Deghosting by joint deconvolution of a migration and a mirror migration

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Summary

This paper focuses on marine receiver deghosting. This step of processing has received renewed interest in recent years because it is a key point in increasing the bandwidth of the final images. Moreover, any non-conventional marine acquisition method, such as over-under streamers, dual-sensor streamers or variable-depth streamers, requires its own receiver deghosting method. I will address the deghosting problem from a novel viewpoint, which leads to a deghosting method adapted to any acquisition method. This deghosting method is optimal in terms of signal-to-noise ratio because it is not performed as a preprocessing stage. It is true amplitude, being able to extract the true deghosted reflectivity, that is the reflectivity that would have been obtained should the water surface be not reflecting. The principle of this method is to perform a migration together with a mirror migration, and to perform a joint deconvolution of these two images. The proposed method is illustrated on a synthetic and a real data example with a variable-depth streamer acquisition.

Introduction

The traditional approach to receiver deghosting is to include the zero-offset receiver ghost into the far-field signature and to perform 1D deconvolution of the dataset at preprocessing stage. $\Delta z$ being the depth of the streamer and $c$ the water velocity, we have:

$$G(f) = 1 - e^{2j\pi \Delta z f/c}$$

(1)

This traditional approach can be refined by taking into account the angle of propagation of the wavefield instead of assuming vertical propagation. The ghost takes the form:

$$G(f, k_x, k_y) = 1 - e^{2j\pi \Delta z \sqrt{f^2/c^2 - k_x^2 - k_y^2}}$$

(2)

This approach raises two problems. The first one is that, while it is easy to take into account non vertical propagation in the inline direction $x$ (parallel to the streamers), it is much more difficult to take into account the crossline direction $y$, due to the coarse $y$ sampling that a multi-streamer acquisition performs. The second problem, is of a more fundamental nature: considering that the deghosting operator is variable, it should not be done as a preprocessing, but should be done after stack. It is a general principle in signal processing that any deconvolution of a redundant measurement with a variable wavelet should be done after stack. We will therefore investigate the possibility of performing the deghosting, not at preprocessing stage, but after stacking.

Optional stacking with a variable wavelet

The optimal solution of the multichannel deconvolution problem:

$$T_n(f) = W_n(f) R(f) + E_n(f), n = 1, .., N$$

(3)

is not the pre-stack deconvolution plus stack formula:

$$\hat{R}(f) = \frac{1}{N} \sum_{n=1}^{N} T_n(f) W_n(f)$$

(4)

but the least squares formula:

$$\hat{R}(f) = \frac{\sum_{n=1}^{N} W_n(f) T_n(f)}{\sum_{n=1}^{N} |W_n(f)|^2}$$

(5)

which corresponds to matched filtering, stacking, and post-stack deconvolution. The only case where the pre-stack deconvolution is valid is when the wavelets $W_n(f)$ do not depend on $n$. The more diversity we have in our wavelets, the more advantage we have to use the least squares formula.

Matched mirror migration

Equation (5) is written for a 1D wavelet. When considering the receiver ghost problem of a streamer acquisition, and in order to take into account all angle of propagation, including the crossline propagation, the multidimensional matched filtering can be realized by a "Matched Mirror Migration". The matched mirror migration is defined as a migration in which the number of receivers is doubled by introducing for each receiver located at $(x_r, y_r, z_r)$ and having recorded the data $d_r(t)$, a mirror receiver at the location $(x_r, y_r, -z_r)$ and consider that this mirror receiver has recorded the data $-d_r(t)$.

The stacked matched mirror image is the equivalent of the numerator of equation (5). What would be the equivalent of the division by the denominator? Spectral whitening by a zero-phase operator is a possible solution but it is not true amplitude, as a white reflectivity assumption has to be made. If we want to be true amplitude, it is very difficult to keep track of the different ghost operators and their evolution through the migration process, especially in depth migration.

