RTM and Kirchhoff angle domain common-image gathers for migration velocity analysis.
Jean-Philippe Montel*, Gilles Lambaré CGGVeritas Massy (France)

Summary
Angle domain common image gathers are recommended in Kirchhoff and reverse time migration for velocity model building in complex area. For these both approaches there is a general agreement that the tomographic ray pairs are fully defined by the reflection and azimuth angle information and the reflection dip and that if the velocity model is correctly updated down to a given horizon, it is not necessary to shoot the tomographic ray pairs upwards through this horizon. We show here through examples and a theoretical analysis that these both statements have to be mitigated when the common image gathers exhibit a significant residual move out on. We also show how to accurately compute the tomographic ray pairs allowing then for an accurate angle domain migration velocity analysis.

Introduction
Common image gathers (CIG) are an important output of prestack depth migration (PreSDM) used for subsurface attribute interpretation and for velocity model building. The CIGs are conventionally computed in the offset domain using Kirchhoff migration, but development of migration in complex media has favoured new types of CIGs such as angle domain Kirchhoff migration (Xu et al., 2001), angle domain wave field continuation migration (Sava and Fomel, 2003) or angle domain reverse time migration (RTM) (Xu et al., 2011). Migration velocity analysis from angle domain CIGs appears then particularly promising for velocity model building in complex area. There is a general agreement that with Kirchhoff or RTM angle domain CIGs (Jousselin et al., 2009; Huang et al., 2010):

1) The tomographic ray pairs are fully defined by the reflection and azimuth angle information and the reflection dip;
2) If the velocity model is correctly updated down to a given horizon, it is not necessary to shoot the tomographic ray pairs upwards through this horizon.

We show here through examples and a theoretical analysis that these two statements have to be mitigated, and how tomographic ray pairs have to be computed.

Figure 1: Comparison of angle domain CIGs for Kirchhoff migration and RTM, (A) versus (B) and (C) versus (D). Comparison of angle domain CIGs for traces recorded at surface (A) and (B) versus traces recorded at 900 m deep (C) and (D). The exact and migration velocity models are shown on the right. The yellow and red curve reported on all the CIGs corresponds to the RMO curve picked on CIG (A).
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**Figure 2**: Definitions of parameters associated to observed event (left), to ray trajectories (center) and to common β migrated event (right) (β states for shot, offset or angle). We have $p^s = p^s + p^r$ and $Δp^s = p^s_β - p^s_γ$, where $p^s_β = p^s_β + p^s_β$ and $p^s = p^s_β + p^s_r$. Note that superscript stands for depth quantities and subscript for surface quantities.

**A canonical example**

We propose to compare angle domain Kirchhoff migration (Xu et al., 2001), and reverse time migration (RTM) (Xu et al., 2011) on a simple 1D model and dataset. For the exact velocity model given on Figure 1, we compute seismograms for a set of planar reflectors and for acquisitions at the surface and at 900 m deep. We then migrate the data using an erroneous migration velocity model (20% slower) and look at the angle domain CIGs in the same depth window (Figure 1). On Figure 1 we clearly see that the curvature of the Kirchhoff and RTM angle domain CIGs differ and that in both cases the curvature also differ for an acquisition at 0 m or at 900 m deep, and this even if the migration velocity model is exact between 0 and 900 m. It is now clear that the second assertion is not always valid and that some theoretical investigations are required. This can be done establishing first the imaging conditions for Kirchhoff and RTM angle domain imaging and then deriving the relations between the tomographic ray pair, the structural dips and the local slope of the residual move out (dRMO), (Figure 2).

**Imaging conditions for angle domain imaging**

Imaging condition for migration (see Xu et al. (2001) or Chauris et al. (2001) for a review) is a necessary condition for a given seismic event (characterized by observed time $T^o(s,r)$) to migrate in depth at position $x$ for a given offset, shot or angle value. For a given shot position $s$ and receiver position $r$, $x$ will be a focusing point in the image if the misfit between the observed time and two-way travel time

$$ΔT(s,x,r) = T^o(s,x,r) - T^o(s,r)$$

(Figure 2) vanishes (travel time condition) and if it is stationary along the summation direction (specularity), i.e. receiver, midpoint or dip angle depending on the type of migration.

Table 1 summarizes in the column “specularity” the various specularity conditions for shot domain, offset domain and angle domain Kirchhoff migration (SDKM, ODKM and ADKM respectively) and for angle domain RTM (ADRTM), $m = ½(r + s)$ and $h = r - s$ denote midpoint and offset, $ϕ$ denotes the direction of the vector sum of the slowness vectors in $x$ of rays $s → x$ and $r → x$, $p^r = p^r + p^r$, and $θ$ denotes the aperture angle(s) between slowness vectors $p^r$ and $p^r$ (Figure 2). Note that for the angle domain RTM described in Xu et al. (2011) focusing is insured through a common shot migration even if forward and backwards propagated wave-fields are internally decomposed into incidence angles. The discrepancy on the moveout of the CIG between Kirchhoff and RTM is due to that Kirchhoff migration uses the propagation direction of Greens function and RTM uses wavefield propagation direction on the receiver side. The propagation direction of Greens function and wavefield propagation direction is coincide only when the velocity model is correct.

