Spatial sampling, migration aliasing, and migrated amplitudes

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ABSTRACT

Seismic migration is a multichannel process, in which some of the properties depend on various grid spacings. First, there is the acquisition grid, which actually consists of two grids: a grid of source locations and, for each source location, a grid of receiver locations. In addition, there is a third grid, the migration grid, whose spacings also affect properties of the migration. Sampling theory imposes restrictions on migration, limiting the frequency content that can be migrated reliably given the grid spacings. The presence of three grids complicates the application of sampling theory except in unusual situations (e.g., the isolated migration of a single shot record). I analyzed the effects of the grids on different types of migration (Kirchhoff, wavefield extrapolation migration, and slant-stack migration), specifically in the context of migration operator antialiasing. I evaluated general antialiasing criteria for the different types of migration; my examples placed particular emphasis on one style of data acquisition, orthogonal source and receiver lines, which is commonly used on land and which presents particular challenges for the analysis. It is known that migration artifacts caused by inadequate antialiasing can interfere with velocity and amplitude analyses. I found, in addition, that even migrations with adequate antialiasing protection can have the side effect of inaccurate amplitudes resulting from a given acquisition, and I tested how this effect can be compensated.

INTRODUCTION

In marine areas, especially deepwater, the recent combination of intensive wide-azimuth data acquisition and advanced seismic imaging has produced migrated images far superior than was conceivable even a decade ago. In land areas, however, this explosion in quality has been more subdued. In areas of complex structures, including mountainous areas, difficulties in seismic acquisition and in correcting for near-surface effects (such as scattering and velocity) have continued to limit our ability to image deep reflectors even when we use advanced imaging techniques. Even land areas of moderate structural complexity challenge our ability to produce reliable migrated amplitudes. Such areas include unconventional reservoirs containing nearly vertical fractures, for which azimuthally dependent moveout and amplitudes can be useful in mapping reservoir properties. There is a reason for these difficulties: Seismic acquisition on land is usually inadequate for sampling near-surface velocity and scattering anomalies that are absent in deepwater marine environments. Moreover, land seismic acquisition is often inadequate for sampling the wavefields we use for imaging. (The same is true for wide-azimuth marine acquisition in deep water, but at least for streamer data, the near-surface problems are hidden by the combined effects of [1] the lower frequencies used for deepwater imaging and [2] the wavefront healing that makes wavefields more continuous at great depths.)

Wavefield sampling during acquisition, and its relationship with seismic imaging, is the subject of this paper. Various authors have explored this issue, particularly in the context of Kirchhoff migration antialiasing (Gray, 1992; Lumley et al., 1994; Abma et al., 1999; Biondi, 2001). However, the sampling requirements for the various seismic arrays used for imaging have not been explicitly investigated. For example, one standard land acquisition technique is “orthogonal,” with given source and receiver inline and crossline spacings — four separate spacings. In orthogonal acquisition, the source lines and receiver lines are mutually orthogonal, or nearly so, with crossline spacings that can easily exceed inline spacings by an order of magnitude. In fact, whereas inline source and receiver spacings are usually much smaller than a wavelength at typical seismic frequencies, crossline spacings for sources and receivers are often much larger than a wavelength. Although these large crossline spacings are significant, they are rarely, if ever, included in migration antialiasing calculations. On the other hand, given the large crossline spacings, seismic imaging by wavefield extrapolation in time or depth (so-called wave-equation migration, or WEM) is considered by some to be inappropriate for land data from orthogonal acquisition — often for the wrong reason — and the imaging method of choice for data acquired orthogonally is usually...
Kirchhoff migration, generally the least accurate depth migration method. Even Kirchhoff migration, however, is subject to sampling requirements, and ignoring those requirements leads to migration noise on migrated stacks and gathers that can result in incorrect analysis of migration velocity and migrated amplitudes.

