Q FACTOR ESTIMATION BY TIME VARIANT SPECTRAL RATIOS

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Summary

A new method to estimate Q factor is introduced in this paper. It takes advantage of spectral decomposition techniques, which are widely used in oil and gas industry nowadays. The connection between these techniques and common Q factor estimation methods are evident, but not appropriately addressed in order to obtain consistent values. The method uses time variant spectral ratios calculated from spectral amplitude gathers, which are generated by using the Generalized Stockwell Transform. Q is computed indirectly by using linear least squares fit for each time over a given frequency bandwidth interactively determined. Instead of estimate Q from linear fit slope, an interpretation stage is added to obtain dissipation times, which must increase with time, giving consistency to results. In this paper, time variant spectral ratios method (TVSR) is tested using a dataset with Q values previously available and a simple example of its potential is done to show how Q sections can be generated using it.

Key words: Q factor, spectral decomposition, time variant spectral ratios.

Introduction

Seismic waves lose energy during propagation. There are many factors affecting this phenomenon. One of them is the intrinsic attenuation. It produces a decrease in the frequency bandwidth of the wave in a progressive way in time. Since the intensity of the attenuation depends on the fluid content of the formations which crosses the seismic pulse, this may be an indicator of the presence of hydrocarbons. Q factor is the magnitude that characterizes intrinsic attenuation, but its use in reservoir characterization is low, but non-existent, due to the complexity that implies to estimate it. On the other hand, spectral decomposition algorithms are used to obtain information about presence of hydrocarbons, to predict thickness and to identify stratigraphic features of interest from seismic data. Other uses include detection of discontinuities and fractures. It is a practice widely accepted in the oil and gas industry and there are many works that demonstrates its usefulness; for example Sinha et al. (2005) uses it to outline stratigraphic features, Miao et al. (2007) find a very good correlation between the data of thickness obtained from wells and the map of the phase component at specific isofrequencies in a heavy oil field, Deng et al. (2007) detects anomalies in the spectral amplitudes associated with presence of gas, Maoshan et al. (2010) uses the spectral amplitudes to estimate absorption coefficients as a direct hydrocarbon indicator.

The natural domain for Q factor estimation is the amplitude-frequency domain (amplitude spectra of the signal). However, a major limitation of this representation is the use of the amplitude spectrum at a given time, not taking into account information of how it evolves temporally. On the other hand, the size of the window used to estimate the spectrum prevents from measuring the attenuation contrasts that might exist between
successive layers. To overcome these difficulties, we extend the amplitude-frequency domain using time as a third dimension: the result is a time-frequency distribution. In this representation, each trace is mapped onto a "gather" of spectral amplitudes: the horizontal axis is the frequency, the vertical one is time, and the amplitudes represent the spectral amplitudes. The connection to any technique of spectral decomposition is obvious. However, there are few works that take advantage of spectral decomposition techniques to compute Q factor. Li et al. (2006) to qualitatively and quantitatively characterize the attenuation in seismic data, uses the wavelet transform and through the analysis of the scalogram (equivalent to Fourier transform amplitude spectrum) is able to estimate Q, using a version of the 'frequency shift method'. However, using the scale parameter with maximum spectral amplitude, they detect the presence of gas in a structure without making use of Q factor explicitly.

This paper proposes a new methodology for Q factor estimation which uses spectral decomposition techniques, generically called Joint Time-Frequency Analysis (JTFA). The ultimate goal is to get a continuous estimate of the temporal evolution of Q. In this way, it can be used for reservoir characterization, in addition to the current usage which is given to the compensation of dispersion and absorption effects. The proposed method allows analysis in 1D, 2D and 3D.

The order of exposition of the issues is as follows. First of all, we obtain naturally the Stockwell Transform, used in JTFA, through a review of some of the techniques currently used in spectral decomposition. It is then presented briefly the method of spectral ratios, core of the methodology, which allows obtaining Q values. Finally, the new technique is presented, with a test performed on real data, and the discussion about its potential, limitations and the lines of work that opens.

