Curvature of a geometric surface and curvature of gravity and magnetic anomalies

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ABSTRACT

Curvature describes how much a line deviates from being straight or a surface from being flat. When curvature is used to interpret gravity and magnetic anomalies, we try to delineate geometric information of subsurface structures from an observed nongeometric quantity. In this work, I evaluated curvature attributes of the equipotential surface as functions of gravity gradients and analyzed the differences between the theoretical derivation and a practical application. I computed curvature of a synthetic model that consisted of representative structures (ridge, valley, basin, dome, and vertical cylinder) and curvature of the equipotential surface, gravity, and vertical gravity gradient (which is equivalent to the magnetic reduction-to-the-pole result) due to the same model. A comparison of curvature of such a geometric surface and curvature of different gravity quantities was then made to help understand these curvature differences and an indirect link between curvature of gravity data and actual structures. Finally, I applied curvature analysis to a magnetic anomaly grid in the Gaspé belt of Quebec, Canada, to illustrate its useful property of enhancing subtle features.

INTRODUCTION

Curvature can be defined as the reciprocal of the radius of a circle that is tangent to the given curve at a point. Curvature is large for a curve that is "bent more" and is zero for a straight line. Smaller circles bend more sharply and hence have higher curvature. Mathematically, curvature of a 2D curve in the x-z plane is a function of first- and second-order derivatives (Thomas, 1972, pp. 473–475; Roberts, 2001):

\[
K = \frac{d^2z/dx^2}{[1 + (dz/dx)^2]^{3/2}}. \tag{1}
\]

Unlike the first-order derivative methods for delineation of lineaments, curvature contains the added dimension of shape. For a 3D surface, curvature is also independent of the surface orientation. However, there is not a straightforward extension of equation 1 for a 2D plane to the curvature at a point on a 3D surface. At a point \((x, y)\), different curvatures may exist. When a mapped surface is created in a gridded form, curvature at a particular point is often characterized by curvature of a local quadratic surface that fits grid values in a least-square sense, and curvature is thus calculated from coefficients of this fitting surface not explicitly from derivatives. The most commonly used quadratic surface has this following form:

\[
z(x, y) = ax^2 + by^2 + cxy + dx + ey + f. \tag{2}
\]

Curvature has been widely used to interpret geophysical data after the publication of Roberts (2001). He selects 11 curvature attributes based on their applicability to seismic interpretation: mean, Gaussian (or total), maximum, minimum, most positive, most negative, dip (profile), strike (tangential), contour (plan), curvedness, and shape index. I reproduce in Appendix A these 11 curvature attributes in Roberts (2001) as functions of the coefficients in equation 2. There may be an infinite number of possible curvature attributes. However, curvature attributes other than the 11 in Roberts (2001) and the differential curvature are rarely applied to geophysical data. Moreover, different curvature attributes are not independent. Many can be derived from a combination of the maximum and minimum curvatures: The mean curvature is their average, the differential is their difference, the Gaussian is their product, the curvedness is their magnitude (divided by \(\sqrt{2}\)), and the shape index is related to an arctangent of their combination, etc. (see Appendix A).

Different groups have directly computed curvature of potential field data. Schmidt and Götze (2003) use many of the curvature...
attributes in Roberts (2001). Cooper (2009, 2013) computes the profile (dip) curvature of gravity and magnetic anomalies for edge detection. He also uses a balanced plan (contour) curvature to enhance ridges in potential field data (Cooper, 2010).

Hansen and deRidder (2006) give a different derivation. They form the so-called curvature tensor (or Hessian), i.e., a $2 \times 2$ matrix, and calculate the two eigenvalues of this matrix in closed form. Their more-positive eigenvalue and more-negative eigenvalue are just the most-positive and most-negative curvatures, respectively. They computed the most-positive and most-negative curvatures of the horizontal gradient amplitude (HGA) of a magnetic reduction-to-the-pole (RTP) result. They used the more positive eigenvalue for linear feature analysis and the more negative eigenvalue for estimating source depth (also see Phillips et al., 2007).