What is the wrong reason, and what might be a correct reason, for the inappropriateness of WEM for orthogonally acquired land data with large crossline spacing? First, the wrong reason, which is often called the flexibility of Kirchhoff migration. Consider migrating a single shot record, in which the recorded data form a sampled portion of a single reflected wavefield recorded on a receiver grid of discrete \((x, y)\) lateral locations. Often the data are recorded with somewhat irregular spacing even when there are nominal inline and crossline increments. All WEM methods assume regular spatial sampling, and the irregular sampling can be seen as preventing the extrapolation of the recorded wavefield from the recording surface to greater depths. Kirchhoff migration, on the other hand, is performed in one input trace at a time, and nothing operationally prevents the application of Kirchhoff migration to irregularly sampled data, even migrating the traces from their exact surface locations. However, Kirchhoff migration theory also requires regular spatial sampling or, alternatively, the explicit use of inverse theory to compensate for irregular sampling, and the simple fact that we can perform standard Kirchhoff migration from an irregular acquisition grid onto a regular migration grid (its flexibility) does not justify its use. So WEM of a single shot record of orthogonally acquired data is no more inappropriate than standard Kirchhoff migration of the same data and can, by binning the unmigrated traces onto a regular grid, produce an image similar to that of a nonantialiased Kirchhoff migration. Either migrated image will be improved if the data are placed, with appropriate moveout corrections, onto the regularly spaced nominal receiver locations. (Kirchhoff migration requires regular spatial sampling because irregular sampling causes incomplete cancellation of migration swings above reflectors, in violation of the principles of WEM.)

Second, a correct reason. A real problem with applying WEM to the shot record is caused by the computational grid \((x, y)\) spacings required by most WEM algorithms. This is easily seen with finite-difference (FD) methods. The FD grid spacings are usually on the order of the desired image grid spacings, i.e., less than one wavelength. FD methods approximate spatial derivatives of the recorded wavefields to continue them from one depth or time to the next. Because the computational \((x, y)\) grid is often much finer than the recording grid in the crossline \((y)\) direction, the methods are forced to assume that the data recorded with large receiver line spacing are zero at most locations. When the migration approximates \(y\)-differentiation by subtracting a nonzero (recorded) sample from a zero (unrecorded) sample, an incorrect value enters the computations. By involving lateral wavenumbers not supported by the crossline receiver spacing, the wavefield computation is aliased, just as a Kirchhoff migration operator can be aliased. The FD operator needs to be antialiased by interpolating the actual recorded data onto the computational grid before the migration. The inevitable effect of this interpolation is a dramatic loss of resolution compared with the migration of a fully sampled recording spread, and the WEM image usually fails to justify the added effort of migrating the unrecorded zero traces. Similarly, phase-shift methods need to be antialiased before migration from a coarse grid of receiver locations onto a fine image grid. This can be done either by interpolating the recorded data as described for FD migration or by restricting the lateral wavenumbers in the crossline direction within the migration operator. The latter choice is analogous to Kirchhoff migration antialiasing, and it is more economical than the former, but still all the unrecorded zero traces need to be migrated. In either case, the highest frequencies to survive the antialiasing and migrate from one crossline to the next with substantial crossline dip might be near the minimum recorded frequency, so that the migration is essentially the combination of 2D (inline) migration and a crossline mix. (This is the case for the acquisition geometry used in the examples.) In addition, the computational effort of migrating data volumes consisting mostly of zero traces can make WEM prohibitively expensive compared with Kirchhoff migration.

Therefore, operator aliasing is possible for any migration method, and its severity is determined by the interaction of the migration operator with the acquisition grid. Previous studies of Kirchhoff migration antialiasing (e.g., Lumley et al., 1994; Abma et al., 1999) failed to account explicitly for the nature of such interaction, which is the focus of this paper.

In the next section, I describe sampling requirements for migration in various domains: migration of wavefields, suitable for WEM, Kirchhoff, and slant-stack migrations, and migration of non-wavefield ensembles of traces, such as common-offset-vector (COV) volumes, suitable for Kirchhoff and slant-stack migrations. Each domain has particular requirements. For example, migration of a single shot record has sampling requirements for the grid of receivers, but no requirement for the grid of shotpoints (which contains only one point). We must remember, in addition, that even the single shot gives rise to a wavefield inside the earth that needs to be sampled adequately on the image grid, giving a second antialiasing requirement that is independent of the acquisition geometry. This requirement also holds for the downward continued wavefield from the receiver locations. As a second example, shot-record migration of all the shots in an entire 3D survey has sampling requirements for each individual shot record and, by reciprocity, sampling requirements for each individual receiver record. These sampling requirements can be fulfilled if we view prestack migration as a combination of focusing in emission and focusing in detection, with a slightly different meaning of these terms from that given by Berkhout (1997). Obeying the requirements imposed separately by the arrays of sources and receivers will provide sufficient (usually pessimistic) conditions for antialiased migration of the survey, with the prospect of reduced migrated amplitudes for certain ranges of offset and azimuth. Migrating other data volumes, such as COV volumes, has different antialiasing requirements, which may be indirectly related to the acquisition geometry. It is often possible to relax the requirements, for example, when migrating straight to stack, but if we do so we cannot be certain of producing accurate amplitudes or even interpretable migrated structures.