**From Fourier Transform to Generalized Stockwell Transform**

Spectral decomposition comprises the set of techniques used to obtain totally or partially the spectrum of a signal evolving on time. Short Time Fourier Transform (STFT), Maximum Entropy Method (MEM), Wavelet Transform (WT), Wigner-Ville Transform (WVT), Matching Pursuit Decomposition (MPD), among others, are examples widely used in oil and gas industry.

To introduce the topic with some formalism and order, we will start with the best known tool for spectral analysis, the Fourier Transform (FT), defined as:

\[
F(h) = \int_{-\infty}^{\infty} e^{-i2\pi ft} h(t) dt
\]  

(1),

where \( f \) is frequency, \( t \) is time and \( h(t) \) is signal being decomposed.

From equation (1), we can establish that to reproduce the full spectrum, it is necessary to have a complete knowledge of the signal \( h(t) \), including all past, present and future information (thus indicate integral limits). In this way, the FT is not suitable for the treatment of non-stationary signals, since that doesn't allow estimating as evolve in time, the amplitude and phase spectra of it.

On the other hand, the numerical implementation of the FT, the Fast Fourier Transform (FFT), introduces two basic problems originated in the need to truncate the signal to be analized.
The first one is the appearance of 'spectral leakage'. To understand it, we consider the case of a sinusoidal signal with a frequency $f_0$. If frequency sampling in the FFT does not include the frequency $f_0$, the energy of the signal is dispersed on frequency samples close to $f_0$, affecting the localization of the estimated spectrum. The effect does not exist in the case where $f_0$ is one of the frequency samples of the FFT (distant to real cases). This effect is reduced to acceptable levels by multiplying the signal by a taper or window function $\psi(t)$, characterized by its length (length to which the signal is truncated) and its shape. The set of techniques involving the use of such windows $\psi(t)$ are called Short Time Fourier Transforms (STFT).

Figure 1 shows a diagram of the elements involving the STFT: we must choose the time $t_0$ where we want to perform the analysis. This time will be the center of the window. The input signal is multiplied by the window function $\psi(t)$ and then the FFT is computed. After that, the estimate of the amplitude spectra is obtained by (2):

$$E(f) = \left| \widehat{F}_n(f) \right|^2$$

Figure 1. Diagram showing how to obtain a spectrum using STFT.

The second problem is time-frequency localization. STFT characterizes it by the center of mass $<t>$ and the mean square deviation $\Delta_{\psi}$ of $\psi(t)$, and the counterparts of its Fourier Transform, $F(\psi)$: the center of mass $<f>$ and the mean square deviation $\Delta_{F(\psi)}$. The time-frequency localization does not depend on the analysis frequency $f_0$, but on the size of $\psi(t)$, which is fixed. This difficulty is a consequence of the principle of uncertainty, which imposes a strict limit to the product between $\Delta_{\psi}$ and $\Delta_{F(\psi)}$:

$$\Delta_{\psi} \times \Delta_{F(\psi)} \geq \frac{1}{4\pi}$$

Equation (3) indicates that we cannot locate simultaneously time and frequency with arbitrary precision. This means that the use of the STFT is not appropriate when we analyze signals with extensive bandwidth because it cannot localize properly low and high frequencies at the same time.

The optimal function $\psi(\alpha)$ in the sense that satisfies the equality in (3), is:

$$\psi(\alpha)(t) = \frac{1}{2\sqrt{\pi \alpha}} e^{-\frac{t^2}{4\alpha}}$$

for any fixed value $\alpha>0$ and where $t$ is time, (Gabor, 1946). STFT with $\psi(\alpha)(t)$ given by (4) is called Gabor Transform and decomposes Fourier Transform defined in (1) exactly, giving the local spectral information contained in this last.
STFT has been generalized in various directions. One of these was the development of multi-window methods where instead of considering a single window \( \psi(t) \), a family of linearly independent tapers \( \{ \psi_m(t) \} \) is used. STFT is estimated for each \( \psi_m(t) \). Spectrum is then obtained by averaging and results, even though they have less bias, have greater variance, decreasing its location. There are several of these families \( \{ \psi_m(t) \} \), such as Hermite functions, spherical prolate wavefunctions, among others.

Wavelet Transform is a multi-resolution decomposition method which constitutes another direction followed in the development of spectral decomposition techniques. It is described below.