Lee et al. (2012) apply the full, profile (dip), and plan (contour) curvatures to the magnetic anomaly, for lineament analysis (dike swarms and fault systems). These three curvatures are defined in Zevenbergen and Thorne (1987), for a surface that is different from the quadratic surface in equation 2 and is not popularly used.

In the magnetic case, like many other transformations, curvature should be computed for magnetic RTP results, not observed magnetic anomalies directly, unless the magnetic inclination is very high (e.g., in Canada: Lee et al., 2012) and remanent magnetization is insignificant or parallel with the present geomagnetic field direction. Mathematically, the magnetic RTP result centers an anomaly over the source and is equivalent to the vertical gravity gradient (VGG). Properties of curvature applicable to the VGG are equally applicable to the magnetic RTP.

The equipotential surface is a gravity quantity and is closely linked to a particular geometric surface of the earth. Curvature of the equipotential surface is a function of the gravity and the gravity gradients and may thus be used to interpret gravity gradients measured by modern gravity gradiometry technologies (Cevallos et al., 2013; Chowdhury and Cevallos, 2013; Li and Cevallos, 2013). However, approximations have to be made when curvature of the equipotential surface is applied to field data. In this work, I will emphasize these approximations and their consequences.

Geophysical applications of curvature involve computing curvature of a geophysical quantity and then interpreting geometric features of subsurface structures from that curvature. Roberts (2001) uses curvature to interpret reflection seismic data. Mathematically speaking, curvature attributes should be computed for a depth horizon, i.e., $z$ in equation 2 is depth. In practice, for a purely qualitative analysis, a time-to-depth conversion is rarely done before curvature attributes are computed; i.e., depth $z$ in equation 2 is replaced by time $t$. For simplicity, “it is assumed that the time scale is interchangeable with depth” (Roberts, 2001, p. 89). This assumption is valid because the two-way time in milliseconds is similar to the depth in meters in magnitude and even in shape.

However, this is not the case for the potential field anomaly: the gravity potential (in m x mGal), the gravity anomaly (in mGal), the gravity gradient anomaly (in Eötvös), or the magnetic anomaly (in nT). The potential field anomaly $\phi$ is a function of $(x, y, z)$. When we compute curvature of the potential field anomaly, the quadratic surface corresponding to equation 2 becomes

$$\phi(x, y, z) = ax^2 + by^2 + cxy + dx + ey + f.$$  (3)

The potential field anomaly is a function of $(x, y, z)$, but we often ignore the observation altitude variation and its effect on the potential field anomaly when we compute curvature of the potential field anomaly. The effect of varying altitudes can be removed by a surface-to-level continuation, but we often do not conduct this process.

Strictly speaking, an anomalous gravity (potential or field or gradient) quantity is a function of the source geometry, the density contrast, and the relative distance and position between an observation point and the source. A structural uplift may produce a gravity anomaly low when the density contrast is negative. A structural uplift with a positive density contrast produces a VGG high over the structure as well as VGG lows outside the structure. Gravity or magnetic anomalies can be produced by a purely lateral variation in lithology (density or magnetic susceptibility). Moreover, a gravity or magnetic anomaly is a total volumetric effect of all possible sources. I will examine in this work the link between curvature of the equipotential surface, field, and VGG anomalies and the actual subsurface structural geometries, using a representative model.

**CURVATURE OF THE EQUIPOTENTIAL SURFACE**

An equipotential surface is a geometric surface on which the gravitational potential is a constant. The best-known equipotential surface of the earth is the geoid that follows a mean sea surface over oceans and defines the elevation. The torsion balance, one of the first geophysical exploration tools, measured gravity gradients. Two of the measured components ($G_{XY}$ and $G_{UV} = (G_{YT} - G_{XZ})/2$, $G_{IJ}$ being the second-order derivatives of the gravity potential) are directly related to curvature of the earth’s equipotential surface and are thus called curvature gradients. Slotnick (1932) derives the differential curvature of the equipotential surface to help understand the theory of the torsion balance and the physical picture of curvature gradients. Cevallos et al. (2013) collectively present four curvatures: mean, Gaussian, differential, and shape index. Below, I give a systematic derivation of the differential curvature and the 11 curvature attributes defined in Roberts (2001), for an equipotential surface.
Derivation

