How Long Should the Sweep Be?
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Summary
A sweep, a sinusoid with a continuously variable frequency, can be defined by its amplitude \( A(f) \) and its sweep rate \( Sr(f) \). Provided the sweep is long enough (longer than 5 or 6 seconds), the amplitude spectrum of the sweep at frequency \( f \) is proportional to \( A(f) \) and to the square root of \( Sr(f) \). Target-oriented sweep design consists in defining \( A(f) \) and \( Sr(f) \) to obtain the desired Signal-to-Noise ratio of the target reflection.

Introduction
Non-linear sweeps were introduced in late 1970s for the purpose of generating a higher proportion of high frequencies to partially compensate for attenuation. The available software only proposed constant amplification over the full frequency range often resulting in a damaging reduction in the low frequency content. This drawback was noticed and more sophisticated electronics were developed to allow more flexibility. An example of such a technique contributing to a remarkable image enhancement was shown by D. Mougenot in 2002.

Sweep analysis
Notations
\( f \)  
Frequency
\( f_s \)  
Start frequency of the sweep
\( f_e \)  
End frequency of the sweep
\( SL \)  
Sweep length
\( Sr(f) \)  
Sweep rate \( (df/dt) \) Can also be expressed as a function of time
Correlation operators are assumed to have an amplitude spectrum of 1 within the signal bandwidth.
A sweep is a signal that can be expressed as
\[
S = A_0(t) \cos(\varphi(t))
\]
Where \( A_0(t) \), the amplitude, is a positive continuous function on interval \( [0 \ SL] \), \( \varphi(t) \), the instantaneous phase, is a continuous function of time on this interval and
\[
f(t) = \frac{1}{2\pi} \int \frac{d\varphi}{dt}, \text{ the frequency, is a continuous function of time on this interval. In the following discussion, it will be assumed that } f(t) \text{ is monotonic and increasing.}
\]
If \( A_0(t) = 1 \), the sweep is called a unit amplitude sweep.

General sweep equation
A unit amplitude sweep is fully defined by its start frequency \( f_{\text{min}} \) and its sweep rate \( sr(t) \).
The instantaneous frequency of a unit amplitude sweep is given by
\[
f(t) = f_{\text{min}} + \int_0^t Sr(\tau) d\tau
\]
Note that the sweep length \( SL \), the sweep rate \( sr \), the start and end frequencies \( f_{\text{min}} \) and \( f_{\text{max}} \) are linked by:
\[
f_{\text{max}} = f_{\text{min}} + \frac{SL}{\int_0^t Sr(\tau) d\tau}
\]
The instantaneous phase of the sweep is
\[
\varphi(t) = 2\pi \int_0^t f(\tau) d\tau + \varphi_0
\]
The sweep equation becomes:
\[
S(t) = \cos \left[ 2\pi \int_0^t \left( f_e + \int_0^t Sr(\tau) d\tau \right) d\varphi + \varphi_0 \right]
\]
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Figure 3: Sweep description
Blue: linear sweep, green: non-linear sweep.

Figure 3a represents the corresponding sweep rate designed to enhance both the low and high frequencies relative to the medium frequency range. Figure 3b represents the corresponding frequency curve from 1 to 12 Hz. Figures 3c represents the corresponding sweeps. Differences in the rate of frequency increase can be seen at the beginning and at the end of the sweep.

Amplitude spectrum
It can be shown, using the stationary phase approximation that the amplitude spectrum of a sweep with sweep rate \( df/dt = Sr(f) \) is

\[
A_f = \frac{1}{2} \sqrt{\frac{1}{Sr(f)}}
\]

(7)
The fundamental result is that the amplitude spectrum of a sweep is proportional to the square root of its inverse sweep rate. Figure 4 represents the square root of the inverse sweep rate (in red) superimposed on the amplitude spectrum of the sweep (in blue). The relatively large discrepancy on the left panel results from the combination of a narrow frequency range (11 Hz), a short sweep length (4 s), and a relatively large taper length (0.8 s). On the right panel, the length is multiplied by 4.

Sweep Design
Abundant experimentation with low energy sources has shown that Signal-to-ambient-Noise ratio can be made proportional to the square root of the order of the summation, regardless of the signal amplitude. (Schissele et al., 2009)

These experiments support the statement that penetration and Signal-to-(ambient)-Noise ratio are two facets of the same frequency-dependent phenomenon:

- If S/N > 1 at frequency \( f \), the reflection is seen at this frequency; the corresponding interface is then “penetrated”.
- If S/N < 1 at frequency \( f \), the reflection is not seen at this frequency; the corresponding reflection is not penetrated.

