Linearized AVO and Poroelasticity for HRS9

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• In this talk, we combine the linearized Amplitude Variations with Offset (AVO) technique with the Biot-Gassmann theory of poroelasticity.

• We first review the theory of linearized AVO.

• We then review the theory of poroelasticity.

• We next derive a linearized AVO equation which is related to the poroelastic properties of the reservoir.

• Finally, we will apply this equation to both model and real data examples.
Consider an interface between two different geological formations, shown on the left.

An incident P-wave on the boundary produces P and S reflected and transmitted waves.

This is called *mode conversion*, and we wish to compute the amplitudes of each ray.
Zoeppritz (1919) derived the amplitudes of the reflected and transmitted waves using the conservation of stress and displacement across the layer boundary, which gives four equations with four unknowns. The solution is as follows:

$$
\begin{bmatrix}
R_P \\
R_S \\
T_P \\
T_S \\
\end{bmatrix} = 
\begin{bmatrix}
-\sin \theta_1 & -\cos \phi_1 & \sin \theta_2 & \cos \phi_2 \\
\cos \theta_1 & -\sin \phi_1 & \cos \theta_2 & -\sin \phi_2 \\
\sin 2\theta_1 & V_{P1} \cos 2\phi_1 & \frac{\rho_2 V_{S2}^2 V_{P1}}{\rho_1 V_{S1} V_{P2}} \cos 2\phi_1 & \frac{\rho_2 V_{S2} V_{P1}}{\rho_1 V_{S1}} \cos 2\phi_2 \\
-\cos 2\phi_1 & V_{S1} \sin 2\phi_1 & \frac{\rho_2 V_{P2}}{\rho_1 V_{P1}} \cos 2\phi_2 & -\frac{\rho_2 V_{S2}}{\rho_1 V_{P1}} \sin 2\phi_2 \\
\end{bmatrix}^{-1}
\begin{bmatrix}
\sin \theta_1 \\
\cos \theta_1 \\
\sin 2\theta_1 \\
\cos 2\phi_1 \\
\end{bmatrix}
$$
Bortfeld (1960), Richards and Frasier (1976) and Aki and Richards (1980) derived a linearized version of the Zoeppritz equations, given as:

\[
R_{PP}(\theta) = a_1 \frac{\Delta V_P}{V_P} + b_1 \frac{\Delta V_S}{V_S} + c_1 \frac{\Delta \rho}{\rho}
\]

where:

\[
a_1 = \frac{\sec^2 \theta}{2}, \quad b_1 = -\frac{4}{\gamma_{sat}} \sin^2 \theta, \quad c_1 = 0.5 - \left[ \frac{2}{\gamma_{sat}} \sin^2 \theta \right], \quad \text{and} \quad \gamma_{sat} = \frac{V_P}{V_S}.
\]
In the linearized approximation, we use averages and differences of the parameters \( (\rho) \), and assume that \( \Delta \rho/\rho < 0.1 \).
Wiggins’ version of Bortfeld-Aki-Richards

A numerically equivalent, but algebraically reformulated, version of the Bortfeld-Aki-Richards equation was derived by Wiggins (1983) and is written:

\[ R_{PP}(\theta) = a_2 R_{P0} + b_2 G + c_2 C \]

where:

- \( a_2 = 1 \),
- \( b_2 = \sin^2 \theta \),
- \( c_2 = \sin^2 \theta \tan^2 \theta \).