When the \((x, y)\) origin is located at a computation point, coefficients of equation 3 can be expressed as the first- and second-order derivatives:

\[
a = \frac{\partial^2 \phi}{\partial x^2}; \quad b = \frac{\partial^2 \phi}{\partial y^2}; \quad c = \frac{\partial^2 \phi}{\partial x \partial y}; \quad d = \frac{\partial \phi}{\partial x}; \quad e = \frac{\partial \phi}{\partial y}.
\]

(4)

The derivation of Slotnick (1932) uses a specifically arranged coordinate system: the origin of the system is located at the observation point, the \(z\)-axis is directed along the normal to the equipotential surface, and the \(x\) and \(y\)-axes are in the tangential plane to the equipotential surface (Figure 1). The two first-order derivatives become zero in this coordinate system, and coefficients \(a, b,\) and \(c\) of equation 4 are thus linked to the (vertical) gravity \((g = G_z)\) and the gravity gradients. In other words, equation 4 becomes

\[
a = -\frac{G_{XX}}{2g}; \quad b = -\frac{G_{YY}}{2g}; \quad c = -\frac{G_{XY}}{g}; \quad d = e = 0.
\]

(5)

It is clearly seen that curvature of an equipotential surface is a function of gradient components \(G_{XX}, G_{YY},\) and \(G_{XY}\). These gradients are thus called curvature gradients. Slotnick (1932, equation 31) gives explicitly the differential curvature of the equipotential surface

\[
K_d = \frac{2\sqrt{G_{XY}^2 + G_{UV}^2}}{g}.
\]

The principal (maximum and minimum) curvatures of the equipotential surface are

\[
K_{\text{max}} = -\frac{G_{XX} + G_{YY} - 2\sqrt{G_{XY}^2 + G_{UV}^2}}{2g}
\]

and

\[
K_{\text{min}} = -\frac{G_{XX} + G_{YY} + 2\sqrt{G_{XY}^2 + G_{UV}^2}}{2g}.
\]

(9)

(10)

The curvedness becomes

\[
K_n = \frac{2G_{XX}^2 + 2G_{YY}^2 + 4G_{XY}^2}{2g}.
\]

(11)

The shape index is

\[
S_l = \frac{2}{\pi} \tan^{-1} \frac{-G_{XX} - G_{YY}}{2\sqrt{G_{XY}^2 + G_{UV}^2}},
\]

(12)

which is dimensionless.

For the equipotential surface, the most positive curvature \((K_+\) and the most negative curvature \((K_-)\) are the same as the maximum and minimum curvatures, respectively; the dip (profile) curvature \(K_{\text{dip}}\) and strike (tangential) curvature \(K_s\) are undefined or unrestricted; and the contour (plan) curvature \(K_c\) becomes zero. All curvature attributes of an equipotential surface are truly a function of curvature gradients \(G_{XX}, G_{YY}, G_{XY},\) and \(G_{UV} = (G_{YY} - G_{XX})/2\) and are not related to \(G_{XZ}, G_{YZ},\) and \(G_{ZZ},\) the three gradient components that are widely used in qualitative interpretation.

