Taking apart beam migration
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The years 2000–2001 sparked a flurry of activity on various flavors of beam migration. Geophysics papers by Yonghe Sun et al. on slant-stack Kirchhoff migration and Ross Hill on Gaussian-beam migration showed the potential of migration methods that combine aspects of Kirchhoff migration with some novel preprocessing. As a result, a number of variant beam-migration methods have arisen in the last few years, some promising great efficiency and some promising great imaging fidelity. On the other hand, because of beam migration’s extra preprocessing, a simple interpretation of beam migration, analogous to that of Kirchhoff migration, has been hard to pin down. In this article, we try to add some intuition to the discussion of beam-migration methods. Our task is challenging since, for the most part, we will describe Gaussian-beam migration, which is possibly the most complicated of the slant-stack migrations. Of course, a successful understanding—even a partial understanding—of this important method will make it easier to understand the entire family of newly emerging beam-migration techniques.

Taking apart beam migration

Different seismic migration methods usually produce similar migrated images, but they do so differently. As a consequence, some methods are easier to understand than others. When we can understand how something works, we tend to use it more effectively than other methods that might be more rigorously justifiable. Perhaps this explains the historical popularity of Kirchhoff migration. We understand the action of Kirchhoff migration by looking at its action on a single input trace because Kirchhoff migration is expressed as the sum of its actions on all the individual input traces. This is not to say that other migration methods, such as wave-equation migration, cannot work on individual traces; rather, they are expressed in terms of downward continuing wavefields consisting of, say, common-shot traces, followed by an imaging operation. Even though the principle of linear superposition allows wave-equation migration to proceed one trace at a time (or even one sample at a time) to build up a complete image, thinking of this action does not help us understand wave-equation migration any better. But swinging out individual traces is how Kirchhoff migration actually works, so our understanding coincides with its action. Our understanding might be incomplete—perhaps we don’t understand why the input traces need to have a derivative applied before migration, or other details—but we can still understand the big picture.

So before taking beam migration apart, we’ll take Kirchhoff migration apart to illustrate the preceding point. In Figure 1, we show the depth-migrated image of a very simple fan model. The model contains some flat reflectors and dipping reflectors with dips of 45°, 60°, and 75°. The reflectors represent density contrasts in a fictitious Earth with a depth-varying velocity given by the function \( v(z) = (1500 + z) \text{m/s} \), where \( z \) is depth. The model data were created with a Kirchhoff modeling program. (Generally, it is not fair to use the same method to migrate seismic data as was used to model the data, but this model is simple enough to use for our ultimate purpose, which is to take Gaussian-beam migration apart. In fact, a Gaussian-beam migration of the same data yields an image indistinguishable from Figure 1, even including the amplitude jitter on the 75° reflector, which is due to a stair-stepping artifact present in the model data.)

We know the Kirchhoff image in Figure 1 was created by adding together partial images from migrating individual input traces, but there is an intermediate step: This prestack migration was performed by migrating a number of common-shot records one record at a time. Figure 2 shows a partial image obtained from migrating one of these common-shot records. To the right of the image, and of images in subsequent figures, is the input (unmigrated) common-shot record. The partial image, which comes from migrating a single wavefield, corresponds to what we might call a basic molecule for shot-record, wave-equation migration. Wave-equation migration, Kirchhoff migration, and beam migration all have different “molecules,” or elementary building blocks, from which the complete image is constructed. The image contains the lim-
ited portion of the subsurface illuminated by the energy sent from the shotpoint and recorded at the receiver locations. A close look at Figure 2 shows that the Kirchhoff migration had a little trouble imaging the top of the steepest reflector (because of a mute applied to the traveltime tables); this is not evident in the sum of all the migrated shot records in Figure 1. Figure 3 shows what we call the basic molecule for Kirchhoff migration, namely the image of a single input trace—here, a near-offset trace swung in all directions. This image looks nothing like geology, but when combined with similar images obtained from all the other input traces, it helps to form the plausible (though maybe not geologically plausible) image in Figure 1.

