Compensating for source and receiver ghost effects in reverse time migration

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Summary

For marine seismic data, the source and receiver ghosts generated by the free surface cause angle dependent frequency and amplitude distortion. For better interpretation and inversion, these unwanted effects are best corrected in a prestack depth image. Based on true amplitude migration theory, we propose to compensate for the ghost effects in a reverse time migration. The numerical examples demonstrate that with the compensation terms, we can generate a wide bandwidth reflectivity map. In particular, the low frequency geological variation is better delineated in the image. We also show that with some simple modifications, the proposed reverse time migration can generate sensible velocity perturbations for seismic inversion.

Introduction

To interpret subtle geologic features, seismic data must contain both the low and high frequencies required for high resolution imaging. High-fidelity, low frequency data provides better penetration to illuminate deep targets, as well as providing greater stability and more important information for seismic inversion. For such reasons it is desirable to get a wide bandwidth seismic image.

To achieve wide bandwidth prestack depth images from marine seismic data, we have to overcome the bandwidth limitation imposed by source and receiver ghosts. Many solutions have been proposed recently. For example, variable-depth streamer acquisition (Soubaras and Whiting, 2011) is emerging to take advantage of the low noise diversity caused by receiver depth variation. As a result, it yields a high quality broadband spectrum. The receiver ghost can be removed in a later processing stage by some novel processing and imaging techniques (Soubaras, 2010; Soubaras and Lafet, 2011). The new method creates an exceptionally sharp and clean wavelet for interpretation.

However, removing the source ghost remains a more difficult task if we assume the sources are excited at a roughly fixed depth (i.e. lack of notch diversity) and with a big shot increment in lateral positions (i.e. severe data aliasing), as what happens mostly in marine tow-streamer surveys.

In this abstract, we propose a method to compensate for source and receiver ghost effects in a Reverse Time Migration (RTM). By studying true amplitude migration theory, we demonstrate that the existence of ghosts distorts both migration spectrum and the amplitude versus angle (AVA) relation. Hence deghosting is a crucial step for AVA analysis. We have developed a theory to remove the ghosts during RTM to obtain a high fidelity, high resolution image. The numerical results show that both reflectivity and velocity perturbation can be well recovered with the proposed RTM.

Theory

Source and receiver ghosts: Let us assume the source depth is $\Delta z_s$ and $v_0$ is the acoustic wave speed of water. The source ghost generated by the free surface reflection is an angle dependent effect, which changes both the wavelet amplitude and spectrum. To simplify our discussion, we assume the surface reflectivity is -1. At the source location, the source ghost acting on the wavefield with the propagation angle $\alpha_s$ takes the form in frequency domain:

$$ G_s(\omega, \alpha_s) = e^{-i\omega \Delta z_s} - e^{-i\omega \Delta z_s} = -2i\sin \frac{\omega \cos \alpha_s \Delta z_s}{v_0} \cdot (1) $$

In frequency and wavenumber domain, we have the following relation

$$ \cos \alpha_s = \sqrt{1 - \frac{v_0^2}{c^2}(k_x^2 + k_z^2)} \cdot (2) $$

Similarly, the receiver ghost can be expressed as

$$ G_r(\omega, \alpha_r) = e^{-i\omega \Delta z_r} - e^{-i\omega \Delta z_r} = -2i\sin \frac{\omega \cos \alpha_r \Delta z_r}{v_0} \cdot (3) $$

