \end{cases},$$

(B-6)

where the wavefield extrapolator $W_{\Delta z}$ is again given by equation 7.

REFERENCES


Gazdag, J., and P. Squazerro, 1984, Migration of seismic data by phase shift and where uses zero velocity so that no lateral propagation occurs.