That is the familiar story of Kirchhoff migration. The complete image comes from building blocks that are non-geologic images of individual input traces. Obvious, but not usually observed, is the fact that the input traces are completely local in space, and the migration operator is completely global in direction. (Applying the same terminology to shot record wave-equation migration, we can say it operates on recorded wavefields that are global in space with an operator that is also global in direction.)

Next, we’ll try to build the analogous story for Gaussian-beam migration. We begin the story by mentioning the flexibility of both Kirchhoff and Gaussian-beam migrations relative to wave-equation migration. Wave-equation migration has trouble imaging data volumes that are not wavefields (common-offset volumes, for example) because it is based on the principle of downward continuing wavefields—the recorded response to a single source—into the Earth. To migrate a common-offset volume, all its individual traces must be migrated separately as separate wavefields, which is a very expensive proposition. Neither Kirchhoff nor Gaussian-beam migrations have this problem. By migrating traces one at a time, Kirchhoff migration can handle data volumes of any description. Gaussian-beam migration is not quite that flexible, but it can handle any data volume for which a slant stack makes physical sense, such as common-shot, common-offset, or common-midpoint. Most implementations are based on Hill’s 2001 common-offset, single-azimuth formulation, suitable for marine streamer acquisition. Here, instead, we continue our discussion using common-shot beam migration.

First, it is instructive to show the Gaussian-beam image of a single shot record (Figure 4). Although the sum of all the common-shot migrations (not shown) is indistinguishable from Figure 1, we can see some differences between the individual migrated shot records in Figures 2 and 4. So, even before getting down to the molecular level, we see that the two migration methods behave differently. Generally, the nongeologic migration swings are more pronounced, and the overall noise level is higher in the Kirchhoff image.

Next we move closer to the molecular level, where the differences will be greater. Above, we described Gaussian-beam migration as a slant-stack migration. To be more precise, it is a local slant-stack migration. Instead of operating all at once on the entire recorded wavefield (like wave-equation migration) or separately on each individual input trace (like Kirchhoff migration), it operates separately on a number of local regions within the input data. Gaussian-beam migration, then, is partially localized in space. The red curves on top of the unmigrated record in Figure 4 illustrate this schematically. The gather of input traces is divided into overlapping regions, whose center locations are called beam centers, separated by roughly one wavelength at a reference frequency, which is usually around 10 Hz. The red curves in Figure 4 show five regions with five beam centers. Where Kirchhoff migration operates on individual traces, beam migration operates on all traces within a region. In our example, we focus on the rightmost, or near-offset, region. Within this region, the taper function in Figure 4 is applied to the traces before the local slant stacks, centered at the beam center, are performed. The taper functions are designed to sum over all regions to unity, so they have considerable overlap. As a result, an input trace in the near-offset region will also contribute to a few beam centers farther from the source location. Slant stacks are performed on the tapered near-offset traces for many p-values (usually about 50 in two dimensions), corresponding to ray takeoff angles from the beam center. In its simplest form, the migration will send data that emerged at the Earth’s surface from a particular direction back into the Earth in the same direction. Put differently, the slant stack acts as a plane-wave decomposition of the recorded data, and the migration sends
Beam migration the (local) plane-wave data back into the Earth in the direction from which it came. This is the fundamental molecular behavior of Gaussian-beam migration, different from that of Kirchhoff migration, which sends a point-source decomposition of the recorded data into the Earth in all possible directions. Gaussian-beam migration, then, is partially localized in space and partially localized in direction. (If this sounds like Heisenberg’s uncertainty principle, the analogy is apt. Ross Hill’s classic papers on Gaussian-beam migration contain relationships between beam center spacing and ray direction spacing that explicitly obey an uncertainty principle.)

Before we illustrate the problem at the ultimate molecular level, we describe one more intermediate level (Figure 5). Here, data from our near-offset beam center have been migrated. This image resembles the Kirchhoff-migrated image of a single trace, with a very important difference. Where the single-trace migration has swung energy equally in all directions, the Gaussian-beam migration has managed to discriminate among image directions. It has placed more energy where the reflectors are, and less where they aren’t, for both the flat and the dipping reflectors. How has it managed to do that? The answer to that question lies in the molecular structure of Gaussian-beam migration.

