True Amplitude Reverse Time Migration: from Reflectivity to Velocity and Impedance Perturbations

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SUMMARY

Conventional prestack depth imaging methods aim at producing a structural image, which delineates the interfaces of the geological variations or the reflectivity of the earth. With the help of broadband acquisition and processing techniques, the bandwidth gap between depth imaging and seismic inversion is reducing. Here, we propose a theory to show how impedance and velocity perturbations can be estimated from the angle domain common image gathers produced by a true amplitude reverse time migration. The near angle stacked image provides the impedance perturbation estimate, while the far angle images can be used to estimate the velocity perturbation. Together with the ghost compensation technique, the method described here can be a useful tool of seismic inversion for marine exploration. Both synthetic and real data examples have demonstrated that the method is reliable and provides additional information for interpreting geological structures and rock properties.
Introduction

Reverse Time Migration (RTM) has become a high end technique to image and interpret subtle and complex geologic features, not only because it correctly handles complex velocities and propagates waves without angle limitations, but also it takes advantage of a complete set of acoustic waves (reflections, transmissions, diffractions, prismatic waves, etc.) to image subsurface structures with reasonably good amplitude (Zhang and Sun, 2009; Xu et al., 2011). Indeed, RTM is conventionally used as a tool for imaging the reflectivity or the interfaces of the subsurface geology. Recently, the concept of ghost compensation proposed by Zhang et al. (2012) brings the possibility of estimating the velocity perturbation of the geology using RTM for seismic inversion. The method extends the low frequency bandwidth of the image by removing the source and receiver ghosts during the conventional RTM. The velocity perturbation can thus be obtained as it is the integration of the stacked reflectivity image. Here, we further extend this theory, whereby we derive a theory for predicting the impedance and velocity perturbations using the Angle Domain Common Image Gathers (ADCIGs) from an RTM and demonstrate our theory through both synthetic and real data examples.

Theory

For a zero-phased and designatured shot record \( Q(x_s, y_s; x, y, t) \) with the shot at \((x_s, y_s, z_s) = (0, 0, 0)\) and receivers at \((x, y, z_r) = (0, 0, 0)\), the RTM based on the theory of true amplitude migration can be summarised as the forward propagation of the source wavefield \( p_{f} \):

\[
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p_{f}(\tilde{x};t) = \delta(\tilde{x} - \tilde{x}_s)f(t),
\]

and the backwards propagation of the receiver wavefield \( p_{g} \):

\[
\left\{ \begin{array}{l}
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p_{g}(\tilde{x};t) = 0, \\
p_{g}(x, y, z = 0; t) = Q(x, y; x, y),
\end{array} \right.
\]

where \( v = v(x, y, z) \) denotes the velocity, \( f(t) \) denotes the source wavelet with a flat spectrum, and \( \nabla^2 \) denotes the Laplacian operator. To obtain a common-shot image with correct migration amplitude, we need to apply the “deconvolution” imaging condition (Zhang et al., 2005). However, in practice, the “cross-correlation” imaging condition:

\[
R(\tilde{x}) = \int p_{g}(\tilde{x};t)p_{f}(\tilde{x};t)dt
\]

is often preferable for reasons of stability. Although this does not appear to be consistent with true-amplitude migration, Zhang et al. (2007) proved that the imaging condition 3 is a proper choice to obtain true amplitude ADCIGs from a wave equation based migration. For this to occur, the forward propagation 1 needs to be modified accordingly (Zhang and Sun, 2009):

\[
\left\{ \begin{array}{l}
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p_{f}(\tilde{x};t) = 0, \\
p_{f}(x, y, z = 0; t) = Q(x, y; x, y),
\end{array} \right.
\]

This equation is different from the conventional wave equation 1 for the forward wavefield, because the source is treated as a boundary condition instead of a right-hand-side forcing term in the propagation. Furthermore, to generate the angle-dependent reflectivity, the following 3D imaging condition provides AVA amplitude versus angle friendly migration amplitude in the subsurface angle domain (Xu et al., 2011):

\[
R(\tilde{x}; \theta; \phi) = \iiint \frac{v(\tilde{x})}{\sin \theta} \delta(\theta - \theta')\delta(\phi - \phi') p_{g}p_{f}d\tilde{x}d\theta'd\phi',
\]

where \( \theta \) and \( \phi \) are the reflection angle and azimuth angle at the imaging location, respectively.