The sampling requirements for migration can be satisfied if we perform migration antialiasing. Antialiasing based on a Nyquist criterion is not strictly a necessary condition (for unaliased migration of recorded events that have nonzero time dip [Biondi, 2001]), but it is sufficient, and its application can lead to migrated volumes with accurate amplitude behavior, assuming the migration method is itself amplitude preserving. However, applying antialiasing to coarsely acquired data will impose some loss of high frequencies during the migration, leading to loss of migrated amplitudes. This amplitude decay needs to be compensated before the migrated
gathers can be analyzed for amplitude as a function of offset, incidence angle, or azimuth angle. As the examples will show, the amplitude compensation can be significant, so that the use of migration antialiasing can compromise the interpretation of migrated amplitudes as a function of offset and/or azimuth.

**MIGRATION ANTIALIASING: ARRAYS OF SOURCES AND RECEIVERS**

Migration antialiasing can be viewed as applying simple principles of sampling theory to acquired seismic data (e.g., Vermeer, 1990). In brief, antialiasing seeks to prevent the migration of aliased spatial frequency components, using a Nyquist relation such as

\[ \frac{f_{\text{max}} \sin \theta}{v(0)} \leq \frac{1}{2\Delta \rho} \tag{1} \]

(Gray, 1992), where \( f_{\text{max}} \) is the maximum allowable frequency, \( \theta \) is emergence angle of a reflected event at the recording surface, \( v(0) \) is the velocity at the recording surface, and \( \Delta \rho \) is the effective spacing on the acquisition grid. This relation holds for zero-offset migration using the exploding reflector model, where \( \Delta \rho \) is the migration grid (usually the common midpoint [CMP] grid) spacing and \( v(0) \) is one-half the actual velocity at the recording surface. For prestack migration, there is a takeoff angle from the source as well as an emergence angle at the receiver, and |\sin \theta|/\( v(0) \) should be replaced by a “migration operator slope” (Abma et al., 1999). In the special case of zero-offset migration, the migration operator slope is |\partial t/\partial \rho|, where \( t \) is the total traveltime and \( \rho \) is a lateral distance between the source-receiver location and the image location. For nonzero-offset migration, the Nyquist relation has a more general expression, described below, than inequality 1. This expression involves operator slope components related to the source and receiver grids. According to relation 1, even frequencies \( f \) that are unaliased on the acquisition grid can cause migration operator aliasing if they fail to obey the inequality for given |\partial t/\partial \rho|, \( v(0) \), and \( \Delta \rho \). As an extreme example, consider the response of a horizontal reflector in a constant-velocity medium to a vertically propagating incident plane wave. The unmigrated reflection event will be perfectly flat, and spatially unaliased. Even these data can cause migration operator aliasing if the acquisition grid spacing is coarse enough and the frequency is high enough. Migration operator aliasing occurs for frequency \( f \) when the relatively steep migration operator cuts through the recorded traces at an angle too large to sample frequencies higher than \( f \) at two points per wavelength, as required by inequality 1 (Figure 1), and it manifests itself as uncanceled migration swings that can appear either random or coherent. Although the reflected wavefield has emerged vertically at the recording surface from the horizontal reflector in this example, migration is unaware of this fact in its action of swinging each recorded sample to all dip angles. This operator aliasing can occur even if the migration grid is much finer than the acquisition grid, preventing image-space aliasing. The migration \((x, y)\) grid must also be fine enough to prevent uncanceled migration swings, so relation 1 must hold on the migration grid as well as the acquisition grid, where now \( \Delta \rho \) is the effective migration grid spacing. (Typically, the migration grid is finer than the acquisition grid, although, for migrating singlefold volumes such as zero-offset or COV volumes, the migration grid can be considered to be the same as the acquisition grid.) To prevent operator aliasing in zero-offset Kirchhoff migration, it is possible to keep the product \( f_{\text{max}} |\partial t/\partial \rho| \Delta \rho \) nearly constant during migration, either by the use of precomputed filter banks (Gray, 1992) or by on-the-fly calculations performed during the migration (Lumley et al., 1994).