We’ve now made it all the way to the fundamental molecular level of Gaussian-beam migration. Kirchhoff migration images from completely localized individual input trace locations into all directions inside the Earth. By contrast, Gaussian-beam migration images from a region around individual beam center locations into a very limited angular range about a single initial direction. Figure 6 illustrates this with a composite plot of three directions, marked by arrows, from the same beam center and the same shot record. One direction (left) shows clear partial images of the dipping events, a second direction (center) shows clear partial images of the flat events (with a bit of curvature most evident on the bottom reflector), and a third direction (right) shows very weak events that shouldn’t be imaged. All the events coincide with events on the single-trace Kirchhoff migration, but the reflectors are much more strongly imaged than the artifacts. To explain why this is so, we show in Figure 7 the same image as in Figure 6 with the dipping events highlighted with a red arrow. On the right, little red and black bars show the effect of the local slant stack on all the reflection events. The three red bars are nearly tangent to the events from the dipping reflectors, and the black bars cut through all the other events. The red bars indicate constructive summation, and the black bars indicate destructive summation or cancellation, i.e., the black bars illustrate that the slant stack has summed across events so that the value of the sum is nearly zero. Partly because of the taper applied to the data before the slant stack and partly because of edge effects, the destructive summation is not complete, and a small amount of residual energy has been migrated to nonreflector locations. Likewise, a small amount of modeling artifact noise appears in the same direction as the partially imaged dipping reflectors.

In summary, what we call the molecular unit (any of the three directions in Figure 6) of Gaussian-beam migration is the image of a very small range of directions from input traces near a beam center location, and the migration of all data from a beam center (Figure 5) is the sum of all these molecular units. The intermediate Gaussian-beam sum of Figure 5 corresponds to the molecular unit of Kirchhoff migration, which is a single migrated trace (Figure 3).

As an aside, the individual local events in Figures 6 and 7...
Beam migration are closely related to curvelets, a term applied to an emerging description of how seismic data can be decomposed. Curvelets can be viewed as molecules, or building blocks, of seismic data. These molecules are even smaller, and therefore more elementary, than those we have discussed so far. (Atoms? Subatomic particles?) Even Kirchhoff migration smiles and beam-migration partial smiles can be decomposed into a set of curvelets.

One nagging question remains about Gaussian-beam migration: If this method is based on rays as Kirchhoff migration is, how do the migrations in individual directions, as in Figure 6, show images that are wider than a very thin raypath? The answer (with apologies for its abuses to V. Cerveny and many other developers of Gaussian-beam theory): Gaussian beams are a kind of fat ray, and the energy spreads away from the thin central raypath into a region around it. Gaussian-beam raytracing creates the partial wavefronts that become broader as the central ray moves away from the source location. In fact, Gaussian beams provide very nice, smooth decay away from the central ray, and this decay is used to build a more complete image, as in Figure 5, from its component molecules in Figure 6. A more detailed answer is available, but it would take us far from the intuitive level of the present discussion.

**Putting it back together again**

Adding together all the individual molecules of beam migration, we have the complete picture. We might not always want to add all the molecules; beam migration has the flexibility to allow us to control the final picture by controlling dip ranges, etc. This flexibility is a two-edged sword that needs to be wielded very carefully. Used in moderation, it can produce modest improvements in signal standout and interpretability; used in excess, it can produce a virtually artificial section with little discernable wavelet character.

In this section, we show several examples, comparing beam-migrated images with images from Kirchhoff and wave-equation migrations. Some images were obtained using Gaussian-beam migration, and some using variant beam methods—see Jilek et al. (2007) for a description of one such method. Alternatively, John Sherwood and coworkers describe one such variant in this issue of TLE. We won’t distinguish among the various beam-migration methods.