Practical applications

Curvatures in equations 6–12 are derived under the specifically arranged coordinate system that satisfies the equipotential surface condition, and these curvatures must be practiced with caution. My understanding is as follows: First, a rigorous computation of these curvatures requires an arrangement of the specific coordinate system for each individual observation point or station. However, it makes no practical sense to do so in interpretation of gravity and gravity gradient grids. Second, it is acceptable to use a fixed specific coordinate system for all stations if we work with the absolute gravity (its average is 980,000 mGal) and the absolute gravity gradient values, respectively. The absolute gravity gradient values are hundreds of Eötvös or fewer and may be thought to be relative to a total average of each corresponding gradient component. Such computed curvature is a curvature perturbation or anomaly, not a curvature itself, of an
equopotential surface. Fourth, one may think that \( g \) in the denominator of equations 6–11 can be observed gravity anomalies. This is incorrect. One simple fact is that gravity anomalies may be negative, positive, and zero, and they may have an arbitrary base (DC) level. Dividing by such anomaly values only amplifies the zero contour lines of the gravity anomaly grid and produces a geologically meaningless result. Besides, dividing by zero becomes singular and impossible.

Curvature attributes of the equipotential surface are thus used to interpret results observed by gravity gradiometers. Modern gravity gradiometry is referenced to the geographic east, north, and vertical down coordinate system; and they are denoted in this work as the \( x \), \( y \), and \( z \)-axes, respectively. The quantity \( g \) in equations 6–11 serves as a scaling factor. The results of the mean, differential, maximum, minimum curvatures, and curvedness are better presented in the unit of per gigameter (per \( 10^9 \) meter), or \( \text{gm} \) for short.

**A SYNTHETIC MODEL AND ITS CURVATURE**

**The model**

To my understanding, curvature finds two major applications in geophysical interpretation: analysis of lineaments (boundaries, edges, faults, ridges, valleys) and identification of shapes (plane, dome, basin, anticline, syncline, saddle). I have designed a synthetic model (Figure 2) to check if and how curvature of different gravity quantities may be used for these two applications. The model consists of a dome, a basin, an anticline, a syncline, and a vertical cylinder (representing an intrusion). The first four are selected because they are said to be identifiable by curvature, and either the top or the base of each represents an edge. In addition, an anticline is defined by a ridge, and a syncline by a valley. A vertical cylinder is chosen because it contains edges in all directions.

All five bodies or structures were scaled to be geologically reasonable. The structures were assigned the same top and bottom depths: 3 and 5 km, respectively. The radii of the cylinder, dome, and basin are 10 km. The anticline and the syncline have a strike length of 30 km and a width of 5 km. The density contrast of the basin and syncline is \(-0.3 \text{ g/cm}^3\), and the density contrast of the other three is \(+0.3 \text{ g/cm}^3\). Figure 3 displays the gravity potential, gravity, and gravity gradient responses of the model in Figure 2.

**Curvature of different quantities**

I use equations A-1–A-12 to compute 12 curvature attributes of each of the following four: (1) the geometric surface consisting of the five structures (Figure 2), (2) the gravity potential (Figure 3a), (3) the gravity (Figure 3b), and (4) the VGG (= \( G_{zz} \)) (Figure 3c). For the equipotential surface, only seven of the 12 curvature attributes are meaningful; and they are functions of curvature gradients \( G_{xx}, G_{yy}, G_{xy} \), and \( G_{uv} \). Equations 6–12 are used to compute seven curvatures of the equipotential surface.

In the real world, we do not normally compute curvature of the gravity potential because it is not observed. However, I like to compare curvature of the gravity potential anomaly and curvature of the equipotential surface. Both use the same second-order derivatives, i.e., the same gravity gradient results. However, the first uses all three components of the anomalous gravity field vector, whereas the latter involves only the absolute vertical gravity and assumes the two absolute horizontal gravity components to be zero.

**SOME OBSERVATIONS ABOUT CURVATURE**

I select some representative curvature results and display them in Figures 4–8. For easy discrimination, all grids in Figures 3–9 are displayed using a linear color stretch for a 99th percentile of each individual grid. Following are my observations and comparisons.