**Poststack migration**

Migration antialiasing was first developed for poststack migration, in which its use is simple. Poststack migration is performed on a grid of CMP locations assuming the data are a recorded wavefield from an exploding reflector (Loewenthal et al., 1976) experiment. The grid of unmigrated data and the migration grid are both the CMP grid, and velocity in relation 1 is replaced by half-velocity. Although the CMP grid is not the recording grid for any of the actual physical wave experiments that comprise the seismic survey, it is the acquisition grid for the fictitious exploding reflector experiment. In addition, the migration grid needs to be spatially dense enough to represent, in an unaliased fashion, the subsurface wavefield produced by the exploding reflectors.

The presence of two spacings in 3D, inline (\( \Delta x \)) and crossline (\( \Delta y \)), poses no special complications if we apply the procedure described by Abma et al. (1999) to handle dips in azimuthal directions.

![Figure 1. Migration operators (red curves) cutting through a flat reflection event. (a) The event is well sampled spatially, and the migration operator samples the wavelet at different points along several traces. The sum of the samples selected by the migration operator is nearly zero. (b) The event is not as well sampled spatially, and the steeper migration operator samples the wavelet at a single point on a single trace. The sum of the samples selected by the migration operator is nonzero (migration operator aliasing). The less steep migration operator samples the wavelet almost adequately, but still at fewer than two points per wavelength.](image-url)
that are neither inline nor crossline; that is, the effective spacing $\Delta \rho$ in azimuthal direction $\phi$ from the inline ($x$) direction is given by

$$\Delta \rho = \max(\Delta x \cos \phi, \Delta y \sin \phi).$$

(2)

**Migrating a single wavefield**

When we migrate an actual wavefield that is the result of a physical experiment (say, a single shot record), the migration process literally focuses the array of receivers onto each image location (Safar, 1985). The antialiasing principle described above applies, using actual surface velocity (not half-velocity) and the actual receiver array. In addition, the image grid needs to be spatially dense enough to represent the subsurface wavefields in an unaliased fashion. For a single shot record, the inline and crossline receiver grid spacings determine the antialiasing criterion, which is inequality 1 with effective spacing $\Delta \rho_s$ determined by the receiver inline and crossline spacings $\Delta x_r, \Delta y_r$ and the azimuthal direction $\phi$ between the receiver location and the image location. As mentioned above, $\Delta y_r$ might exceed $\Delta x_r$ by an order of magnitude or more for orthogonally acquired data, and the antialiasing requirement 1 can have a severely limiting effect on frequencies passed by the migration operator in the crossline direction. If, for example, the surface velocity $v(0)$ is 2500 m/s and the migration operator has a desired time slope of sin 30°/v(0) = 1/2 · 2500 = 0.0002 s/m, a spread of receivers with inline spacing 25 m and crossline spacing 250 m allows a maximum frequency of 100 Hz to be migrated in the inline direction, but only 10 Hz crossline.

Next, we consider the theoretical extension of the preceding paragraph to the antialiasing migration of all shot records in a 3D survey using WEM, Kirchhoff, or slant-stack migration. This leads to an unnecessarily pessimistic criterion for Kirchhoff and slant-stack migration, so I then present a different formulation, appropriate for those methods, which allows us to relax the antialiasing requirements.

**Migrating an entire survey of wavefields**

When we perform prestack migration on a 3D survey using WEM, we are considering recorded wavefields from several different sources, each at a different location. The ultimate goal of the migration might be to analyze migrated common image gathers (CIGs) for velocity and/or amplitudes, and we must account for inline and crossline spacings of sources and receivers. Here, I use the migration imaging condition to help provide the antialiasing criterion. The imaging condition typically involves the crosscorrelation of the downward-continued source and receiver wavefields at subsurface image locations (Claerbout, 1971). For an individual shot record, it takes a form as an integral over frequency such as

$$I(x; x_s) = \int U(x; x_s; f)D^*(x; x_s; f)df,$$

(3)

where $I$ is the image at subsurface location $x$ from an experiment with a source at $x_s$, $U$ is the downward continued wavefield from the receivers, $D^*$ is the complex conjugate of the downward continued wavefield from the source, and $f$ is frequency. The full image is the summation of expression 3 over all shots, possibly binned into gathers indexed by surface offset, subsurface reflection angle, or subsurface azimuth angle. Wavefield $U$ was originally recorded on the (possibly coarse) inline/crossline receiver grid, and its downward continuation and imaging should take that fact into account, even if the subsurface image grid has much finer lateral spacings. For each trace and each image location, there are azimuthal directions $\phi_s, \phi_r$ for raypaths from source and receiver locations to the image location, and raypath emergence angles $\theta_s, \theta_r$ with corresponding migration operator slope components $[\partial r_s/\partial \rho_s, \partial r_s/\partial \rho_r]$ from source and receiver locations (Figure 2). Antialiasing can be accomplished by considering the combination of focusing in emission and focusing in detection. Focusing in emission focuses the array of source locations onto the image locations; it will require the frequency restriction of source wavefield $D$. Focusing in detection focuses the array of receivers onto the image locations; it will require the frequency restriction of receiver wavefield $U$. The separate focusing of the source and receiver arrays will impose the more restrictive of the two conditions on any trace to be migrated. That is,