**Wavelet Transform**

The main disadvantage of the STFT is that fixed the size of the window \( \psi(t) \), high and low frequencies cannot be localized optimally at the same time. To locate components of high frequency is required to use a window of small length, in contrast to low-frequency components, where the use of large windows is necessary. This problem is the one that leads to the development of multi-resolution spectral decomposition methods. Let be \( \psi(t) \) a window that satisfies the so-called 'admissibility condition':

\[
\int_{-\infty}^{\infty} \psi(t) \, dt = 0
\]  

Equation (5) allows introducing an extra degree of freedom, the scale parameter, which adds flexibility in the time-frequency localization. The introduction of this parameter leads to the definition of the Integral Wavelet Transform (IWT): 

\[
(W\psi h)(b,a) = \left| a \right|^{-\frac{1}{2}} \int_{-\infty}^{\infty} h(t) \psi(\frac{t - b}{a}) \, dt
\]  

In equation (6), \( a \) is so called scale parameter and \( b \) represents time where transform is calculated. IWT is defined through a series of convolutions between signal \( h(t) \) and the complex conjugate of \( \psi(t) \), where \( a \) characterizes each member of a family of functions. Scale parameter stretches and compresses the mother window \( \psi(t) \), in order to localize both low and high frequencies at the same time, at the expense of adding an extra dimension in the signal analysis space. What we get is a spectrum for each time as a function of \( a \) values. In an opposite direction to what happens with STFT which time-frequency localization is fixed, if the scale parameter is small IWT locates high frequency events, if it is large it does so with low frequency events.

Energy density \( W_{\psi}(b,a) \) of IWT is called scalogram and is the equivalent of the STFT amplitude spectrum. It is defined as:

\[
E_{\psi}(b,a) = \left| W_{\psi} h(b,a) \right|^2
\]  

Equation (7) contains information on \( a \), but not on instantaneous frequency \( f \). Although sometimes the inverse multiplicative of \( a \) is identified with central frequency \( f_c \), this is not strictly true. We can only define an approximate relation between the 'instantaneous pseudo-frequency' \( f \), the center frequency \( f_c \) and the temporal sampling interval \( \Delta_t \) of the signal:
\[ f = \frac{f_c}{a\Delta f} \]  
(8)

This represents a disadvantage of the IWT versus the STFT, because \( E_{\psi}(b,a) \) is representative of a range of frequencies rather than a specific one.

In its implementation, Morlet window is commonly used in the IWT. It is defined as:

\[ \psi(t) = \frac{1}{\sqrt{\sigma^2 \pi}} e^{\frac{t^2}{2\sigma^2}} e^{-\frac{i^2}{2\sigma^2}} \]  
(9),

where \( 2\pi\sigma_0 \) is chosen to satisfy the admissibility condition (5).

The wavelet transform is an example of JTFA, since that gives direct information on values of amplitude and phase spectra in the time-scale parameter plane. However, the fundamental problem of this technique is its incapacity to identify each transform component with a frequency, rather than with a range of them. The next step is to overcome this difficulty by introducing the Stockwell Transform, which we describe below.

Stockwell Transform

Also called S Transform is defined as the IWT of a signal \( h(t) \) multiplied by a phase factor:

\[ s(\tau, f) = W_\psi(b = \tau, a = f^{-1})(h(t)) \]  
(10)

\[ \psi(t, f) = \frac{|f|}{\sqrt{2\pi}} \exp\left(-\frac{t^2 f^2}{2}\right) \exp(i2\pi ft) \]  
(11)

The scale parameter \( a \) is defined as the inverse of the frequency of the phase factor. This factor separates mother wavelet \( \psi(t) \) into two parts: a Gaussian function located in time and a complex exponential function which selects a specific frequency.

In order to increase the resolution of the S Transform, we can define the Generalized Stockwell Transform or Generalized S Transform through the following window:

\[ \psi(t, f) = \frac{|f|}{k\sqrt{2\pi}} \exp\left(-\frac{(t - \tau)^2 f^2}{2k^2}\right) \exp(i2\pi ft) \]  
(12);

where \( k \) controls the number of oscillations in the window, so that it controls the time-frequency resolution, increasing it with respect to the IWT.