**Migrated dips and dRMO**

It is possible to connect the expressions of the dip in the migrated image $∂x_β/∂x$ and the local slope of the RMO (dRMO) in the CIG, $∂x_β/∂r$, $∂x_β/∂h$, or $∂x_β/∂θ$, to the ray trajectories ($θ, ϕ$) and to the local slopes of the observed time $Δp^s$ through a first order analysis of the focusing in the vicinity of the focusing point $x$ (Chauris et al. 2002). The two last columns of Table 1 show the expression resulting from this analysis for SDKM, ODKM, ADKM and ADRTM. The expressions for the common shot or common offset were given in Chauris et al. (2002),
and Chauris (2000) addressed the case of angle domain Kirchhoff migration also used in Chauris et al. (2001). However, expression given in Table 1 for this case differs while the case of angle domain RTM as proposed by Xu et al. (2010) had never been given. These two last cases, both in angle, exhibit original features which deserve a special treatment.

The key modification with respect to the shot and offset domain cases is that the expressions of the dip contain the slope misfit term $\Delta p_{sr}$. While in the shot and offset domain cases it was possible to trace ray upwards just knowing the dip of the event and the shot or offset value it is no more the case in the angle domain cases, Kirchhoff and RTM. Moreover while in the shot and offset domain cases slope misfit term $\Delta p_{sr}$ was computed from the dRMO after the ray tracing step it is no more possible in the angle domain cases. In fact ray tracing and computation of slope misfits have to be computed jointly.

### Tracing tomographic ray pairs for angle domain CIGs

Tracing tomographic ray pairs for angle domain CIGs can be seen as an optimization problem. For this process our constraints are the specularity condition, the dip picked on a common angle migrated section and the dRMO picked on the angle domain CIG. Our unknown parameters are the directions of the rays characterized by angles $(\theta, \phi)$ (Figure 2) and the slope misfits $\Delta p_{sr}$.

A non-linear local optimization scheme can be implemented for finding ray trajectories and slope misfits that fit the specularity condition, the observed dips and the dRMO.

Let’s consider a smoothed version of Marmousi (Billette et al., 2003) synthetic case study. We perform an ADKM using as velocity model 1.2 times the slowness of the exact model. dRMOS are picked on CIGs (Figure 3A), while the dips are picked on Common Angle Sections (CAS) (Figure 3B). They are used as input for our iterative kinematic demigration.

As starting point for the iterative process (iteration 0 on Figure 3) we assume $\Delta p_{sr} = 0$, i.e., the null dRMO assumption (see column dRMO in Table 1). Then ray tracing can be done using the simplified equation of the dip $\partial z_{mig}/\partial s_{xy} = p_{sr} p'_{sr}$ (the same as for common shot or common offset imaging) (ray trajectories in blue on Figure 3B).

Figures 3C and D show however that the resulting demigrated event does not fit with observed data. Two iterations later in our kinematic demigration process Figures 3E and F show that we have now well recovered the observed event but also that the ray trajectories have very significantly evolved (ray trajectories in red on Figure 3B) causing significant changes in CMP and offset values.

<table>
<thead>
<tr>
<th>Specularity</th>
<th>dip</th>
<th>dRMO</th>
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<tbody>
<tr>
<td><strong>SDKM</strong></td>
<td>$\Delta p_{sr} \frac{\partial s_{mig}}{\partial r} \big</td>
<td>_{s} = 0$</td>
</tr>
<tr>
<td><strong>ODKM</strong></td>
<td>$\Delta p_{sr} \frac{\partial s_{mig}}{\partial m} \big</td>
<td>_{h} = 0$</td>
</tr>
<tr>
<td><strong>ADKM</strong></td>
<td>$\Delta p_{sr} \frac{\partial s_{mig}}{\partial \phi} \big</td>
<td>_{\theta} = 0$</td>
</tr>
<tr>
<td><strong>ADRTM</strong></td>
<td>$\Delta p_{sr} \frac{\partial s_{mig}}{\partial \theta} \big</td>
<td>_{s} = 0$</td>
</tr>
</tbody>
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Table 1: Expressions of specularity, dip and dRMO for shot, offset, and angle domain Kirchhoff migration (SDKM, ODKM and ADKM respectively) (Xu et al., 2001), and angle domain RTM (ADRTM) (Xu et al., 2010). Note that $p'_{sr} = \partial T'_{sr}/\partial x$ and $\partial s_{mig}/\partial \mu$ represent acquisition quantities when $\mu$ is a surface parameter $(s, r, m$ and $h)$ and paraxial quantities when $\mu$ is a depth parameter $(x, y, z, \theta$ and $\phi$. Note that the slope misfit term $\Delta p_{sr}$, written in red appears in the expression of the dip condition for both angle domain CIGs, ADKM and ADRTM. This imposes to compute jointly tomographic rays and slope misfits.
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Figure 3: Kinematic demigration as a joint optimization process. (A) the picked dRMO in the CIG, (B) the picked dip and the initial (blue) and final (red) demigrated rays. (C) and (D) show the recovered observed event for the initial demigration (iteration 0) while (E) and (F) show it for the final demigration (iteration 2) in the CMP and common offset sections, COS, respectively.

Conclusion

With a canonical example and a theoretical analysis we have shown that angle domain Kirchhoff (Xu et al., 2001) and RTM (Xu et al., 2010) reveal interesting specificities with important implications for velocity model building. Indeed when residual move out is significant:
1) Their angle domain CIGs do not exhibit the same curvature.
2) It is necessary to shoot the tomographic rays (at least for the source side in angle domain RTM) until the surface even if the exact velocity model is used between the acquisition surface and the redatuming level.
3) Tomographic rays cannot be traced directly from the structural dips (as in ODKM and SDKM), and reflection and azimuth angles, but require to be traced the additional information of the slope of the RMO and a non-linear optimization step.

This last step is particularly important for non-linear tomographic methods (Guillaume et al. 2008) which aim at updating velocity models with a limited number of picking steps.

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REFERENCES