In order to have a true amplitude deghosting, we consider separately the two components of the matched mirror migration: the normal migration that migrates the
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The conventional deconvolution method, (Robinson and Treitel, 1964) can be stated as: given a trace \( d(t) \), find a minimum phase wavelet \( a_{\min}(t) \) and a reactivity \( r(t) \) such as:

\[
d(t) = a_{\min}(t) \ast r(t) \quad (6)
\]

This problem is mathematically ill-posed, which means that if we construct a \( d(t) \) from given \( a_{\min}(t) \) and \( r(t) \), we are unable to rederive \( a_{\min}(t) \) and \( r(t) \) from \( d(t) \), even in the noiseless case. This is why we must add the statistical assumption that the reactivity \( r(t) \) is white.

It is a very reasonable assumption to consider a ghost wavelet as a minimum phase signal, or at least a marginally minimum phase signal. We can likewise consider that the mirror migration gives the same reactivity as the migration but distorted by a ghost wavelet which is maximum phase. We can then consider the following problem:

Considering \( d_1(t) \) and \( d_2(t) \) two given signals, find a signal \( r(t) \), a normalized minimum phase operator of given length \( g_{\min}(t) \) (normalized meaning \( g_{\min}(0) = 1 \)), and a maximum phase normalized operator of given length such as:

\[
d_1(t) = g_{\min}(t) \ast r(t) \\
d_2(t) = g_{\max}(t) \ast r(t) \quad (7)
\]

In an intuitive way, we can say that we have a binocular vision of the reflectivity \( r(t) \): one image, \( d_1(t) \), the migration, is colored by a normalized minimum phase distortion, and the other, \( d_2(t) \), the mirror migration, is colored by a normalized maximum phase distortion. We want to recover \( r(t) \) in true color, that is without distortion.

Although the joint deconvolution problem as stated by equation (7) looks like the conventional deconvolution model stated by equation (6), it has totally different mathematical properties. It is a well-posed problem, which means it has a unique solution, even when the minimum phase and maximum phase properties are marginally respected (meaning the operators have perfect spectral notches). If we take a normalized minimum phase \( g_{\min}(t) \), a normalized maximum phase \( g_{\max}(t) \) and a reactivity \( r(t) \) and construct \( d_1(t) \) and \( d_2(t) \), then we can uniquely and perfectly reconstruct \( g_{\min}(t), g_{\max}(t) \) and \( r(t) \) from \( d_1(t) \) and \( d_2(t) \). No assumption is needed on the amplitude spectrum of \( r(t) \), which is arbitrary and unknown. This can be checked in the cepstral domain.

The joint deconvolution can be done in a least square sense in the case we have noise in our data, and can also be done in a multichannel way, where a given number of traces \( d_1(t, x) \) and \( d_2(t, x) \) are linked to a reactivity \( r(t, x) \) through common operators \( g_{\min}(t) \) and \( g_{\max}(t) \).

1D synthetic data example

The simple but tricky synthetic example shown in Figures 1 to 7 proves that the joint deconvolution is a well-posed problem, even where there are notches in the reflectivity and the ghosts operator. Figure 1 shows a reactivity \( r(t) = w(t) + w(t + 125\Delta t_g) \) where \( w(t) \) is an airgun signature, and \( \Delta t_g = 16\text{ms} \). The amplitude spectrum of the reflectivity shown in Figure 2 between 0 and 125 Hz, is obviously non white, having perfect notches at 0, 31 and 93 Hz. We model a migration with a ghost \( G_{\min}(t) = 1 - e^{j\pi t/16\Delta t_g} \), the delay of the ghost being \( \Delta t_g = 12\text{ms} \) and a mirror migration with \( G_{\max}(t) = \overline{G_{\min}(t)} \). The common amplitude spectrum of the ghost operators is shown in Figure 3. The spectra of the ghosts have notches at 0, 62 and 125 Hz. Figure 4 and 5 shows the migration and the mirror migration and Figure 6 the common spectrum of the two migrations.