Before showing data examples using beam migration, we offer the following disclaimer: Our intention is to show the imaging capabilities of beam migration, so we’ve chosen examples that generally favor beam migration in steep-dip performance (versus wave-equation migration) or signal-to-noise preservation (versus Kirchhoff migration). We emphasize, however, that beam migration’s superiority is not universal. While beam migration rarely appears noisier than Kirchhoff migration, the structural image produced by Kirchhoff migration can sometimes appear more correct than the structural image produced by beam migration, and the same is true for wave-equation migration. Similarly, beam migration rarely produces steep dips on an image that are more convincing than those produced by reverse-time migration (although it can happen). In short, the advantages of beam migration, like any migration method, can be exploited to add to the interpretability of our seismic data.
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We first show a comparison on a familiar 2D synthetic data set, released by BP in 2004, comparing images from beam, wave-equation, and reverse-time migration on one of the two salt bodies in that model. The irregular shape (sedimentary “keyhole” in the middle of the salt structure) and steeply dipping salt present an imaging challenge for all methods. Figure 8 shows images from one-way, wave-equation (left), beam migration (center), and reverse-time migration (right). The image quality of beam migration is comparable to that of the other two methods, with steep-dip accuracy better than that of one-way migration and slightly inferior to that of reverse-time migration. As an aside, we mention one of the perceived advantages of reverse-time migration over all other methods, namely its ability to image “prismatic,” or double-bounce, events. Usually this is true, but, depending on reflector geometry, both one-way and beam methods can sometimes image these events. In this example, the left side of the right flank is imaged from double-bounce energy recorded far to the left of the salt body. Evidence for this assertion is supplied by a beam image for which the angle control was relaxed. Instead of preventing imaging when the subsurface opening angle exceeds 120°, this limitation was disabled in Figure 9, which shows a close-up of the bottom of the salt.

Figure 9. "Prismatic," or double-bounce, energy imaged by beam migration. The blue arrows track very wide-angle reflection energy as it travels down from the left, reflects from the bottom of the sedimentary keyhole, reflects again from the steeply dipping salt flank, and travels upward.

Figure 10. Gulf of Mexico subsalt images from wave-equation migration (top) and beam migration (bottom). Subsalt reflectors are slightly cleaner on the beam image, although beam migration has swung noise into some areas.
Figure 11. Gulf coast (onshore) comparison of Kirchhoff migration (top) and beam migration (bottom). The beam image is cleaner throughout, and more interpretable in zones of complex faulting.

body. The blurring comes from imaging very wide-angle reflection and refraction events, which image at all points along the source-to-reflector-to-receiver travel paths. In Figure 9, this wide-angle energy reflects twice—once at the base of the keyhole and once at the steeply-dipping salt flank. We thank our colleague Yu Zhang for pointing this out.

Our first real data example is from the Gulf of Mexico, courtesy of CGGVeritas. Figure 10 compares a wave-equation migrated image (top) with a beam-migrated image (bottom). Overall, structural differences are minor, but circled areas in-
Beam migration indicates where the beam image is somewhat cleaner than the wave-equation migrated image. In a few areas of poor signal, the beam migration has swung noise into the image, reminiscent of Kirchhoff migration.

On a Gulf Coast land data set comparison (Figure 11), the structural differences between the Kirchhoff image (top) and the beam image (bottom) are again slight, with the beam migration cleaner. In a few areas, such as the complex fault zone to the right of the right tip of the salt body, the beam migration can be interpreted more easily. Also, the beam image has better definition on the steeply dipping salt flanks.

On a North Sea data set, courtesy of CGGVeritas Central North Sea Data Library (Figure 12), we show a beam migration (left) and wave-equation migration (right) of a salt body, with better steep-dip preservation, and, in this case, better subsalt event continuity on the beam-migrated image. On this project, early velocity model updates used Kirchhoff migration, and beam migration was used for later model updates. It is possible that this process resulted in a final velocity model that was biased in favor of beam migration over wave-equation migration. In fact, in describing how beam migration works, we have ignored the whole issue of using ray methods versus wave methods for velocity estimation. Several authors have pointed out that methods such as ray-based seismic tomography can provide optimal velocity models for ray-based (Kirchhoff or beam) migrations, but these models might be less than adequate for wave-equation migration. Only when wave-equation-based velocity estimation is better understood will we be able to consider this issue in detail.

