Variable-depth streamer: deep towing and efficient deghosting for extended bandwidth
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Summary
This paper starts by revisiting the receiver deghosting problem, showing that in order to be optimal in terms of signal-to-noise ratio, it should not be performed as a pre-processing stage as it is done usually. A new deghosting algorithm is described, based on computing a migration together with a mirror migration, and performing a joint deconvolution of these two images. It is true amplitude, being able to extract the true deghosted reflectivity, that is the reflectivity that would have been obtained should the water surface be not reflecting. The paper then describes how this new deghosting method allows the revisiting of the slant streamer technique, which was experimented in the 1980’s. The principle is to create a notch diversity at stack level, the deghosting being done by the stack. We revisit this acquisition technique in two ways: firstly, we adapt the depth profile of the streamer to modern streamer length, leading to variable depth streamer rather than slant streamer. Secondly, recognizing that the stack performs an imperfect deghosting, leaving a residual ghost, we perform the residual deghosting with our new deghosting technique. A variable-depth streamer dataset was acquired offshore NW Australia and processed with the proposed deghosting algorithm, showing a very wide bandwidth, with low frequencies down to 2.5 Hz and high frequencies up to 110 Hz.

Introduction
The traditional approach to receiver deghosting is to include the zero-offset receiver ghost into the far-field signature and to perform 1D deconvolution of the dataset at preprocessing stage. \( \Delta z \) being the depth of the streamer and \( c \) the water velocity, we have:

\[
G(f) = 1 - e^{2j\pi2\Delta z f / c} \tag{1}
\]

This traditional approach can be refined by taking into account the angle of propagation of the wavefield instead of assuming vertical propagation. The ghost takes the form:

\[
G(f, k_x, k_y) = 1 - e^{2j\pi2\Delta z \sqrt{k_x^2 + k_y^2 - k_0^2}} \tag{2}
\]

This approach raises two problems. The first one is that, while it is easy to take into account non vertical propagation in the inline direction \( x \) (parallel to the streamers), it is much more difficult to take into account the crossline direction \( y \), due to the coarse \( y \) sampling that a multi-streamer acquisition performs. The second problem, is of a more fundamental nature: considering that the deghosting operator is variable, it should not be done as a preprocessing, but should be done after stack. It is a general principle in signal processing that any deconvolution of a redundant measurement with a variable wavelet should be done after stack. We will therefore investigate the possibility of performing the deghosting, not at preprocessing stage, but after stacking.

Optimal stacking with a variable wavelet
The optimal solution of the multichannel deconvolution problem:

\[
T_n(f) = W_n(f) R(f) + E_n(f), n = 1,..,N \tag{3}
\]

is not the pre-stack deconvolution plus stack formula:

\[
\hat{R}(f) = \frac{1}{N} \sum_{n=1}^{N} T_n(f) W_n(f) \tag{4}
\]

but the least squares formula:

\[
\hat{R}(f) = \frac{\sum_{n=1}^{N} W_n(f) T_n(f)}{\sum_{n=1}^{N} |W_n(f)|^2} \tag{5}
\]

which corresponds to matched filtering, stacking, and post-stack deconvolution. The only case where the pre-stack deconvolution is valid is when the wavelets \( W_n(f) \) do not depend on \( n \). The more diversity we have in our wavelets, the more advantage we have to use the least squares formula.

Matched mirror migration
Equation (5) is written for a 1D wavelet. When considering the receiver ghost problem of a streamer acquisition, and in order to take into account all angle of propagation, including the crossline propagation, the multidimensional matched filtering can be realized by a "Matched Mirror Migration". The matched mirror migration is defined as a migration in which the number of receivers is doubled by introducing for each receiver located at \((x_r, y_r, z_r)\) and having recorded the data \( d_i(t) \), a mirror receiver at the location \((x_r, y_r, -z_r)\) and consider that this mirror receiver has recorded the data \(-d_i(t)\).

The stacked matched mirror image is the equivalent of the numerator of equation (5). What would be the equivalent of the division by the denominator? Spectral whitening by a zero-phase operator is a possible solution but it is not true amplitude, as a white reflectivity assumption has to be made. If we want to be true...
amplitude, it is very difficult to keep track of the different ghost operators and their evolution through the migration process, especially in depth migration.