Figure 6 shows why this simple synthetic example is tricky: the amplitude spectrum of the given data, is full of notches: at frequencies 62 and 125 Hz these notches are due to the ghosts, and should be deconvolved by the deghosting, but the notches at 31 and 93 Hz should be left untouched as they belong to the reflectivity, while at 0 Hz there is a double notch, one of which should be deconvolved. The input of the joint deconvolution of the image and mirror image, being shown in Figure 4 and 5, the output is the two ghost operators which amplitude spectra are shown in Figure 3. They match the theoretical ones to numerical accuracy, having extracted the 3 ghost notches, and discarded the 3 reflectivity notches. The deghosted output is superimposed in Figure 1, and is again exact to numerical accuracy.

In order to check the robustness to noise, Figure 7 shows the spectra of the estimated ghosts when noise is added to the migration and mirror migration at a level of 20% of the maximum amplitude and the deghosting is done in multichannel mode on 100 traces. The deghosted traces are extremely noisy and are not shown, but this is expected when deconvolving noisy data by an almost perfect notch. However, we can see that the ghosts are adequately estimated.

![Fig. 1: Reflectivity and estimated deghosted migration.](image-url)
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Having checked that the joint minimum phase-maximum phase deconvolution is a well-posed problem, we must now check that we are able to perform with it true amplitude deghosting, that is retrieve the migration that can be obtained with a non-reflecting water surface. The velocity model used in this 2D dataset was a vertical gradient and the wavelet used for the pre-stack modeling of the shots was an actual airgun wavelet. The streamer geometry considered was a variable-depth streamer. The modeling of the shots was done twice: one with a reflecting water surface (ghosted data) and one without (unghosted data). Noise of the same level were added to both sets of shots. The shots with the ghost were processed by deterministic designature, migration (shown in Figure 8) and mirror migration (shown in Figure 9). The joint minimum phase maximum phase deconvolution was applied on the migration and mirror migration in a windowed multichannel version, the result being shown in Figure 10. The deghosted image of Figure 10 can be compared to Figure 11, which is the reference migration, computed by migrating the unghosted data through designature and migration. The close match between the deghosted output in Figure 10 and the reference migration in Figure 11 proves the true amplitude nature of the proposed deghosting algorithm. The very good low frequency content of the deghosted data can also be noted, the reflectors showing as black on grey background. Also, the deghosted image has a low level of noise, similar to the reference image, due both to the nature of the deghosting algorithm and to the variable-depth streamer acquisition that prevents perfect notches in the stacked images.

Real data example

The proposed deghosting algorithm was used to process a variable-depth streamer acquisition. The data was migrated in depth, just after an aggressive source designature whitening in the bandwidth [2.5 Hz, 110 Hz]. Figure 12 shows the migration, Figure 13 the mirror migration. The residual causal ghost can easily be seen in the migration on the water-bottom, as well as the anticausal ghost on the mirror migration. Figure 14 shows the deghosted output after application of the joint deconvolution, with no spectral shaping applied. Confirming the findings on synthetic data, the broadband nature of the deghosted image, both in low and high frequencies, together with its non-noisy nature, can be verified.

Conclusion

We have described a new deghosting method that it is not performed as a preprocessing stage, but at imaging stage to ensure the best signal-to-noise ratio. The method jointly deconvolves the migration and the mirror migration of the data. This deghosting method was shown to be true amplitude, retrieving on ghosted modeled data the migration of the unghosted modeled data. It can be used with various acquisition methods but is particularly well-suited for variable-depth streamer acquisitions.

2D synthetic data example

Having checked that the joint minimum phase-maximum
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Fig. 8: Migration.

Fig. 9: Mirror migration

Fig. 10: Deghosted migration

Fig. 11: Reference migration

Fig. 12: Migration

Fig. 13: Mirror migration

Fig. 14: Deghosted migration