Reverse time migration: True amplitude RTM has been developed in Zhang and Sun (2009) and Xu et al. (2011). To migrate a shot record $Q(x, y, z, t)$ with the shot at $(x_s, y_s, z_s = 0)$ and receivers at $(x, y, z = 0)$, we have to compute the wavefields originating at the source location and observed at the receiver locations. Because the source wavefield expands as time increases and the recorded wavefield is computed backward in time, we denote them by $p_x$ and $p_y$ respectively in the following acoustic wave equations:

$$ \left[ \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \Delta \right] p_x(\vec{x}, t; \vec{x}_s) = 0, \quad (4) $$

$$ p_x(x, y, z = 0; t; \vec{x}_s) = \delta(\vec{x} - \vec{x}_s) \int_{\mathbb{R}} f(t') dt', $$

and

$$ \left[ \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \Delta \right] p_y(\vec{x}, t; \vec{x}_s) = 0, \quad (5) $$

$$ p_y(x, y, z = 0; t; \vec{x}_s) = Q(x, y, z, \vec{x}_s; t), $$

where $v = v(\vec{x})$ is the velocity, $f(t)$ is the source signature, and $\Delta = \partial^2_x + \partial^2_y + \partial^2_z$ is the Laplacian operator.
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It has been proven that wave propagation (4) and (5) together with the following 3D imaging condition provide AVA friendly migration amplitude in the subsurface angle domain (Xu et al., 2011)

\[ R(\tilde{x}, \theta; \phi) = \int \frac{v(\tilde{x})}{\sin \theta} \delta(\theta' - \theta) \delta(\phi' - \phi) p_\alpha p_r d\theta' d\phi', \]

(6)

where \( \theta \) and \( \phi \) are the reflection angle and azimuth angle at the imaging location, respectively.

**Compensating for the ghost effects in RTM**: A defect of the aforementioned true amplitude RTM theory is that the ghost effects are ignored. Since in RTM, we propagate an artificial source wavefield, it is straightforward to compensate for the source ghost during the migration. To achieve this, we need to modify the boundary condition in (4) to the following

\[ \hat{p}_S(x, y, z = 0; \alpha, \tilde{x}_s) = \delta(\tilde{x} - \tilde{x}_s) \frac{f(\omega)}{i\omega G_r(\omega, \alpha, \alpha_s)}, \]

(7)

where \( \hat{p}_S(\omega) \) represents the Fourier transform of \( p_s(t) \).

A similar idea can be applied to the receiver side to compensate for the receiver ghost

\[ \hat{p}_R(x, y, z = 0; \alpha, \tilde{x}_r) = \frac{Q(x, y; x_s, y_s; \omega)}{G_s(\omega, \alpha_s)} \]

(8)

However, when seismic data is aliased, directly applying compensation formula (8) may cause instability, so some regularization remedy must be applied during the wave propagation.

In theory, imaging condition (6) provides angle dependent reflectivity (Bleistein et al., 2001). Compensating ghost effects leads to reliable low frequency components in the image. Such information can be used to estimate velocity, as demonstrated in waveform inversion methods (Tarantola, 1984). To obtain velocity perturbation from an RTM, we just need to modify boundary condition (8) and imaging condition (6) as follows (Jin et al. 1992)

\[ \hat{\delta}(\tilde{x}) = \int p_b(\tilde{x}; t, \tilde{x}) p_r(\tilde{x}; t, \tilde{x}) d\tilde{x}, \]

(10)

**Numerical experiments and examples**

The first example is designed to prove the reliable amplitude and spectrum response of RTM after compensating for the ghost effects. A similar test was done in Zhang and Sun (2009) without ghosts in the seismic data. Here we assume both source and receiver ghosts are recorded. Figure 1 shows a 2D single shot record over five horizontal reflectors at different depths, with the shot in the center of the section and the receivers out to an offset of 7500m on either side. The shot and the receiver depth are 10m and 15m, respectively, and the water velocity is 1500m/s. In the modeling, we assume the reflectivity is uniform at all reflection points over all reflection angles. Due to the existence of ghosts, both the wavelet amplitude and spectrum are distorted across travel time and lateral distance, in addition to the effect of geometrical spreading. If we use the conventional true amplitude RTM formulation (4) and (5), stack all the migrated common image shot gathers to generate subsurface offset gathers (Sava and Fomel, 2003), and then convert them to subsurface angle domain common image gathers (CIGs), as shown in Figure 2, we end up with a distortion in the spectrum of the migrated image (left) and an incorrect AVA trend (right). After we have compensated for the ghost effects on both source and receiver sides, the wavelets on the migrated angle gather have wider and more balanced frequency bandwidth and appear much sharper (Figure 3, left). Also, the normalized peak amplitudes along reflectors in the reflection angle domain converge well which indicates the reflectivity is well recovered and the AVA relation is more reliable (Figure 3, right).