In order to have a true amplitude deghosting, we consider separately the two components of the matched mirror migration: the normal migration that migrates the receivers and the mirror migration that migrates the mirror receivers. The normal migration stacks coherently the primary events, the ghosts events being imperfectly stacked in such a way that the migration has a residual ghost wavelet that is causal. The mirror migration stacks coherently the ghosts events with their polarity reversed, in such a way that the primary events are imperfectly stacked in such a way that the mirror migration has a residual ghost wavelet that is anticausal. The proposed deghosting method uses this “binocular vision” of two images of the same reflectivity with a different viewpoint to extract the true amplitude deghosted migration, that would have been obtained by a conventional migration if the water-surface was non-reflective.

Minimum-maximum phase joint deconvolution

The conventional deconvolution method, (Robinson and Treitel, 1964) can be stated as: given a trace $d(t)$, find a minimum phase wavelet $a_{\text{min}}(t)$ and a reflectivity $r(t)$ such as:

$$d(t) = a_{\text{min}}(t) \ast r(t)$$

(6)

This problem is mathematically ill-posed, which means that if we construct a $d(t)$ from given $a_{\text{min}}(t)$ and $r(t)$, we are unable to rederive $a_{\text{min}}(t)$ and $r(t)$ from $d(t)$, even in the noiseless case. This is why we must add the statistical assumption that the reflectivity $r(t)$ is white.

It is a very reasonable assumption to consider a ghost wavelet as a minimum phase signal, or at least a marginally minimum phase signal. We can likewise consider that the mirror migration gives the same reflectivity as the migration but distorted by a ghost wavelet which is maximum phase. We can then consider the following problem:

Considering $d_1(t)$ and $d_2(t)$ two given signals, find a signal $r(t)$, a normalized minimum phase operator of given length $g_{\text{min}}(t)$ (normalized meaning $g_{\text{min}}(0) = 1$), and a maximum phase normalized operator of given length such as:

$$d_1(t) = g_{\text{min}}(t) \ast r(t)$$

$$d_2(t) = g_{\text{max}}(t) \ast r(t)$$

(7)

In an intuitive way, we can say that we have a binocular vision of the reflectivity $r(t)$: one image, $d_1(t)$, the migration, is colored by a normalized minimum phase distortion, and the other, $d_2(t)$, the mirror migration, is colored by a normalized maximum phase distortion. We want to recover $r(t)$ in true color, that is without distortion.

Although the joint deconvolution problem as stated by equation (7) looks like the conventional deconvolution model stated by equation (6), it has totally different mathematical properties. It is a well-posed problem, which means it has a unique solution, even when the minimum phase and maximum phase properties are marginally respected (meaning the operators have perfect spectral notches). If we take a normalized minimum phase $g_{\text{min}}(t)$, a normalized maximum phase $g_{\text{max}}(t)$ and a reflectivity $r(t)$ and construct $d_1(t)$ and $d_2(t)$, then we can uniquely and perfectly reconstruct $g_{\text{min}}(t), g_{\text{max}}(t)$ and $r(t)$ from $d_1(t)$ and $d_2(t)$. No assumption is needed on the amplitude spectrum of $r(t)$, which is arbitrary and unknown. This can be checked in the cepstral domain.

The joint deconvolution can be done in a least square sense in the case we have noise in our data, and can also be done in a multichannel way, where a given number of traces $d_1(t,x)$ and $d_2(t,x)$ are linked to a reflectivity $r(t,x)$ through common operators $g_{\text{min}}(t)$ and $g_{\text{max}}(t)$.

1D synthetic data example

The simple but tricky synthetic example shown in Figures 1 to 3 proves that the joint deconvolution is a well-posed problem, even where there are notches in the reflectivity and the ghosts operator. A reflectivity $r(t) = w(t) + w(t + \Delta t_g)$ is used for this example, where $w(t)$ is an airgun signature, and $\Delta t_g = 16$ms. The amplitude spectrum of the reflectivity shown in Figure 1 between 0 and 125 Hz, is obviously non white, having perfect notches at 0, 31 and 93 Hz. We use for the migration a ghost $G_{\text{min}}(t) = 1 - e^{2j\pi 2\Delta t_g f}$, the delay of the ghost being $\Delta t_g = 12$ms and for the mirror migration $G_{\text{max}}(t) = G_{\text{min}}(t)$. The common amplitude spectrum of the ghost operators is shown in Figure 2. The spectra of the ghosts have notches at 0, 62 and 125 Hz. Figure 3 shows the common amplitude spectrum of the migration and the mirror migration.