Windows \( \psi(t, f) \) in equations (11) and (12) do not satisfy the admissibility condition (5), which is the reason why they are not IWT. However, their temporal average values are equal to the Fourier transform of \( h(t) \):

\[ \langle s(\tau, f) \rangle = \int_{-\infty}^{\infty} s(\tau, f) d\tau = F(h) \]  
(13),

from where we deduce that to recover the signal, an Inverse Fourier Transform can be applied to the equation (13). The S Transform identifies each scale parameter \( a \) with a specific frequency and improves spectral decomposition resolution. These facts constitute the advantage of the S Transform over the IWT.
Generalized Stockwell Transform constitutes a robust multiresolution method of JTFA which produces low variance and unbiased estimates for time-frequency distributions. For these reason, it is a natural choice to work with in what follows.

**Q factor estimation**

The objective of this paper is to introduce a new method to estimate Q factor. Before that, it is necessary to understand how it is estimated with a traditional technique, the spectral ratio method, represented by equation (14):

$$\ln \left[ \frac{S(f)}{R(f)} \right] = \frac{\pi \Delta t_{wt}}{Q} f - \ln(G)$$

\hspace{1cm} (14),

\[
S(f) \text{ is the amplitude spectrum of a pulse emitted by a source, which is registered with amplitude spectrum } R(f) \text{ at a receiver location after traveling a time } \Delta t_{wt}. \ G \text{ represents energy losses resulting from effects that are frequency independent. The linear term in frequency describes losses due to energy absorption in the medium which are parameterized by } Q. \text{ We assume that the medium where the seismic wave travels is homogeneous. Also, } Q \text{ is frequency independent in the bandwidth of interest and its estimation is done using a zero offset seismic section. Under these hypotheses, spectral ratios, first member of equation (14), represent a straight line with slope } p \text{ so that:}
\]

$$Q = \frac{\pi \Delta t_{wt}}{p}$$

\hspace{1cm} (15).

Basically, we must estimate \( S(f) \) and \( R(f) \). The first one can be estimated from different approximations depending on data available and \( R(f) \) is calculated in a window centered at the time of interest. On the other hand, we must determine the frequency bandwidth to be used in the linear regression, because attenuation effects are progressive in time (Picotti, Carcione, 2006).

Note that Q represents an average value over \( \Delta t_{wt} \). For this reason Q in equation (15) can be called Q equivalent (Q_{eq}). In these terms, dissipation time \( \Gamma^{(n)} \) is defined as:

$$\Gamma^{(n)} = \frac{\Delta t_{wt}^{(n)}}{Q_{eq}^{(n)}}$$

\hspace{1cm} (16),

where \( n \) identify estimate values as time increases. Also, we have,

$$\Gamma^{(n-1)} \leq \Gamma^{(n)}$$

\hspace{1cm} (17),

which constitutes a strong constraint on the possible Q values at \( \Delta t_{wt} \), because dissipation cannot decrease when \( \Delta t_{wt} \) increases.

It is necessary to clarify that seismic data which are used to estimate Q must be prepared in a careful way, because every process that violates the stationarity requirement produces biased results (Anicich, 2008).

**Q factor estimation by time variant spectral ratios**

Having developed the necessary concepts, we will describe a new method to estimate \( Q \), which is schematically shown in Figure 2 and we will call Time Variant Spectral Ratios method (TVSR).

Given a zero-offset section and a location where we want to determine \( Q \), do the following steps:
1- Estimate the Generalized S Transform of every trace in a selected neighborhood (e.g. a 3 x 3 traces area around the center CDP).

2 - Build a supergather of spectral amplitudes adding all S transform obtained in 1.

3 - Model the source amplitude spectrum (reference spectrum in the spectral ratio method) calculating the average value of the traces of the supergather of spectral amplitude on a shallow time window.

4 - Interpret mutes at high (external mute) and low frequencies (internal mute): both determine the bandwidth used in linear regressions at each time. These mutes take account of the progressive decrease of the bandwidth in time.

5 - Estimate the spectral ratios gather by using the reference spectrum estimated in step 3.