\( R_{P0} = \frac{1}{2} \left[ \frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\rho} \right] \) = the intercept,

\( G = \frac{\Delta V_P}{2V_p} - \frac{4}{\gamma_{sat}^2} \frac{\Delta V_S}{V_s} - \frac{2}{\gamma_{sat}^2} \frac{\Delta \rho}{\rho} \) = the gradient,

\( C = \frac{\Delta V_P}{2V_p} \) = the curvature.
The Smith-Gidlow-Fatti (S-G-F) equation

Smith and Gidlow (1987), Gidlow et al. (1992) and Fatti et al. (1994) reformulated to linearized zero-offset $P$, $S$-wave and density reflectivity. Their equation is written:

$$ R_{PP}(\theta) = a_3 R_{P0} + b_3 R_{S0} + c_3 R_D $$

$$ a_3 = 1 + \tan^2 \theta, \quad b_3 = \frac{-8 \sin^2 \theta}{\gamma_{sat}^2}, \quad c_3 = \frac{1}{2} \tan^2 \theta - \frac{2 \sin^2 \theta}{\gamma_{sat}^2}, $$

$$ R_{S0} = \frac{1}{2} \left[ \frac{\Delta V_S}{V_S} + \frac{\Delta \rho}{\rho} \right], \text{ and } R_D = \frac{\Delta \rho}{\rho}. $$
Comparing the three equations

Although all three of the previous equations give exactly the same value for $a$ at a given angle, they each have advantages:

- The intercept and gradient terms from the Wiggins equation require a knowledge of angle but not $\gamma_{sat}$. Also, this equation can be used for qualitative crossplot analysis.

- The other two formulations both require a knowledge of angle and $\gamma_{sat}$, but give us estimates of physical parameters.

- The $\Delta V_P/V_P$, $\Delta V_S/V_S$ and $\Delta \rho/\rho$ terms from the original B-A-R equation can be used to invert for $P$ and $S$ velocity and density (Buland and Omre (2003)).

- The $R_{P0}$, $R_{S0}$ and $R_D$ terms from the S-G-F equation can be used to invert for $P$ and $S$ impedance and density (Simmons and Backus (1996), Hampson et al. (2005)).
Intercept vs Gradient Analysis

Top: Crossplot with interpreted zones.
Bottom: Zones on seismic, where:

- Pink = Top of Gas
- Yellow = Base of Gas
- Blue = Hard streak
Shuey’s version of Bortfeld-Aki-Richards

Shuey (1985) rewrote the Bortfeld-Aki-Richards equation using $V_p$, $\rho$, and $\sigma$, and only the form of the gradient term ($G$) is changed:

$$R_{pp}(\theta) = a_2 R_{p0} + b_2 G + c_2 C$$

where:

$$G = R_{p0} \left[ D - 2(1 + D) \frac{1 - 2\sigma}{1 - \sigma} \right] + \frac{\Delta \sigma}{(1 - \sigma)^2},$$

$$D = \frac{\Delta V_p / V_p}{2 R_{p0}}, \quad \sigma = \frac{\sigma_2 + \sigma_1}{2}, \quad \Delta \sigma = \sigma_2 - \sigma_1.$$
Shuey’s Equation

Instead of simply using algebra to re-arrange terms, Shuey made use of the differential form given by:

\[ \Delta V_S = \frac{\partial V_S}{\partial V_P} \Delta V_P + \frac{\partial V_S}{\partial \sigma} \Delta \sigma \]

- This means that Shuey’s equation will give slightly different values than the Bortfeld-Aki-Richards equation.

- As shown in the next slide, the values are close for small changes (i.e. the \( \Delta p/p \) terms approximately 0.1 or less).
This figure shows a comparison between the Aki-Richards and Shuey equations for a typical gas sand.
The elastic constant formulation

- So far we have considered only parameterizations of the linearized Zoeppritz equations that involve $P$-wave velocity, $S$-wave velocity, Poisson’s ratio and density.
- However, we know that the velocities are functions of more fundamental constants, as given here:

$$
V_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{K + (4/3)\mu}{\rho}}, \quad V_S = \sqrt{\frac{\mu}{\rho}}
$$

where: 
- $\lambda = $ the first Lamé constant,
- $\mu = $ the second Lamé constant, or shear modulus, and
- $K = $ the bulk modulus, or reciprocal of compressibility.