![Figure 1](image)

Figure 1: A shot record over five horizontal reflectors in a medium with \( v = 1500 \text{m/s} \). Both source and receiver ghosts are recorded.

In the second example, we apply RTM to the 2004 BP 2D model (Billette and Brandsberg-Dahl, 2005). The synthetic data was generated by high order finite-difference acoustic modeling using both velocity and density models, with shot spacing 50m, receiver spacing 25m and 12000m maximum offset. Both source and receiver ghosts are recorded with \( \Delta x = \Delta z = 12.5 \text{m} \). In Figure 4, we compare the conventional stacked RTM image (left), the RTM image with both source and receiver ghost compensation (middle), and the impedance model (right), which is the product of the velocity and density. It is clear that the new RTM image (middle) has much more balanced energy over the migration bandwidth, which better delineates the salt boundaries, reflectivity polarity and the textures of the geological layering. Also, the new image shows more low frequency energy, well correlated to the trend of impedance variations (right). This test shows that the compensated low frequencies derived from deghosting are quite reliable.

Next, we use our true amplitude RTM to compensate for the ghosts and to produce velocity perturbation instead of reflectivity, as described by the modified boundary
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condition (9) and imaging condition (10). Here we used the Sigsbee2a model to generate the synthetic data set, with the source depth at 25ft. We migrate the data using a constant gradient background velocity (Figure 5, left). The RTM image with source ghost compensation is shown in the middle of Figure 5. The right picture of Figure 5 is the real velocity perturbation, derived by subtracting the migration velocity from the exact modeling velocity. With the ghost compensation, the RTM image does a good job in reconstructing the original velocity perturbation, as predicted by the theory. In Figure 6, we choose one imaging location, and compare the velocity perturbation computed from RTM (left) with its real curve (right). The overall match of the two is quite good, except for a scaling difference, which can be calibrated by a linear search.

Figure 2. Left: A migrated angle domain CIG using conventional true amplitude RTM. Right: The AVA curves picked from the left.

Figure 3. Left: A migrated angle domain CIG from true amplitude RTM with source and receiver ghost compensation. Right: The AVA curves picked from the left.

Figure 4. Left: The conventional stacked RTM image. Middle: The stacked RTM image with source and receiver ghost compensation. Right: The exact impedance model.
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Finally, Figure 7 shows a comparison between conventional and deghosted RTM images for a shallow water data set from the Central North Sea. The source and receiver depths are 6m and 8m, respectively. We highlight wavelets at about 4.2km depth from each image as wiggle plots. The sidelobes are significantly reduced after deghosting. Also, the RTM with deghosting has retained lower frequencies which has increased the bandwidth giving the image a more continuous and textured appearance.

**Conclusions**

Directly compensating for the source and receiver ghost effects in a prestack depth migration is attractive and can improve our ability in interpreting geological structures and rock properties. We have shown that this can be incorporated in an RTM. When seismic data has noise and is poorly sampled, a simple deghosting on the receiver may not work well in the full bandwidth, but with some stabilization strategy, it helps to compensate for the angle dependent low frequency loss. Back propagating an inverse source ghost can effectively remove the ghost on the source side. All together, the proposed method improves the imaging power of RTM. As we show in this abstract, our method produces a sharper wavelet and more balanced amplitudes. Both angle dependent reflectivity and velocity perturbation can be output from a true amplitude RTM for interpretation and inversion.

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References


