Pre-stack deghosting for variable-depth streamer data
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SUMMARY

Variable-depth streamer acquisition is an acquisition technique aiming at achieving the best possible signal-to-noise ratio at low frequencies by towing the streamer very deeply, but by using a depth profile varying with offset in order not to limit the high frequency bandwidth. Previous papers have shown how a joint deconvolution allows the post-stack deghosting of such variable-depth streamer acquisitions, and the purpose of this paper is to show how this can be generalized to a multi-channel joint deconvolution that allows pre-stack though post imaging deghosting of such acquisitions. After computing migrated gathers as well as mirror migrated gathers, a deghosted imaging deghosting of such acquisitions. Post-stack deghosting, the two inputs being the migration and the mirror migration being recovered as

\[ G_{\text{min}}(t) * R(t) = g_{\text{min}}(t) * r(t) \]
\[ G_{\text{max}}(t) * R(t) = g_{\text{max}}(t) * r(t) \]

Although the joint deconvolution problem as stated by equation (3) looks like the conventional deconvolution model, it has totally different mathematical properties. It is a well-posed problem, which means it has a unique solution, even when the minimum phase and maximum phase properties are marginally respected (meaning the operators have perfect spectral notches) (Soubaras, 2010).

A mathematical proof can be sketched as following: suppose we have two solutions \((g_{\text{min}}, g_{\text{max}}, r)\) and \((G_{\text{min}}, G_{\text{max}}, R)\) of problem (3), we therefore have:

\[ G_{\text{min}}(t) * R(t) = g_{\text{min}}(t) * r(t) \]
\[ G_{\text{max}}(t) * R(t) = g_{\text{max}}(t) * r(t) \]

therefore:

\[ R(t) * r^{-1}(t) = G_{\text{min}}^{-1}(t) * g_{\text{min}}(t) \]
\[ R(t) * r^{-1}(t) = G_{\text{max}}^{-1}(t) * g_{\text{max}}(t) \]

and:

\[ G_{\text{min}}^{-1}(t) * g_{\text{min}}(t) = G_{\text{max}}^{-1}(t) * g_{\text{max}}(t) \]

The l.h.s. of equation (6) is causal because a minimum phase signal is causal and has a causal inverse. The r.h.s is anticausal because a maximum phase signal is anticausal and has an anticausal inverse. The only way a signal can be both causal and anticausal is by having a non-zero value only at lag 0, therefore being proportional to a Dirac function \(\lambda \delta(t)\). Because of the normalizations, \(\lambda\) must be 1. Therefore:

\[ G_{\text{min}}(t) = g_{\text{min}}(t) \]
\[ G_{\text{max}}(t) = g_{\text{max}}(t) \]
\[ R(t) = r(t) \]

which proves that the joint deconvolution has an unique solution.

The joint deconvolution is used to deghost post-stack by using, in overlapping space-time windows, the migration image as \(d_1(t)\) and the mirror migration image as \(d_2(t)\), the deghosted migration being recovered as \(r(t)\).
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PRE-STACK DEGHOSTING BY MULTICHANNEL JOINT DECONVOLUTION

The joint deconvolution allows to deghost after stack a variable depth streamer acquisition. The deghosting can also be performed before stack, on common image gathers (that is before stack but after imaging), by using a multichannel joint deconvolution model. The multichannel joint deconvolution considers the data $d_1(t,h)$, which is the pre-stack gather after normal imaging and $d_2(t,h)$ the pre-stack gather after mirror imaging. $h$ is the offset dimension or any other dimension of a pre-stack gather such as angle. Noting the reflectivity gather by $r(t,h)$ we can write the following multichannel joint deconvolution model, by introducing normalized minimum and maximum phase operators $g_{\min}(t,h)$ and $g_{\max}(t,h)$

\[
\begin{align*}
  d_1(t,h) &= g_{\min}(t,h) * r(t,h) \\
  d_2(t,h) &= g_{\max}(t,h) * r(t,h)
\end{align*}
\]

Written as this, the multichannel deconvolution problem is already well-posed, as it consists of $N_h$ separate joint deconvolutions. However, solving the problem this way results in amplification of the noise due to each ghosts notch. In order to take advantage of the notch diversity, we must couple these otherwise independant problems in $h$. This can be done by imposing to the reflectivity gather $r(t,h)$ a parametrical variation in $h$:

\[
r(t,h) = \sum_{i=0}^{p} a_i(t)T_i(h)
\]

where $T_i(h)$ is typically a set of orthogonal polynomials.