Note that: 
$$
K - \lambda = \frac{2}{3} \mu
$$
The elastic constants

- We can extract the three terms $\mu$, $\lambda$ and $K$ from the density and velocities using the following equations:

$$\mu = \rho V_S^2$$

$$\lambda = \rho V_P^2 - 2\rho V_S^2 = \rho V_P^2 - 2\rho \mu$$

$$K = \rho V_P^2 - \frac{4}{3}\rho V_S^2 = \rho V_P^2 - \frac{4}{3} \mu$$

- The first two equations above are the basis for the lambda-mu-rho (LMR) method, although in practice we use $P$ and S-impedance instead of velocity, which leads to $\lambda\rho$ and $\mu\rho$ rather than $\lambda$ and $\rho$. 

Gray et al. (1999) used an approach similar to Shuey (1985) to re-formulate the Aki-Richards’ equation for the elastic constants.

That is, they used the differential forms shown below:

\[
\Delta \lambda = \frac{\partial \lambda}{\partial V_P} \Delta V_P + \frac{\partial \lambda}{\partial \mu} \Delta \mu + \frac{\partial \lambda}{\partial \rho} \Delta \rho
\]

\[
\Delta K = \frac{\partial K}{\partial V_P} \Delta V_P + \frac{\partial K}{\partial \mu} \Delta \mu + \frac{\partial K}{\partial \rho} \Delta \rho
\]
Using the differential forms shown on the previous slide (and a lot of algebra), Gray et al. (1999) derived two new equations, one for $\lambda$, $\mu$ and $\rho$, and one for $K$, $\mu$ and $\rho$:

\[
R_{pp}(\theta) = \left( \frac{1}{4} - \frac{1}{2\gamma_{sat}^2} \right) \sec^2 \theta \frac{\Delta \lambda}{\lambda} + \frac{1}{\gamma_{sat}^2} \sec^2 \theta - 2\sin^2 \theta \left( \frac{1}{2} \Delta \mu \frac{\Delta \lambda}{\lambda} + \left( \frac{1}{2} - \frac{1}{4 \sec^2 \theta} \right) \Delta \rho \frac{\Delta \rho}{\rho} \right)
\]

\[
R_{pp}(\theta) = \left( \frac{1}{4} - \frac{1}{3\gamma_{sat}^2} \right) \sec^2 \theta \frac{\Delta K}{K} + \frac{1}{\gamma_{sat}^2} \sec^2 \theta - 2\sin^2 \theta \left( \frac{1}{3} \Delta \mu \frac{\Delta K}{K} + \left( \frac{1}{2} - \frac{1}{4 \sec^2 \theta} \right) \Delta \rho \frac{\Delta \rho}{\rho} \right)
\]

These equations are given on page 244 of the textbook by Avseth et al. Note the similarity of the two equations.
Russell et al. (2003) asked the question: “For the porous reservoir rock, which term is more applicable, $\lambda$ or $K$?”

As we showed, it doesn’t matter when each term is expanded for porous media.

We thus replaced these terms with a more general term $f$, which reduces to either $\lambda$ or $K$.

The theory was developed by Biot (1941) for $\lambda$ and $\mu$, and Gassmann (1951) for $K$ and $\mu$. A good summary is found in Krief et al. (1990).
General equation for P-wave velocity

By equating Biot and Gassmann’s formulations, the general equation for $P$-wave velocity can be written:

$$V_{P_{\text{sat}}} = \sqrt{\frac{f + s}{\rho_{\text{sat}}}}$$

$f = \text{fluid/porosity term} = \alpha^2 M$ (often called $K_{\text{pore}}$)  
($\alpha$ is the Biot coefficient and $M$ the fluid modulus)

$s = \text{dry skeleton term}$

$$= K_{\text{dry}} + \frac{4}{3} \mu = \lambda_{\text{dry}} + 2 \mu$$
Extracting the fluid term

Using the seismic velocities and density, we can extract the fluid term as shown below:

\[ f = \rho V_P^2 - c(\rho V_S^2) = f + s - c\mu \]