6 – Perform a linear regression at each twt, according to the equation (14) to obtain \( \Gamma \) and \( Q \).

7 – Apply a median filter on estimated \( \Gamma \) and decimate its sampling interval: this is because the levels of dissipation can be distinguishable on windows large enough, otherwise \( \Gamma \) estimates are too noisy (White, 1992).

8 - Interpret interactively \( \Gamma \), which should be increasing in twt (equation 17).

9 - Built Q model by using equation (16).

10 - Repeat steps 1 to 9 at each selected location.

11 - Eventually, to get a Q section or volume, interpolate results.

![Figure 2. Q factor estimation workflow.](image)

**Test data**

To validate the methodology against real data, a seismic line of a 3D cube was investigated. These data was preconditioned by application of short wave refraction static corrections, first, second residual and “trim” statics; NMO correction with final RMS velocities and a front mute to eliminate NMO stretch (Dasgupta and Clark, 1998). Radon
transform was applied before median stack to eliminate multiples. Processes involving operator design and application by sliding windows were avoided, because they distort the amplitude spectrum by its non stationary nature, affecting spectral ratios. Source signature was not available because seismic data was acquired with dynamite.

In a first test, Q was estimated at CDP 953380, where previous values (see Table 1) were obtained using the traditional scheme of spectral ratios (Anicich, 2008). Table 1 shows for each traveltime $t^{(n)}$ (surface datum were use to reference traveltimes which implies $\Delta twt=tw_{t^{(n)}}=t^{(n)}$), $Q_{eq}^{(n)}$ and $I_{-}^{(n)}$ values derived from equations (14), (15) and (16) where minimum squares adjustment was done including only data within $(f_{1}^{(n)}, f_{2}^{(n)})$.

<table>
<thead>
<tr>
<th>$\Gamma^{(n)}$ (sec)</th>
<th>$Q_{eq}^{(n)}$</th>
<th>$t^{(n)}$ (ms)</th>
<th>$f_{1}^{(n)}$ (Hz)</th>
<th>$f_{2}^{(n)}$ (Hz)</th>
</tr>
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<tbody>
<tr>
<td>0.0111</td>
<td>81</td>
<td>900</td>
<td>12</td>
<td>55</td>
</tr>
<tr>
<td>0.0139</td>
<td>93</td>
<td>1300</td>
<td>15</td>
<td>42</td>
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<td>1700</td>
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<td>42</td>
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<td>0.0176</td>
<td>119</td>
<td>2100</td>
<td>10</td>
<td>35</td>
</tr>
<tr>
<td>0.0161</td>
<td>155</td>
<td>2500</td>
<td>12</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 1. $Q$ estimate at CDP 953380 (Anicich, 2008).

STFT was computed using a window of 500 milliseconds long, centered at $t^{(n)}$. Without having specific horizons to do the analysis and taking into account that the original
objective was to compensate for intrinsic attenuation and dispersion, time sampling was defined from 900 ms to 2500 ms each 400 ms.

At $t^{(n)}$ equal to 2500 ms an inversion of $\Gamma^{(n)}$ is observed; it constitutes a violation of condition (17). Another interesting point is the oscillating behavior of $f_1^{(n)}$. These two problems cannot be removed consistently unless additional information on general attenuation trend is available.

Using TVSR, time-frequency distribution was estimated at the same CDP (figure 4a). It is observed how frequency bandwidth progressively decreases with traveltime. A neighborhood of 3 x 3 traces was selected to build a smoother time-frequency distribution (figure 4b), and a reference signature $S(f)$ was estimated using a shallow data window between 0 and 300 milliseconds (blue line at the top of the figure 4b). Frequency bandwidth of Table 1 (black lines in figure 4b) was used in the linear adjustment of spectral ratios (figure 4c), just for comparison purposes. It was originally estimated at the frequencies where spectrum falls -20dB with respect to its maximum, however small variations on the obtained values were allowed in order to enhance linear fit. Attenuation trend can be seen in figures 4a and 4b. It constitutes an advantage of the method because it allows seeing time windows where dissipation is small enough to produce random fluctuations in $\Gamma$ and where attenuation becomes important. For example, a strong contrast in attenuation can be seen at 1100 ms approximately in figures 4a and 4b.