$$f_{\text{max}} \leq \frac{1}{2 \max \left[ 2 \left| \partial r_s/\partial \rho_s \right| \Delta \rho_s, 2 \left| \partial r_s/\partial \rho_r \right| \Delta \rho_r \right]},$$

(4)

where $\Delta \rho_s, \Delta \rho_r$ are calculated using equation 2 on the source and receiver grids separately with azimuth angles $\phi_s, \phi_r$ both measured from the same direction (for definiteness, say the receiver inline direction). This is different from the 3D prestack antialiasing formula,

$$f_{\text{max}} \leq \frac{1}{2 \left| \partial r/\partial \rho \right| \Delta \rho},$$

(5)

of Abma et al. (1999). (The extra factor of 2 in the denominator of inequality 4 reconciles relations 4 and 5 for the zero-offset case, analogous to the use of half-velocity in migrating CMP stacked data.) Inequality 5 has only one grid, the CMP grid, which is appropriate for zero-offset migration or the migration of a single COV volume, whereas inequality 4 takes into account source and receiver grids. Even when there is only one underlying $(x, y)$ grid, however, the two formulas are still slightly different. That is because Abma et al. (1999) use an effective grid spacing $\Delta \rho$ determined by the azimuthal direction between the image location and the source/receiver midpoint (a sort of “average azimuth” shown in their Figure 5), whereas the present formulation retains separate grid spacings $\Delta \rho_s, \Delta \rho_r$ determined by two separate azimuthal directions between image location and source/receiver locations. (Effectively,
Abma et al. [1999] assume that the propagation of the recorded wavefield is from an image location to the source/receiver midpoint location, whereas I assume that the propagation is from the source location to the image location, then from the image location to the receiver location. Therefore, their formulation is correct only for the exploding reflector experiment and the present formulation is slightly more rigorous.) Typically, one of the two azimuthal directions used by the present formulation will produce a criterion more conservative than the average of the two, and the present formulation will restrict high frequencies more than that of Abma et al., except for zero-offset migration.

If the data are sparsely sampled, inequality 4 will yield a pessimistic antialiasing criterion that will limit or prevent the use of WEM. This is because of the large values for $\Delta \rho_r, \Delta \rho$, that appear in the denominator of the right side of equation 4 for most of the image locations in the survey (i.e., those not directly below the intersection of source and receiver lines). Seismic data interpolation (e.g., Trad, 2009) can be used to mitigate this problem to some degree, but the amount of interpolation needed to reduce the source and receiver line spacings to acceptable sizes for WEM is usually prohibitive. Sometimes, interpolation is used to refine the receiver grid to an acceptable size, and the remaining migration aliasing due to large shot line spacing is assumed to be suppressed by the power of stack.

**Migrating an entire survey, as an ensemble of traces from different physical experiments**

The preceding formulation, based on migrating a set of 3D recorded wavefields, has produced an extremely conservative antialiasing criterion that is appropriate for WEM but severely limits its applicability if the data were acquired sparsely. This criterion can be relaxed when performing Kirchhoff migration of 3D data volumes. For this to happen, the volumes to be migrated must be indexed on a single $(x, y)$ grid that is typically denser than the acquisition geometry; a COV volume is an example. In principle, volumes of traces sharing the exact same offset vector between source and receiver locations are very sparse, with inline/crossline live trace spacings equal to source inline/crossline spacings. However, COV volumes are typically filled with traces with similar, but not identical, offset vectors in such a way as to produce one trace at each CMP location. Within each volume of a nominal COV, the range of inline (crossline) offsets is the inline (crossline) shot spacing (Cary, 1999; Li, 2008). These volumes come from many different physical experiments, so there is no hope of applying WEM efficiently to them. However, their inline/crossline $(x, y)$ sampling is regular, and we can apply Kirchhoff migration to them, just as rigorously as we apply Kirchhoff migration to 2D common-offset records (Bleistein et al., 2000). The grid $(x, y)$ spacings are specified as the CMP spacings. As long as the COV volumes are singlefold (i.e., exactly one trace at each CMP location), the migration CMP grid can be considered to be the same as the acquisition grid, with one source and one receiver at every CMP location except at the edges of the volumes. Then we can apply either criterion 4 (with $\Delta \rho_s, \Delta \rho_r$ determined from equation 2 using, respectively, azimuth angles $\phi_s, \phi_r$) or the slightly less rigorous criterion 5 of Abma et al. (1999) when migrating any trace. For COV migration, the two criteria will produce similar maximum frequencies for small offsets, i.e., until the offset-to-depth ratio approaches unity.