- The constant \( c \) must be chosen so that the term \( s - c\mu \) is equal to zero. This gives us the following relationship:

\[ c = \left[ \frac{V_P^{dry}}{V_S^{dry}} \right]^2 = \gamma_{dry}^2 \]
Physical meaning of the fluid term

Noting that $\rho V_S^2 = \mu$ and dividing both sides of the previous equation through by this term, we find that:

$$\frac{f}{\mu} = \left( \frac{V_P}{V_S} \right)_{sat}^2 - \left( \frac{V_P}{V_S} \right)_{dry}^2$$

$$= \gamma_{sat}^2 - \gamma_{dry}^2$$

- As expected, this implies that the greater the difference in the two velocity ratios, the stronger the fluid effect.
- Conversely, when the two velocity ratios are identical, (i.e. for non-porous rocks) the fluid term equals zero.
Table of values

Here is a table of values for the various ratios:

<table>
<thead>
<tr>
<th>(Vp/Vs)^2</th>
<th>Vp/Vs</th>
<th>σdry</th>
<th>Kdry/μ</th>
<th>λdry/μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.000</td>
<td>2.000</td>
<td>0.333</td>
<td>2.667</td>
<td>2.000</td>
</tr>
<tr>
<td>3.333</td>
<td>1.826</td>
<td>0.286</td>
<td>2.000</td>
<td>1.333</td>
</tr>
<tr>
<td>3.000</td>
<td>1.732</td>
<td>0.250</td>
<td>1.667</td>
<td>1.000</td>
</tr>
<tr>
<td>2.500</td>
<td>1.581</td>
<td>0.167</td>
<td>1.167</td>
<td>0.500</td>
</tr>
<tr>
<td>(3) 2.333</td>
<td>1.528</td>
<td>0.125</td>
<td>1.000</td>
<td>0.333</td>
</tr>
<tr>
<td>2.250</td>
<td>1.500</td>
<td>0.100</td>
<td>0.917</td>
<td>0.250</td>
</tr>
<tr>
<td>2.233</td>
<td>1.494</td>
<td>0.095</td>
<td>0.900</td>
<td>0.233</td>
</tr>
<tr>
<td>(2) 2.000</td>
<td>1.414</td>
<td>0.000</td>
<td>0.667</td>
<td>0.000</td>
</tr>
<tr>
<td>(1) 1.333</td>
<td>1.155</td>
<td>-1.000</td>
<td>0.000</td>
<td>-0.667</td>
</tr>
</tbody>
</table>

Note in the above table that (1) corresponds to $K-\mu-\rho$, (2) to $\lambda-\mu-\rho$ and (3) to a poroelastic clean sand.
The generalized form

- Using the generalized equation for P-wave velocity, we can re-formulate the Aki-Richards’ equation using the differential form shown below:

\[ \Delta f = \frac{\partial f}{\partial V_P} \Delta V_P + \frac{\partial f}{\partial \mu} \Delta \mu + \frac{\partial f}{\partial \rho} \Delta \rho \]

- The new equation is shown in the next slide.
A generalized formulation

The re-formulation of the Aki-Richards equation using \( f, \mu \) and \( \rho \) is given by:

\[
R_{PP}(\theta) = a_4 \frac{\Delta f}{f} + b_4 \frac{\Delta \mu}{\mu} + c_4 \frac{\Delta \rho}{\rho}
\]

where:

\[
a_4 = \left( \frac{1}{4} - \frac{\gamma_{dry}^2}{4 \gamma_{sat}^2} \right) \sec^2 \theta
\]

\[
b_4 = \frac{\gamma_{dry}^2}{4 \gamma_{sat}^2} \sec^2 \theta - \frac{2}{\gamma_{sat}^2} \sin^2 \theta
\]

\[
c_4 = \frac{1}{4} - \frac{1}{4} \sec^2 \theta, \quad \gamma_{sat}^2 = \left[ \frac{V_S^2}{V_P^2} \right]_{sat} \quad \text{and} \quad \gamma_{dry}^2 = \left[ \frac{V_S^2}{V_P^2} \right]_{dry}
\]
Some observations