Figure 3 shows why this simple synthetic example is tricky: the amplitude spectrum of the given data, is full of notches: at frequencies 62 and 125 Hz these notches are due to the ghosts, and should be deconvolved by the deghosting, but the notches at 31 and 93 Hz should be left untouched as they belong to the reflectivity, while at 0 Hz there is a double notch, one of which should be deconvolved. The output of the joint deconvolution are the two ghost operators which amplitude spectra are superimposed in Figure 2. They match the theoretical ones to numerical accuracy, having extracted the 3 ghost notches, and discarded the 3 reflectivity notches. The deghosted output spectrum is superimposed in Figure 1, and is again exact to numerical accuracy.

![Fig. 1: Spectra of reflectivity and estimated deghosted migration.](image-url)
From slant to variable-depth streamer

Although this deghosting method can be used with any kind of acquisition geometry, it is particularly adapted to acquisitions exhibiting pre-stack notch diversity. The pre-stack notch diversity prevents perfect notches being present on the post-stack data. One such acquisition is the slant streamer where the streamer depth increases linearly with depth. However, with current streamer lengths, such a configuration does not ensure sufficient notch diversity for shallow events, which do not use the whole length of the streamer. There is the need to optimize the depth profile in order to ensure diversity for all reflectors depth. One exemple of such variable-depth streamer configuration consists in a linear slope for the first 2 km of the streamer, going from 7.5m to 37.5m depth, followed by an horizontal portion of 6 km. The first 2 km of the streamer ensures the notch diversity, while the horizontal portion provide an enhancement of the frequencies below 10 Hz. The deghosting algorithm ensures good balancing between the different frequencies.

Variable-depth streamer: synthetic data

The velocity model used in this 2D dataset was a vertical gradient and the wavelet used for the pre-stack modeling of the shots was a actual airgun wavelet. The streamer geometry considered was the variable-depth streamer configuration described above. The modeling of the shots was done twice: one with a reflecting water surface (ghosted data) and one without (unghosted data). Noise of the same level were added to both sets of shots. The shots with the ghost were processed by deterministic designature, migration (shown in Figure 4) and mirror migration (shown in Figure 5). The joint minimum phase maximum phase deconvolution was applied on the migration and mirror migration in a windowed multichannel version, the result being shown in Figure 6. The deghosted image of Figure 6 can be compared to Figure 7, which is the reference migration, computed by migrating the unghosted data through designature and migration. The close match between the deghosted output in Figure 6 and the reference migration in Figure 7 proves the true amplitude nature of the proposed deghosting algorithm. The very good low frequency content of the deghosted data can also be noted, the reflectors showing as black on grey background. Also, the deghosted image has a low level of noise, similar to the reference image, due both to the nature of the deghosting algorithm and to the variable-depth streamer that prevents perfect notches in the stacked images.

Variable depth streamer: real data

A variable-depth streamer acquisition as described above was performed offshore NW Australia to test its potential for broadband imaging. The data was migrated in depth, just after source designature whitening in the bandwidth $[2.5\, \text{Hz},110\, \text{Hz}]$. Figure 8 shows the migration, Figure 9 the mirror migration. The residual causal ghost can easily be seen in the migration on the water-bottom, as well as the anticausal residual ghost on the mirror migration. Figure 10 shows the deghosted output after application of the joint deconvolution, with no spectral shaping applied. Confirming the findings on synthetic data, the broadband nature of the deghosted image, both in low and high frequencies, together with its non-noisy nature, can be verified. Figure 11 shows the spectra of the 3 migrations.

Conclusion

We have described a new deghosting method that is not performed as a preprocessing stage, but at imaging stage to ensure the best signal-to-noise ratio. The method jointly deconvolues the migration and the mirror migration of the data. This deghosting method was shown to be true amplitude, retrieving from ghosted modeled data the migration of the unghosted modeled data. This deghosting method allows the processing of variable-depth streamer acquisitions, an adaptation of the slant streamer to modern streamer length. The potential of such an acquisition and processing has been proven on a real dataset, where a broadband $[2.5\, \text{Hz} - 110\, \text{Hz}]$ image has been obtained.

References

Variable depth streamer

Fig. 4: Migration

Fig. 5: Mirror migration

Fig. 6: Deghosted migration

Fig. 7: Reference migration

Fig. 8: Migration

Fig. 9: Mirror migration

Fig. 10: Deghosted migration