**The same curvature of different quantities may show different features**

Figure 4 shows the mean and differential curvature attributes of five different quantities. These two curvature attributes are selected because many others (maximum, minimum, Gaussian, curvedness, shape index, etc.) can be derived from them. Most of the conclusions about curvature found in publications are established for curvature of a geometric surface. Figure 4 indicates that properties about curvature of a geometric surface are not directly applicable to curvature of a gravity quantity, and features seen in one gravity quantity may not be generalized to another gravity quantity. There is only one anomaly centered over each body or structure in the mean curvature of the equipotential surface (Figure 4c) or the gravity potential anomaly (Figure 4e). However, additional closed lineaments surround the central anomalies in the mean curvature of the VGG (Figure 4i).
Figure 3. The responses to the synthetic model in Figure 2 of (a) the gravity potential, (b) the gravity, and gravity gradient components: (c) $G_{zz}$, (d) $G_{xx}$, (e) $G_{xy}$, and (f) $G_{uv}$.

Figure 4. The mean curvature ($K_m$) of (a) the geometric surface, (c) the equipotential surface, (e) the gravity potential, (g) the gravity, and (i) the VGG. The differential curvature ($K_d$) of (b) the geometric surface, (d) the equipotential surface, (f) the gravity potential, (h) the gravity, and (j) the VGG.
The above observation has the following theoretical explanation. The gravity potential, field, and the gradient anomalies are not linearly proportional to the source geometry, but they are a function of the geometry of the source, the density contrast, as well as the distance and relative position between an observation station and the source. The potential, field, and gradient are inversely proportional to the distance, in power one, two, and three, respectively. Anomalies can be produced by a purely lateral density variation, and a sign change in the density contrast reverses the sign of the anomalies (i.e., all grids in Figure 3) completely.

The mean curvature of the equipotential surface (equation 7 and Figure 4c) is the VGG (Figure 3c) divided by a big constant. According to Cevallos et al. (2013), the mean curvature of the equipotential surface is useful for interpretation of gravity gradient data. For qualitative interpretation, the VGG (G\textsubscript{zz}) is routinely used, and the mean curvature of the equipotential surface does not add new information.

The most positive and most negative curvatures

The maximum curvature of the geometric surface (Figure 5a) is effective at delimiting faults and lineaments. However, a better edge-type display is derived by searching all possible normal curvatures for the most positive and most negative values. The most positive and most negative curvatures are readily obtained by setting coefficients \( d \) and \( e \) in equation 2 or 3 to zero in equations for the maximum and minimum curvatures, respectively. These two coefficients are zero for an equipotential surface (see equation 5), and the maximum curvature and the most positive curvature of an equipotential surface are thus identical (Figure 5c). Moreover, these two curvatures are different from the most positive curvature of the gravity potential only by a big constant scaling factor.

For a geometric surface, the most positive curvature defines the curvature that has the greatest positive value and will show anticlinal and domal features. It can be a negative number, and negative values indicate a bowl feature. The most negative curvature defines the curvature that has the greatest negative value and will in general highlight synclinal and bowl features. It can be a positive number, and positive values indicate a dome feature. The most positive curvature and the most negative curvature reveal almost every single lineament contained within a surface. Figure 5b displays the most positive curvature of the geometric surface, and it supports this understanding. However, the most positive curvatures of the equipotential surface (Figure 5c), the gravity (Figure 5d), and the VGG (Figure 5f) delineate the geometric lineaments very inaccurately. This is again because the potential, the gravity, and the VGG are a function of not only the geometry of a causative body but also other factors.

Dip and strike curvatures reveal ridges and valleys

The dip (profile) curvature is a measure of the rate of change of dip in the maximum dip direction. Extracting the curvature in a direction perpendicular to the dip curvature, i.e., along strike, gives the strike (tangential) curvature. The average of the dip and strike curvatures results in the mean curvature, as they are orthogonal curvatures. The magnitude and direction of faults are preserved with these two attributes. They tend to exaggerate any local relief and may enhance subtle features. The dip and strike curvatures of the geometric surface together reveal all edges, ridges, and valleys (Figure 6a and 6b). The dip and strike curvatures of the equipotential surface are undefined (unrestricted). There are gravity and VGG highs over ridges and lows over valleys (Figure 3b and 3c), and these highs and lows are sharpened in their dip curvature (Figure 6c and 6e) and particularly their strike curvature (Figure 6d and 6f).