**Slant-stack migrations**

Next, I discuss antialiasing of slant-stack migrations, such as beam migration (Hill, 1990, 2001). These consist of two parts: slant stack of the recorded data into $(p_x, p_y)$ beam components and mapping the beam components in the subsurface. Antialiasing these migrations requires that the slant stack be antialiased and that the migration avoid image-space aliasing as required for all other migration methods. For shot-record beam migration (Gray, 2005), the slant stack is expressed in the frequency domain as the 2D Fourier transform,

$$S(p; f) = \int WU(x; f) \exp[2\pi if \cdot (x - L)] dx, \quad (6)$$

where $S$ is the slant data with ray-parameter vector $p = (p_x, p_y)$, $W$ is a weight term, $U$ is the recorded wavefield, $f$ is frequency, and $L$ is the over receiver locations x centered around a receiver “beam center” location L. The slant stack will be aliased if, for data $U$ with a particular time slope vector $p$, it produces nonzero values for a vector $p'$ that is significantly different from $p$. Such a situation is depicted in Figure 1b, where the flat data (data time slope equals zero) have been undersampled by the steep slant operator (operator time slope is much greater than zero). When this happens, the migration will incorrectly map the spurious nonzero slant-stacked data to image locations unrelated to their actual origins at reflectors in the subsurface. The unaliased slant stack requires

$$f_{\text{max}}|p| \leq \frac{1}{2\Delta \rho}, \quad (7)$$

where $\Delta \rho$ satisfies equation 2 with receiver inline and crossline grid spacings $\Delta x_r$ and $\Delta y_r$. This is a condition similar to inequality 1. Ray parameter vector $p$ has for its components the individual time slopes $\partial t/\partial x_r$ and $\partial t/\partial y_r$. For a slant operator with nonzero slope, a large crossline grid spacing $\Delta y_r$ will typically cause $|\partial t/\partial y_r|/|\partial t/\partial x_r|$ to be large, forcing $f_{\text{max}}$ to be small. Therefore, antialiasing for 3D shot-record beam migration of land data suffers from the same problem as 3D WEM for orthogonally acquired data, namely, frequency restriction resulting from the large receiver line spacing. The antialiasing criteria for single-shot beam migration and shot-record beam migration of an entire 3D survey are the same as for 3D WEM, namely, inequalities 1 (for single-shot migration) and 4 (for the migration of an entire survey). For marine data recorded at the seafloor, shot-record beam migration can be replaced by receiver-record migration using the shot spacings in $x$ and $y$, which are usually finer than the receiver spacings. Still, the receiver spacings in $x$ and $y$ remain an issue for seafloor cable and node acquisitions when migrating an entire 3D survey; as with WEM, it might be impossible to avoid migration operator aliasing entirely while maintaining adequate frequency content without relying on the power of stacking all the images together in the hope of canceling aliasing noise.

In contrast, when beam migration is performed on COV volumes (Hill, 2001) with a single live trace at each CMP $(x, y)$ location, antialiasing the slant stack proceeds almost as for COV Kirchhoff migration. The slant stack is expressed as equation 6, where now $U$ is the unmigrated COV data, $p$ is a midpoint ray parameter vector, and the integral is over CMP locations x. Ray parameter vector $p$ is the sum of source and receiver ray parameter vectors
time slopes \(\frac{\partial t}{\partial \rho_x}\) and \(\frac{\partial t}{\partial \rho_y}\), into condition 4. We must remember, however, that beam migration applied to COV volumes of land data acquired along an irregular surface with substantial near-surface velocity variations suffers from slant-stacking problems unrelated to migration operator aliasing, as described by Gray (2005).