The sweep design problem consists in obtaining an adequate S/N ratio over as wide a frequency range as possible. “Adequate” is quite subjective; it means neither too high nor too low. It refers to a reference acquisition, which, for a given target reflection, provides acceptable S/N ratio at a reference frequency, \( f_{ref} \). If the subscript \( ref \) is used for the reference acquisition and the subscript \( new \) for the new acquisition, the factors multiplying the reference S/N ratio in the new acquisition, (with hopefully a wider frequency range) are the following:

- **Acquisition parameters.** Their effect can be described by the frequency-independent scalar:

\[
A_{acq} = \frac{PF_{new} \cdot D_{new} \cdot Nv_{new} \cdot \sqrt{N_{ref} \cdot Sd_{ref} \cdot Ra_{ref}}}{PF_{ref} \cdot D_{ref} \cdot Nv_{ref} \cdot \sqrt{N_{ref} \cdot Sd_{ref} \cdot Ra_{ref}}}
\]

(8)

Where \( PF \) is the Peak force, \( D \) the drive, \( Nv \) the number of vibrators, \( Nr \) the number of receiver per SP, \( Sd \) the source density and \( Ra \) the receiver area (Meunier, 2011, p. 116, equation 24). Note that this scalar does not take into account the effect of the sweep rate (which is what we are trying to determine.)

- **Ambient Noise.** This is the denominator in the S/N ratio. We assume that its behavior can be described by its Power spectrum density, \( N(f) \):

\[
A_s = \sqrt{\frac{N_{ref}}{N_{new}(f)}}
\]

(9)

- **Absorption.** The amplitude of the target reflection, \( t \), is assumed to be described by a single “average” quality factor \( Q_a \). Its effect is given by:

\[
A_{abs} = \exp\left( -\pi t \frac{f - f_{ref}}{Q_a} \right)
\]

(10)

- **Source radiation.** This results in 6 dB/octave amplification when seismic data represent particle velocity or pressure.

\[
A_r = \frac{f}{f_{ref}}
\]

(11)

- **Vibrator limitations.** We assume that these limitations can be described by the ratio, \( \rho(f) \), of the admissible-to-nominal amplitudes (actual sweep amplitude)

\[
A_l = \frac{\rho_a(f)}{\rho_{n}(f_{ref})}
\]

(12)
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- **Migration effects.** If the noise is uncorrelated and there is no anti-alias filter in the operator, the migration effect on signal amplitude is comparable to the effect of the sum in a Fresnel zone and the effect on noise to the effect of the sum in the migration aperture. This results in a 6-dB/octave reduction. In case of inadequate spatial sampling, the migration operator may become aliased; the effect of anti-alias filtering of the operator is to reduce the noise summation domain and in turn reduce noise (amplitude and migration accuracy) for the relevant frequencies. Moreover, in the very low frequency range, ambient noise is often correlated on large distances; its sum has therefore a higher amplitude than if it were fully uncorrelated. This may result in a complex expression that can be expressed as $m(f)$:

$$A_n = \frac{m_{\text{new}}(f)}{m_{\text{ref}}(f)}$$

Note that when the new and the reference spatial samplings are different, $m_{\text{new}}$ and $m_{\text{ref}}$ are different.

Figure 5 represents the amplification necessary to compensate for the combination of the above effects in the case of the vibrator and the target depth used in the example below and for various quality factors (Qa). The case of the vibrator and the target depth used in the source amplitude spectrum squared) must be:

$$S_{\text{new}} = \frac{S_{\text{ref}}}{(A_{\text{aqg}}A_nA_{\text{ars}}A_{\text{rs}}A_{\text{m}})^2}$$

The design procedure includes the following steps:

1. **Choice of $f_{\text{ref}}$.** It is obtained by the analysis of vintage data; Ideally, $f_{\text{ref}}$ is a frequency for which S/N ratio of the vintage seismic image is adequate; that is, neither too low nor too high. It is also convenient to avoid very low frequencies where $\rho_b$ is not 1.

2. **Determination of $S_{\text{ref}}$ directly from reference acquisition parameters.**

3. **Computation of $A_{\text{aqg}}$ from the reference and new acquisition parameters using equation 8.**

4. **Determination of $N_{\text{new}}(f)$ and $N_{\text{ref}}$.** It is necessary to have an ambient noise analysis in the area. Ideally, this analysis should cover various noise conditions; $N_{\text{new}}(f)$ is then the median Power Spectral Density of the noise. In general, $N_{\text{ref}}$ can be taken as $N_{\text{new}}(f_{\text{ref}})$.

5. **Determination of $\rho_b$.** It is obtained from the vibrator characteristics as follows:

   *The vibrator pump can move the mass to its full stroke above frequency, $f_{\text{flow}}$; its maximum stroke is large enough to generate its nominal force above frequency $f_{\text{stroke}}$."

   $$\frac{f_{\text{stroke}}}{f_{\text{flow}}} = \left(\frac{f_{\text{stroke}}}{f_{\text{flow}}}ight)^k$$

   $\rho_b(f) = \left(\frac{f_{\text{stroke}}}{f_{\text{flow}}}ight)^k$ for $f < f_{\text{flow}}$

   $\rho_b(f) = 1$ for $f > f_{\text{flow}}$

   It is often advantageous to also apply amplitude limitation when frequency exceeds a given threshold, $f_c$, to minimize the adverse effects of servovalve limitations, overpressure and baseplate flexing while maintaining a higher drive level below this threshold:

   $\rho_b(f) = (f_c/f)^k$ for $f > f_c$

   with exponent $k$ in the range of $1/4$ to $1/2$.