But neither the dip curvature nor the strike curvature of the gravity or VGG outlines the edges of the geometric surface well.

Shape identification

Roberts (2001, Figure 6) uses a combination of the mean curvature \( K_m \) and the Gaussian curvature \( K_g \) for a qualitative shape classification of a geometric surface: It is a dome when \( K_g > 0 \) and \( K_m > 0 \); a bowl (basin) when \( K_g > 0 \) and \( K_m < 0 \); a ridge (anticline) when \( K_g = 0 \) and \( K_m > 0 \); and a valley (synform) when \( K_g = 0 \) and \( K_m < 0 \). Chopra and Marfurt (2007) use the most positive and most negative curvatures instead. The shape index that ranges from −1 to +1 allows a more quantitative shape description: bowl (−1), valley (−0.5), flat (0.0), ridge (+0.5), and dome (+1) (Roberts, 2001). These characteristic numbers are derived and work perfectly for geometric surfaces (Figure 7a). However, if I apply them directly to interpret the shape index of the equipotential surface (Figure 7b), the gravity (Figure 7c), and the VGG (Figure 7d), a shape classification produces many false indications that are due to interference from neighboring bodies. The complexities increase with the order of the vertical derivative. The five bodies or structures in this synthetic model are separated widely, and it makes a classification more difficult when applying to field data. Another interesting observation may be that the shape index of the geometric surface of the vertical cylinder is about −0.5; but the shape index, at the center of the cylinder, of its equipotential surface, gravity, or VGG is about +1.0. This is because the vertical cylinder in this model has a positive density contrast and produces a dome-shaped anomaly.

Curvature for edge detection

The HGA of gravity (or VGG) is widely used to detect boundaries, edges, and faults. Different curvature attributes have also been proposed to serve this type of interpretation. Hansen and deRidder (2006) use the more positive eigenvalue (i.e., the most positive curvature) of the HGA of the magnetic RTP result (equivalent to the VGG). Cevallos et al. (2013) use the zero contour lines of the Gaussian curvature of the equipotential surface. Cooper (2013) uses the zero contours of the dip curvature of the gravity anomaly.

I apply these edge detectors to the synthetic model, and the results are shown in Figure 8. The technique of Hansen and deRidder (2006) requires a computation of the third-order derivatives of the magnetic RTP result and is thus sensitive to noise in observed data. Even so, the most positive and most negative curvatures produce complex and inaccurate edges (Figure 8a and 8b). Lines in Figure 8c and 8d are the zero contours of the Gaussian curvature of the equipotential surface and the Gaussian curvature of the VGG, respectively. Contour lines in Figure 8c delineate true edges well but produce many false ones too. Contour lines in Figure 8d are complex and will be too difficult to tell true edges from false ones. The false indication is related to the fact that curvature is a function of horizontal not vertical derivatives and thus suffers from strong interference from neighboring sources. To overcome the problem (i.e., to remove false indications), Cevallos et al. (2013) interpret
the zero contours of the Gaussian curvature of the equipotential surface together with the mean curvature of the equipotential surface.

To an outstanding contrast, the maxima of the HGA of the gravity (Figure 8e) and the maxima of the HGA of the VGG (Figure 8f) outline the true edges sharply and accurately, accompanied by no false edges. The HGA of the gravity anomaly is related to a special line (not surface) curvature— the curvature of the plumb (or gravity field) line (Torge, 2001, pp. 59–62; Cevallos et al., 2013):

$$K_f = \frac{\sqrt{G_{XZ}^2 + G_{YZ}^2}}{g}.$$  \hspace{1cm} (13)

Like curvature of the equipotential surface, by definition, the gravity and two horizontal gravity gradients in equation 13 are absolute. In practice, we may use the absolute gravity and relative gradients. The curvature of the plumb line and the HGA of the gravity anomaly thus differ by a constant scaling factor and make no difference for a qualitative analysis.