The following comments can be made about the general formulation:

- If we substitute $\gamma_{\text{dry}}^2 = 2$ into the previous formulation, we obtain the Gray et al. (1999) expression for $\lambda, \mu, \rho$.
- If we substitute $\gamma_{\text{dry}}^2 = 4/3$ into the previous formulation, we obtain the Gray et al. (1999) expression for $K, \mu, \rho$.
- Since we never have a situation in which $\gamma_{\text{sat}}/\gamma_{\text{dry}} < 1$, the scaling coefficient for the fluid term will always be positive or zero.
- The fluid term equals zero if we are dealing with a dry or non-porous rock.
- For a constant $\gamma_{\text{dry}}^2 = s/\mu$, $\Delta \mu/\mu$ is identical to $\Delta s/s$. 
A model example

On the right are the modeled AVO curves in which the top layer is a wet sand and the bottom layer is a gas sand. The Aki-Richards and $f\cdot\mu\cdot\rho$ curves are very close. $\gamma_{\text{dry}}^2 = 2.333$ for each layer, but the $\mu$ and $K$ values varied.
The general 3-term formulation

Note that all of the equations we have discussed can be written in the same form:

\[ R_{pp}(\theta) = ap_1 + bp_2 + cp_3, \]

where \( a, b, \) and \( c \) are functions of \( \theta \) and \( V_P^2 / V_S^2 \) (dry or wet), and \( p_1, p_2, \) and \( p_3 \) are functions of \( V_P, V_S, \rho, \sigma, f, \) or \( \mu. \)

- A summary of all the equations is shown on the next slide.
- We can implement a weighted least-squares approach to extract the \( p \) terms from pre-stack gathers.
# Three term summary

<table>
<thead>
<tr>
<th>Method</th>
<th>Need to know</th>
<th>Able to compute</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>Wiggins</td>
<td>$\theta$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Shuey</td>
<td>$\theta$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>B-A-R</td>
<td>$\theta$</td>
<td>$\theta, \gamma_{sat}^2$</td>
</tr>
<tr>
<td>S-G-F</td>
<td>$\theta$</td>
<td>$\theta, \gamma_{sat}^2$</td>
</tr>
<tr>
<td>$f-\mu-\rho$</td>
<td>$\theta, \gamma_{sat}^2, \gamma_{dry}^2$</td>
<td>$\theta, \gamma_{sat}^2, \gamma_{dry}^2$</td>
</tr>
</tbody>
</table>
We applied the $f-\mu-\rho$ method to a Class 3 gas sand from Alberta. The super-gathers are shown above, with the zone of interest highlighted. Since the far angle is at 30°, the density term extraction is considered unreliable.
Here is the fluid extraction ($\Delta f/f$) with a picked event at the zero-crossing of the gas sand. We used a dry velocity ratio squared of 2.333.
Here is the rock skeleton extraction ($\Delta\mu/\mu$) with a picked event at the zero-crossing of the gas sand.
Here is a comparison of the delta fluid result (top) with the delta shear modulus result (bottom).

Note the change in polarity at the gas sand when comparing the two results.
Conclusions

In this talk, we combined the linearized Amplitude Variations with Offset (AVO) technique with the Biot-Gassmann theory of poroelasticity. This gave us a way to extract fluid and skeleton effects from a reservoir using pre-stack angle gathers, from a knowledge of the dry and saturated velocity ratios. One caution is that it is not clear what “dry” means for rocks such as shales and fractured carbonates. More research is needed.
References


Gassmann, F., 1951, Uber die Elastizitat poroser Medien, Vierteljahrsschrift der Naturforschenden Gesellschaft in Zurich, 96, 1-23.


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