For marine acquisition, both the source ghost \( G_s \) and receiver ghost \( G_r \) effects must be taken into consideration and can be easily compensated during the wave propagation of the RTM. Zhang et al. (2012) modified the boundary conditions in 2 and 4 as follows:
where \( \alpha_s \) and \( \alpha_r \) denote the propagation angles at the surface for the source and receiver, respectively, and \( \hat{p}_s(\omega) \) denotes the Fourier transform of \( p_s(t) \); and demonstrated that compensating ghost effects leads to reliable recovery of low frequency components in the image.

Here, we take one step further and use RTM to estimate velocity or impedance perturbations. Considering the acoustic equation with both velocity \( v(\vec{x}) \) and density \( \rho(\vec{x}) \) variations as follows:

\[
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \rho \frac{1}{\rho_0} \frac{\partial}{\partial t} \right) p(\vec{x}; t; \vec{x}_s) = \delta(\vec{x} - \vec{x}_s) \delta(t).
\]

For given initial velocity \( v_0(\vec{x}) \) and density \( \rho_0(\vec{x}) \) models, the perturbed wavefield \( \delta p(\vec{x}; t; \vec{x}_s) = p - p_0 \) satisfies the following equation:

\[
\left( \frac{1}{v_0^2} \frac{\partial^2}{\partial t^2} - \rho_0 \frac{1}{\rho_0} \frac{\partial}{\partial t} \right) \delta p(\vec{x}; t; \vec{x}_s) = \left( \frac{2\delta v}{v_0} \frac{\partial^2}{\partial t^2} - \left( \frac{\partial \rho}{\rho_0} \right) \frac{\partial}{\partial t} \right) p_0(\vec{x}; t; \vec{x}_s),
\]

where \( \delta v = v - v_0 \) and \( \delta \rho = \rho - \rho_0 \) denote the velocity and density perturbations, respectively. Using the similar method developed in Jin et al. (1992) and Foragus and Lambare (1997), we obtain the ray-based relation between the perturbed geological models and wavefield:

\[
\frac{\delta v}{v_0} + \cos^2 \theta \frac{\delta \rho}{\rho_0} = 32\pi \iiint \frac{\delta p(\vec{x})}{v_0} \cos^2 \theta \cos \alpha_s \cos \alpha_r \left[ A_s A_r e^{-i\omega(t - \tau_s - \tau_r)} Q \delta(\theta - \theta_0) d\vec{x}, d\vec{x}_s, d\omega \right],
\]

where \( A_s (A_r) \) is the amplitude of the Green's function from the source (receiver) to the image point, \( \tau_s (\tau_r) \) is the traveltime between the source (receiver) and the image point. In the context of RTM, equation 9 can be rephrased by modifying the imaging condition 6 as follows

\[
\sin^2 \theta \frac{\delta v}{v} + \cos^2 \theta \frac{\delta (v \rho)}{v \rho} = \iiint \frac{\delta p(\vec{x})}{v_0} \cos^2 \theta \delta(\theta - \theta_0) \frac{\hat{p}_s(\omega)}{i\omega} \frac{\hat{p}_s(\omega)}{\hat{p}_s(\omega)} \delta(\vec{x} - \vec{x}_s) d\vec{x}, d\vec{x}_s, d\omega \delta \theta.
\]

Equation 10 tells us that if we output subsurface angle gatherers with a proper imaging condition, the near angle images predict the impedance perturbation \( \delta(v \rho)/(v \rho) \), while the far angle images can be used to estimate the velocity perturbation \( \delta v/v \). Therefore, we can separate the effects of velocity and density on the stacked image by outputting ADCIGs.