Dissipation times $\Gamma$ obtained using spectral ratio gather (figure 4c) were filtered by a median operator of 150 milliseconds window length and was decimated to 100 milliseconds sampling rate. After that, raw $Q_{eq}$ was estimated (right panel of Figure 6, red
circles). We can note that as time increases, frequency bandwidth decreases and thus, where dissipation is small, $\Gamma$ has significant fluctuations. These fluctuations are not consistent with conservation of energy, expressed by equation (17), so they must be eliminated through interactive interpretation of dissipation time.

Linear piecewise interpretation of $\Gamma$ is shown on the left panel of Figure 6. Blue circles describe $\Gamma$ model, which was used to obtain Q model (left and right panel respectively). This step allows giving to Q function a smooth aspect. At the bottom of the section, where fluctuations in $\Gamma$ are significant, the behavior was approximated with a single almost constant function, which says to us that no significant dissipation is observed with such a fluctuating behavior.

Table 2 shows a comparison between the results obtained using the TVSR method and the estimates of $Q_{eq}$ earlier done. Values of $\Gamma^{(n)}$, $Q_{eq}$, $\Gamma^{(n)}$ (TVSR) and $Q_{eq}$ (TVSR). When we use TVSR for Q computation, $\Gamma$ inversion at 2500 ms, is easily removed by interpretation. This step allows us to see the trend that follows the dissipation time as traveltime increases, enabling us to interpret it and improving its resolution.

An extension of TVSR method can be attenuation analysis at regularly spaced CDP locations, like velocity analysis. Results of this type of analysis can be interpolated and smoothed to obtain a smooth Q section. In this case frequency bandwidth and $\Gamma$ were interpreted every 30 CDPs. Figure 6 shows, just like an example, how lateral attenuation

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**Figure 5.** Interactive analysis of dissipation time.

**Table 2.** $Q$ factor estimated at CDP 953380 TVSR.
strength can be detected using TVSR, enabling us to choose smoothing parameters to see details in the appropriate scale to our particular purposes (not a particular one in this case). In this way Q anomalies can be identified and delimited to characterize reservoir rocks.

![Figure 6. Section of Q factor superimposed on the seismic amplitudes.](image)

**Conclusions**

A new method was introduced to estimate Q factor using spectral decomposition techniques. Choice of Generalized S Transform was done in order to optimize time-frequency localization allowing, at the same time, a regular frequency sampling. It can be said that one of its advantages is the visualization of the attenuation trend behavior. This allows establishing intervals of homogeneous attenuation behavior, as the analyst is able to clearly see the trend of this phenomenon in the gathers of spectral amplitudes. The same type of display is very useful to determine frequency bandwidth for Q estimation: internal and external mutes can be interpreted as a function of time, representing a progressively decrease frequency content.

Equation (17) places a strong restriction under the assumptions of the method. This is used for attenuation trend interpretation. Rather than estimate Q directly, it can be done indirectly by making use of a reliable Γ interpretation obtained from raw values. Sampling interval in time (dt) should be chosen large enough as to capture significant variations in Γ. Note that dt was arbitrary determined when a median filter was applied followed by a decimation. These two processes can be adapted to the particular objective of the study and quality of the seismic data.

At the interpretation stage is where geological markers can be introduced to allow a further refinement of Γ. Interactive interpretation of Γ gives consistency and eliminates statistical fluctuations due to low dissipation levels or noisy data. The stage of interpolation involves interpretation of several control points in the area of interest. The result is a Q section or cube that can be used in reservoir characterization. However, when we are talking about processing and imaging, we need to be careful to
compensate for attenuation and dispersion using a spatial variable Q field, because diffraction information can be loss for subsequent processing steps.

The fundamental limitation of TVSR method is the use of sections stacked as approximation to zero offset sections, basic assumption of the technique of spectral ratios, as it is presented in this work. However, the method of spectral ratios is easily extensible to constant offset panels (Dasgupta and Clark). Consequently, a version of the method introduced here can be generalized to be applied to pre-stack seismic data.

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