**Migrated amplitudes**

Antialiasing modifies, and possibly distorts, migrated amplitudes. Any antialiasing restriction alters the integration limits in the integral over frequency in building the image using equation 3; by doing this, it modifies the migrated amplitude at each image location. In a typical nonsymmetric case [orthogonal acquisition with (source inline spacing) < (receiver crossline spacing) < (source crossline spacing)], the amplitude diminution will be azimuth dependent: greatest in the source crossline direction and least in the source inline direction. This results in a migration-induced amplitude-versus-azimuth function that modulates the reflection coefficient-versus-azimuth function, often a desired output of the migration. Failure to compensate for the amplitude distortion produced by the antialiased migration will result in incorrectly estimated azimuthal properties. Fortunately, we can compensate for the amplitude distortion, at least approximately, by recognizing that the peak value of the band-limited migrated wavelet is proportional to the integral of the source spectrum in the frequency domain (Bleistein et al. [2000], p. 235). When we apply migration antialiasing, we reduce the maximum frequency present in the spectrum during migration, and we do this by reducing the upper integration limit in equation 3 or in the frequency integral in an alternative migration formula such as equation 5.1.48 of Bleistein et al. (2000). Approximating the source amplitude spectrum as a flat function from a lower frequency near zero to the upper limit makes the peak value of the wavelet in time roughly proportional to the maximum frequency. Thus, the migrated amplitude (peak value) is roughly proportional to the quotient of the maximum frequency allowed by antialiasing divided by the maximum frequency requested in the migration. We can compensate for the loss of amplitude caused by antialiasing by dividing by this quotient. If the maximum antialiasing frequency is near the low end of the recorded spectrum, we cannot apply this compensation because it will blow up low-frequency noise; instead, we must accept the fact of unreliable migrated amplitudes.

**EXAMPLES**

Several simple examples illustrate the preceding points. The acquisition geometry is the same for all examples: orthogonal acquisition with inline receiver spacing 25 m, inline source spacing 50 m, receiver line spacing 200 m, and source line spacing 400 m (Figure 3). This geometry is perfectly regular, using spacings that are typical for many land surveys. The CMP spacings in \((x, y)\) are 12.5 and 25 m, respectively.

Figure 4 shows the maximum frequency allowed by migration antialiasing, in many different situations, for migrating a single trace onto a depth slice at depth 2500 m. The velocity is 2500 m/s. Figure 4a shows the maximum frequency for a COV-style migration, in which the \((x, y)\) spacings are the CMP spacings. This figure corresponds to Figure 4a of Abma et al. (1999), with different acquisition parameters, seismic velocity, and depth. The source and receiver (small black squares) have \((x, y)\) coordinates \((\pm 1125, 0)\). The black curves are the migration ellipses for three impulses, corresponding approximately to migrated dips of 15° (innermost ellipse), 30° (middle ellipse), and 45° (outermost ellipse). (At this depth, the ellipses are nearly circular, and the equal-dip contours for those dips nearly overlie the ellipses in the figure.) The maximum frequencies range from approximately 25 (darkest) to 100 Hz (white). The sparse acquisition shown in Figure 3 does not produce enough recorded traces to yield singlefold unmigrated COV volumes with a limited range of inline and crossline offsets, so a large amount of effort is required to produce the COV volumes. This effort is usually the combination of some different forms of seismic data spatial interpolation, for example least-squares interpolation (Trad, 2009) and nearest neighbor interpolation applied to normal-moveout corrected traces. The major goal of the interpolation is to produce fine spatial sampling of the unmigrated traces to allow the COV migration operator to image a wide range of dips with a minimum of antialiasing. (In some cases [e.g., Gray et al., 2006], interpolation can raise the maximum migration frequency enough to allow the imaging of previously invisible steep reflectors, but that is of less concern here than its influence in regularizing migrated amplitudes.) For migrated dips less than 10°, interpolation will succeed reasonably well in regularizing amplitudes; there is little maximum frequency variation and, therefore, little modulation of migrated amplitudes by migration antialiasing. Larger azimuthal variation of maximum frequency, and consequently migrated amplitudes, can be seen at the larger dips. A side effect of seismic data interpolation is a spatial smearing of amplitudes upon migration; it is necessary to keep this in mind when analyzing migrated amplitudes, even at low dips.

In Figure 4b, the COV geometry is the same as in Figure 4a and the source and receiver locations have been rotated azimuthally by approximately 27°. We are now migrating a trace with the same offset but a different azimuth — therefore, a different
COV volume — from that in Figure 4a. This figure shows a greater range of azimuthal variation of migrated amplitude than that in Figure 4a for all dips; if this variation is not taken into account, amplitudes on migrated gathers as functions of offset and azimuth (AVO and AVAZ) will be interpreted incorrectly.