**Effects of depth on curvature**

All results in Figures 3–8 are computed for the model in Figure 2, and all bodies/structures in the model have a top depth of 3 km. The source depth affects the magnitude and the shape of the resulting potential field anomaly. A source of any geometry produces potential

![Image](image-url)

**Figure 5.** (a) The maximum curvature ($K_{\text{max}}$) and (b) the most positive curvature ($K_+$) of the geometric surface, (c) the maximum curvature (the same as the most positive curvature) of the equipotential surface, (d) the most positive curvature of the gravity anomaly, (e) the maximum curvature and (f) the most positive curvature of the VGG.

![Image](image-url)

**Figure 6.** (a) The dip (profile, $K_{\text{dip}}$) curvature and (b) the strike (tangential, $K_s$) curvature of the geometric surface, (c) the dip curvature and (d) the strike curvature of the gravity, (e) the dip curvature and (f) the strike curvature of the VGG.
Figure 7. The shape index ($S_i$) of (a) the geometric surface, (b) the equipotential surface, (c) the gravity, and (d) the VGG.

Figure 8. Results of different edge detectors applied to the synthetic model. They are (a) the most positive curvature ($K_+$) and (b) the most negative curvature ($K_-$) of the HGA of the VGG, (c) the zero contours of the Gaussian curvature ($K_g$) of the equipotential surface, (d) the zero contours of the Gaussian curvature of the VGG, (e) the HGA of the gravity, and (f) the HGA of the VGG.

Figure 9. The curvature results when the top depth of the model in Figure 2 is increased from 3 to 13 km. (a) The most positive curvature ($K_+$) and (b) the shape index ($S_i$) of the equipotential surface, and (c) the most positive curvature and (d) the shape index of the VGG. They should be compared with Figures 5c, 7b, 5f, and 7d, respectively.
field responses as those of a point mass, when the observation-source distance is large enough. The source depth affects curvature of the potential field anomaly in a similar manner.

I have increased the top depth of the model in Figure 2 from 3 to 13 km; then I computed the gravity potential, the field, and the gradient responses; and different curvature attributes. Figure 9 displays the most positive curvature (the same as the maximum curvature) and the shape index of the equipotential surface and the most positive curvature and the shape index of the VGG. Figure 9a shows the most positive curvature of the equipotential surface for a top depth of 13 km may be compared with Figure 5c depicting the most positive curvature of the equipotential surface for a top depth of 3 km, Figure 9b compared with Figure 7b for the shape index of the equipotential surface, Figure 9c with Figure 5f for the most positive curvature of the VGG, and Figure 9d with Figure 7d for the shape index of the VGG.

The comparison suggests that a capability of delineating a target decreases with the depth. All curvatures become smoother and interference between bodies becomes stronger when the depth gets larger. The shape index of the equipotential surface directly over the ridge and the valley for the top depth of 13 km (Figure 9b) is close to 1.0 (indicating a dome) and −1.0 (a bowl), changing from 0.5 to −0.5 in Figure 7b.

The gravity or magnetic anomaly is a volumetric effect of many sources. This volumetric effect intensifies with depths. In this model test, bodies are distributed horizontally only. In the real world, sources are also stacked vertically, and a gravity quantity or its curvature cannot be directly related to a particular source. The level of approximation or assumption we have to make in interpretation of curvature is really similar to those used in the gravity or magnetic depth estimation.

APPLICATIONS TO THE GASPÉ AEROMAGNETIC ANOMALY

In this section, I apply curvature analysis to an excellent data set in the extreme northern Appalachian Mountains that has been compiled and interpreted by the Geological Survey of Canada (Pinet et al., 2008). Gravity data and high-resolution aeromagnetic data, coupled with extensive field mapping and susceptibility measurements, allow comparison of potential field transformation results with known geologic structures (Figure 10). This example shows that curvature is useful in interpretation of gravity gradient and magnetic anomalies with a wide range of wavelengths and magnitudes. Curvature is especially good at enhancing subtle features such as those created by folded or faulted sedimentary layers with slight contrasts in magnetic susceptibility.