6. **Determination of $Q_a$.** It is estimated from vintage data. Ideally, VSP data will allow observation of the upper frequency range and provide a more reliable estimate.

**Example**

The reference acquisition is a 12-line acquisition with a 50-m group interval and 200-m source-and-receiver-line intervals. It used a linear sweep from 10 to 70 Hz over 12s. The new acquisition is a 100-line acquisition with a 25-m group interval and 100-m source-and-receiver-line interval. Their relevant parameters are:

<table>
<thead>
<tr>
<th>$P_f$ (kN)</th>
<th>$D$ (%)</th>
<th>$N_v$</th>
<th>$S_d$ ($/\text{km}^2$)</th>
<th>$N_r$</th>
<th>$R_a$ ($\text{m}^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref</td>
<td>200</td>
<td>70</td>
<td>4</td>
<td>100</td>
<td>2400</td>
</tr>
<tr>
<td>New</td>
<td>300</td>
<td>80</td>
<td>1</td>
<td>400</td>
<td>40000</td>
</tr>
</tbody>
</table>

Analysis of the reference acquisition leads to

$f_{\text{ref}} = 50$ Hz and $Q_a = 90$ for the target at time $t_t = 1.4$ s.

The noise Power Spectral Density is given by Figure 6. In this case, seismic noise is larger than instrument noise in the [4-40 Hz] frequency range.

**Figure 5:** Relative amplification required to obtain a flat spectrum

**Figure 6:** Power Spectral density of Noise ($N$). The blue curve
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represents total ambient noise. The black curve represents the noise floor (instrument noise.) It is larger than seismic noise below 4 Hz and above 40 Hz.

The vibrator characteristics lead to
\[ f_{\text{flow}} = 7.5 \text{ Hz}, \quad f_{\text{stroke}} = 5 \text{ Hz}, \] we use \( f_c = 50 \text{ Hz} \) and \( k = 1/3 \)

The corresponding sweep amplitude is shown in Figure 7.

Anti-aliasing and Low frequency noise correlation are neglected in \( A_w \) where \( (A_w = f_{\text{ref}}/f) \). In this case, the source radiation factor and the migration factor exactly cancel each other.

Equation 14 becomes:

\[
S_{\text{new}} = \left( 1.75 \sqrt{N/N(50)} \exp \left( -\frac{\pi f}{Q} \left( f - 50 \right) \right) \right)^{5} \frac{1}{\rho_s(f)} \frac{50}{f} \left( \frac{f}{50} \right)
\]

Figure 8 represents the increase in S/N ratio (relative to the reference acquisition) to obtain a flat spectrum of the target reflection. It is the square root of \( S_{\text{new}} \). To choose the sweep length, it is convenient to separately analyze the time necessary to extend the sweep range toward low frequencies and high frequencies.

This is shown in Figure 9 in which the blue curve gives the sweep time above and the red curve the sweep time below 10 Hz. Extending the reference low-frequency limit to 4 Hz only costs 1 minute/km². Starting at 2 Hz requires a significantly larger effort (1/2 hour/km²). Likewise, a 60-Hz high-frequency limit is quite affordable (14 minutes/km² spent between 10 and 60 Hz); it would actually require less sweep time than the reference acquisition (20 minutes/km²). However, an 80-Hz high-frequency limit results in a significant cost increase (1 hour and 48 minutes/km² spent between 10 and 80 Hz).

Such an increase may seem unreasonable. However, taking advantage of simultaneous acquisition techniques such as V1 (Postel et al., 2008) or ISS (Howe et al., 2008) will make this increase affordable in terms of time spent per km². In 2010, P. Pecholcs reported a simultaneous acquisition experiment in which a total of more than 160 hours of simultaneous vibrations were acquired in a single day. In the present example, for a flat spectrum stretching from 2 to 80 Hz on the target reflection, this productivity would make it possible to record 78 km² in a day!

In addition, the trace density increase of the new acquisition will result in better organized noise reduction in the low frequency extension of the spectrum and better spatial sampling to take full advantage of pre-stack migration and properly image steeper dips in higher frequency range.

Conclusions

Bandwidth extension in vibroseis data involves the use of very low sweep rates in the edges of the data bandwidth. One consequence can be a significant lengthening of sweep time. Many factors other than the sweep rate affect the Signal-to-ambient-Noise ratio. It is necessary to take them all into account to optimize the benefit of sweeps with variable sweep rates. The combination of bandwidth extension with simultaneous acquisition techniques can limit or even avoid acquisition time increase.

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