Figure 4c and 4d shows the effects of using the actual acquisition grid (no data interpolation), using the same source and receiver locations as for Figure 4b and the same CMP grid for migration as for Figure 4a and 4b. Now, very few of the CMP grid locations are occupied by live traces. In Figure 4c, the source and receiver inline and crossline spacings are considered (focusing in emission and detection), and antialiasing criterion 4 is used. This will be the case if we are performing an antialiased WEM on the full survey, migrating to CIGs for velocity or amplitude analysis. The maximum frequencies range from approximately 2 Hz (darkest) to 45 Hz (white). In Figure 4d, the source inline and crossline spacings are ignored in criterion 4, as if the survey consisted of a single shot record (focusing in detection only). This will be the case if we are performing an antialias WEM on the full survey, but migrating straight to stack and ignoring errors due to coarse source sampling. The maximum frequencies range from approximately 2 Hz (darkest) to 88 Hz (white). With no interpolation applied to the unmigrated data beforehand, there is a substantial loss of frequency content in both these cases; in the case of Figure 4c, little recorded energy will migrate from one shot or receiver line to the next.

Figure 5a and 5b shows depth slices from migrated impulses corresponding to Figure 4b and 4d. The three elliptical events in each figure have dips of approximately 15°, 30°, and 45°. In these migrations, the only amplitude factor applied is the modulation due to antialiasing, so the peak amplitude of the migrated wavelet is nearly proportional to the maximum frequency allowed by antialiasing. These figures show how the amplitude modulation caused by antialiasing, shown in Figure 4, affects the migrated wavelet. Normally, migration will combine this amplitude modulation with other amplitude factors, such as obliquity and geometrical spreading compensations.

Together, Figure 4a, 4c, and 4d provides a consistent and unsurprising conclusion for interpreting migrated amplitudes at image depths approximately equal to source-receiver offset for a typical fine or sparse acquisition using a true-amplitude migration. That is, given fine enough acquisition/interpolation geometry (e.g., COV) and simple geologic structure, we can easily and reliably interpret migrated AVO and AVAZ on horizontal reflectors but not on steeply dipping reflectors; but given a coarse geometry, we cannot reliably interpret migrated AVO and AVAZ at all even when the migration grid is fine. Figure 5b (coarse geometry) corroborates this conclusion. Figures 4b and 5a (fine geometry), however, contradict the first part of the conclusion. Even for gently dipping reflectors (on and inside the innermost ellipse, on the order of 10° in this example) imaged from a fine acquisition geometry, traces from similar offsets but different azimuths (0° from inline in Figure 4a and 27° from inline in Figures 4b and 5a) will contribute different migrated amplitudes to an image. This is most easily seen at points near the tops of the ellipses in Figure 4a and 4b, where maximum frequencies are lower in Figure 4b than in Figure 4a. This difference is caused by migration antialiasing, and it is independent of actual reflection coefficients. If the amplitude difference is not removed, it will cause interpretable wobble in AVAZ and, by smearing azimuthal antialias amplitude modulation differently for different offsets, in AVO. This modulation is only one of many, some recognized (e.g., Duren, 1991) and some not, that can compromise amplitude analysis of unmigrated and migrated data. The net result of all these modulation factors is some variation in spatial sampling, migrated amplitudes.
offset migration. Depending on the type of migration (e.g., migration of a single wavefield that is the medium’s response to a single point source excitation, or migration of a single COV volume), the antialiasing condition can be restrictive. WEM and slant-stack migrations are subject to migration operator aliasing just as Kirchhoff migration is, so the condition applies to all migration methods. It also applies equally to migration of land and marine surveys, although the deep targets of some marine surveys involve small enough reflection emergence angles and low enough frequencies that the condition has little effect. However, for targets at shallow and moderate depths, typical coarse acquisition spacings require antialiasing. The examples have shown that, in such cases, antialiasing can produce a subtle but significant migrated amplitude modulation, and failure to compensate for this can produce incorrect interpretation of migrated AVO and AVAZ. The compensation proposed here, a simple division, will fail when the antialiasing is severe, indicating the need for a stable improvement based on inverse theory.

CONCLUSIONS

I have investigated the effects of migration antialiasing on migrated amplitudes. I have presented a condition for antialiasing, depending on source and receiver inline and crossline spacings as well as inline and crossline spacings of the migrated image, which generalizes an earlier condition that is strictly valid only for zero-offset migration.

REFERENCES

Vermeer, G., 1990, Seismic wavefield sampling: SEG.