**Numerical and real data examples**

The first example is designed to prove the reliability of impedance and velocity perturbation estimation using RTM after ghost compensation. Here, we construct a horizontally invariant geological model to 4km in depth. Figure 1 left presents the shot record with the shot in the centre of the section and the receivers spread to both sides with maximum offset of 12km. Both the source and receiver ghosts are recorded. Setting the background velocity and density to 2000m/s and 2000kg/m³, respectively, we use the ghost compensated RTM formulation 2, 4 and 6 to propagate the wavefields, apply imaging condition 10 to generate the subsurface offset gathers (Sava and Fomel, 2003), and then convert them to ADCIGs, as shown in Figure 1 right. Based on our theory, the 0° trace directly estimates the impedance perturbation while velocity perturbation can be calculated using any nonzero degree traces. However, a far angle trace will better ease the numerical errors than a near angle trace, when separating the velocity perturbation from the impedance perturbation using equation 10. We thus select the 0° and 66° traces for the purpose of estimating the impedance and velocity perturbations. After simple rescaling, one could see from Figure 2 that the estimated perturbations (red) match with the exact perturbations (blue) reasonably well. The mismatch of the velocity perturbation at depths more than 3km is due to the lack of large reflection angles over the limited offset range and the stretch of an event on the far angle CIG.

In the second example, we apply RTM to the BP2004 2D model (Billette and Brandsberg-Dahl, 2005). The synthetic data is generated by high order finite-difference acoustic modeling using both velocity and density models, with shot spacing 50m, receiver spacing 25m and 8km maximum offset. Both source and receiver ghosts are recorded at 12.5m depth. In Figure 3, we compare the impedance
perturbation output from a ghost-compensated RTM (top right) with the exact impedance perturbation (top left), which is the product of the model velocity and density perturbations. For detailed investigation, we present the estimated and exact impedance at a given location, indicated by the blue dashed line in the top images. The estimated impedance perturbation reproduces almost the exact solution, except at the sharply changing impedance boundaries, for example, at the water bottom or at the top of salt boundary. This is due to the large model perturbation which doesn’t fully satisfy the linearization assumption in deriving equation 8 and the lack of very low frequencies in seismic data. This result demonstrates the applicability of our theory to geological models with complex structures.

Figure 1: A shot record (left) and a migrated ADCIG (right) using the true amplitude RTM. The CIG figure demonstrates the selected 0° and 66° traces (dashed blue lines).

Figure 2 The impedance (top) and velocity (bottom) perturbations for the synthetic experiment, which present the exact perturbations (blue) and computed perturbations from the true amplitude RTM (red).

Figure 3 The exact (top left) and estimated (top right) impedance perturbations for the numerical experiment using BP2004 model. The detailed plot (bottom) of the exact (blue) and estimated (red) impedance perturbations at the given location (vertical dashed blue lines).

In Figure 4 we present ghost-compensated RTM images, a stacked reflectivity image (left) and an impedance perturbation section (middle), for a shallow water dataset from the Central North Sea. The source and receiver depths are 6m and 7m, respectively. The low frequency components are compensated in both images, which give the sections a more continuous and textured appearance characteristic of broadband data. In particular, we compare the estimated impedance perturbation...
from RTM against the filtered well-log data (right) at a given location (the red vertical line in the middle image). It can be seen that the estimated impedance provides a good approximation to the measured data. The ghost-compensated RTM is not only a technique to achieve an image with broad bandwidth, but also a useful tool to estimate the impedance or velocity of the real-world subsurface geology.

**Figure 4.** Ghost-compensated RTM images from a Central North Sea dataset: the stacked reflectivity image (left) and the impedance perturbation section (middle). The comparison between the measured impedance using filtered well-log (black) and the estimated impedance from RTM (red) at the given location (the vertical red line in the middle image) is shown on the right.

**Conclusions**

We have developed a theory of true amplitude RTM to delineate the impedance and velocity perturbations of subsurface structures. The near angle stacked image provides the impedance perturbation estimate, while far angle images can be used to estimate the velocity perturbation. Together with the ghost compensation technique, the method described here can be a useful tool of seismic inversion for marine exploration and used to bridge the gap between full waveform inversion (Tarantola, 1984) and reverse time migration. Both synthetic and real data examples have shown the method is reliable and provides additional geological information for interpretation and inversion.

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**References**

Zhang, Y., Xu, S., Bleistein, N. and Zhang, G., 2007, True amplitude angle domain common image gathers from one-way wave equation migrations: Geophysics, 72, S49-58.