Inverting for $a_i(t)$, $g_{\min}(t,h)$ and $g_{\max}(t,h)$ gives a reflectivity model $r(t,h)$ that can be considered as the deghosted gather. It is however better to use $r(t,h)$, $g_{\min}(t,h)$ and $g_{\max}(t,h)$ to produce a ghost model and a mirror ghost model:

\[
\begin{align*}
  G_1(t,h) &= g_{\min}(t,h) * r(t,h) - r(t,h) \\
  G_2(t,h) &= g_{\max}(t,h) * r(t,h) - r(t,h)
\end{align*}
\]

which can be subtracted to the gather $d_1(t,h)$ and mirror gather $d_2(t,h)$. The two deghosted images can be combined, thus reinforcing the pre-stack image produced by the direct arrival by the pre-stack image produced by the arrival which is reflected by the water-surface.

SYNTHETIC DATA EXAMPLE

Figure 1 shows a synthetic shot gather for a variable-depth streamer acquisition. The reflectivity consists in three pairs of events at 1, 2 and 3 seconds. The fact that the reflectivity has double events guarantees that it is not white. Random noise has been added to the shot gather. Figure 2 and 3 shows a migrated gather and migrated mirror gather. These have been computed with a non-exact velocity model, as can be seen by the residual moveouts.

On the gather, the black events correspond to the upgoing wavefield and the white events to the downgoing wavefield (ghosts). On the mirror gather, we can check that the black events have exactly the same moveout curve as the black events on the normal gather, the perturbing white events being precursors. This is due to the fact that on the mirror gather the black events correspond to the downgoing wavefield and the white ones to the upgoing wavefield.

Figure 4 shows the deghosted gather computed from the gather and the mirror gather by pre-stack joint deconvolution. Figure 5 shows the synthetic shot gather computed by modeling the data with no reflecting water-surface and no random noise added. Figure 6 is the migration of this unghosted shot record and this can be considered as the reference gather, that is what a perfect deghosting should produce. The very good match between the deghosted gather in Figure 4 and the reference gather in Figure 6 can be checked. Figure 7 shows a RMO curve picked on the event at 3 seconds and Figure 8 the AVO response along this curve for the deghosted gather in black and the reference gather in red. The deghosted gather reproduces very well the AVO response of the reference gather.

REAL DATA EXAMPLE

A variable-depth data acquisition was performed in the Gulf of Mexico. The bandwidth achieved on this dataset was six octaves [2.5 Hz - 160 Hz]. Figure 9 shows the migration and Figure 10 the mirror migration. Pre-stack deghosting was done on this dataset and Figure 11 shows the stack of the pre-stack deghosted gathers. The effect of the pre-stack deghosting can be seen on the interleaved gathers. Amplitudes variations and residual moveout have been preserved on the deghosted gathers.

CONCLUSION

We have described how the joint deconvolution can be used to perform the pre-stack receiver deghosting of variable-depth streamer data. The method uses the common image gathers after migration and mirror migration. This deghosting method is true amplitude, preserves AVO behaviour and residual moveout curves. Tests on synthetic data has shown that it can recover the AVO curve of the reference unghosted data.
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Fig. 1: Shot record
Fig. 2: Migrated gather
Fig. 3: Mirror migrated gather
Fig. 4: Deghosted gather

Fig. 5: Unghosted shot record
Fig. 6: Reference gather
Fig. 7: RMO curve
Fig. 8: AVO for Fig. 6-7 (black=deghosted, red=reference)
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Fig. 9: Migration and gathers.

Fig. 10: Mirror migration and gathers.

Fig. 11: Deghosted migration and gathers.
REFERENCES