Our final example is from offshore Vietnam, where beam migration was used to image near-vertical fractures in granitic basement. Figure 13 compares sections imaged by Kirchhoff migration in 2002 (left) and by beam migration in 2006 (right). Hints of very steep crisscrossing fractures visible on the Kirchhoff image are very clear on the beam image. On the beam depth slice shown in Figure 14 (right), we can see that these features are confined to the basement and are therefore not migration artifacts.

But there’s a catch
There’s always a catch. In fact, there are two catches with beam migration.

The first catch is philosophical, but it has practical implications. Depth imaging using one-way and two-way versions of the wave equation has become commonplace. Over the decades, the industry has built an excellent library of one-way wavefield extrapolators, and excellent numerical approximations to the two-way wave equation are available. Meanwhile, the efficiency of using one-way or two-way wave-equation methods on large data volumes has increased dramatically with the increased speed of computation and data access. While beam methods are themselves very accurate, flexible, and efficient, they are complicated. Gaussian-beam migration, the most complicated member of the family, requires theoretical and practical experience in ray tracing as well as willingness to deal with ray traveltimes and amplitudes whose values are complex numbers. It requires juggling those quantities with local slant-stacked data that are also complex-val-
Figure 13. Offshore Vietnam depth images: Kirchhoff (left) and beam (right). The basement reflector is about halfway down the section at a depth of approximately 3000 m. The nearly vertical events within the basement reservoir are evident on the beam image, but are hard to discern on the Kirchhoff image.

Figure 14. Offshore Vietnam depth slices: Kirchhoff (left) and beam (right). The near-vertical fractures are almost completely lost on the Kirchhoff image, but they are seen to be confined to the granite basement on the beam image.
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ued. Getting comfortable with the complicated interactions among complex-valued quantities and complex operators can take a long time, writing a working program can take a long time, and maintaining the program can be difficult. As good as beam migration can be, is it worth the investment to write one?

The second catch is practical, but it has philosophical implications. Almost all working beam-migration programs rely on rays. Part of the flexibility of beam migration lies in the ability to control the rays in some ways, such as turning off rays that exceed a predetermined maximum angle, or using the ray information to provide dip control or opening angle control during the migration. However, rays provide a less complete description of a wavefield than one-way and two-way wavefield approximations. In particular, rays might exit the model, or they might be turned off when the ray angle is too large. When this happens, the wavefields associated with these rays become zero, which never happens with one-way and two-way approximations. If the wavefield associated with a single ray vanishes, the problem might not be serious, but if many neighboring wavefields vanish, shadow zones can be a serious limitation in imaging. Figure 15 shows that this can happen. Another issue with rays (which is sometimes stressed more than it needs to be) is the requirement for a smooth velocity function. Ray theory usually requires that the seismic velocity be a smooth function of position, and a large body of experience suggests that a smoothing distance on the order of a wavelength at the dominant frequency is usually close to the right amount to apply to obtain a good image. Even though our ability to estimate seismic velocity usually has an uncertainty exceeding that amount, some consider the need for velocity smoothing to be undesirable.

Conclusions

"Beam migration" refers to a family of methods that combine a local slant stack of unmigrated traces into different directions with a migration operator that is tailored for each direction. Gaussian-beam migration is a specialized method that relies on Gaussian-beam ray tracing; it is a rigorous method that can be understood as a local plane-wave migration. We have identified its individual “molecules” as components from a small area and a small set of emergence directions, and we have shown the relationship between these molecules and the more familiar molecules of Kirchhoff migration. This understanding allows us to recognize the power and flexibility of beam migration.

As our analysis and examples have shown, beam migration can provide cleaner images than Kirchhoff migration, and does not suffer from the dip limitations of one-way wave-equation migration. However, the complicated nature of beam migration and its reliance (usually) on rays are disadvantages relative to all wave-equation methods.


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