Figure 11a shows aeromagnetic anomalies in the Gaspé Peninsula of Québec, Canada, which is publicly available from the Geological Survey of Canada. The survey was flown at 120-m clearance in 2004, and the data were gridded using an interval of 75 m. The grid covers an area of 47 × 85 km², and the magnetic inclination at the center of the grid is as high as 72°. This makes a magnetic RTP unnecessary unless there is strong remanent magnetization, and its direction is significantly different from the present geomagnetic field direction. I thus treat the magnetic anomaly grid in Figure 11a as equivalent to the VGG and compute its curvature.

According to Pinet et al. (2008), the aeromagnetic anomaly grid area in Figure 10 is part of the Eastern Gaspé belt subbasin. This area is geologically complex and is characterized by faults exhibiting various types of movements. The east–west-trending Grand Pabos fault is a right-lateral strike-slip fault that has long been recognized as a major structural feature in the southern part of the Gaspé Peninsula. The Grande-Rivière fault and the Marcil-Sud fault are located in the...
northern part of the area in which relatively short-wavelength magnetic anomalies are associated with magmatic dikes and stocks. The Mont-Alexandre syncline centers on a magnetic depression and is characterized by composite, relatively high amplitude, magnetic anomalies on each flank that are associated with shallow mafic volcanic rocks. The southern part of the study area is characterized by a highly variable magnetic fabric that differs strongly from the more subdued magnetic signal of the Gaspé Peninsula. The strong magnetic variations in the southern grid area are due in part to shallow volcanics and also, possibly, to intrusive bodies at depth. Magnetic susceptibility measurements (Pinet et al., 2008) indicate that the sources of short-wavelength linear magnetic anomalies are either high magnetic susceptibility contrasts related to magmatic rocks or subtle susceptibility contrasts within the sedimentary succession. Within the Gaspé belt, Pinet et al. (2008) note that magnetic lineaments often tracked sedimentary contacts, presumably due to concentrations of magnetic minerals. These anomalies have very low amplitudes and require enhancement to interpret.

Magnetic anomalies in Figure 11a are displayed using a linear color stretch of 90th percentile, and the actual values range from −725 to 962 nT. The subtle features can be clearly seen after application of the automatic gain control (Figure 11b) or coherence filter (Figure 11c), using a 3×3 window. A majority of the automatic gain control results fall in a very narrow range. They range from −1.5 to 1.5 for all data points and from −1.00028 to −0.99964 for an 80th percentile. The coherence is computed by crosscorrelating a central window with the neighboring eight ones, i.e., in eight different directions (Li, 2009). Low coherence values correspond to discontinuities. I have computed all 12 curvature attributes of the magnetic anomaly grid using equations in Appendix A and show here the results of the differential curvature (Figure 11d), mean curvature (Figure 11e), maximum curvature (Figure 11f), curvedness (Figure 11g), and strike curvature (Figure 11h).

The differential curvature and curvedness reveal very similar features, and major features or discontinuities in them are also traceable in the maximum curvature. The mean and strike curvatures are quite similar. For lineament analysis and particularly enhancement of small faults and subtle features, all five curvatures in Figure 11d–11h, the automatic gain control (Figure 11b), and the coherence (Figure 11c) may have similar effects.

Curvature is sensitive to noise in data because curvature is related to second derivatives. A 3×3 spatial mean filter is applied to the magnetic anomaly grid in Figure 11a before its curvature is computed. To be consistent, the same mean filter is also applied before transformations are performed to produce results in Figure 11b and 11c. Curvature may change significantly with sizes

Figure 11. The total magnetic intensity (TMI) anomaly in the Gaspé belt of the north Québec Appalachians and its different transformation results. (a) TMI anomaly, (b) automatic gain control, (c) coherence, (d) differential curvature, (e) mean curvature, (f) maximum curvature, (g) curvedness, and (h) strike (tangential) curvature. A mean filter of a 3×3 window has been applied to panel (a), and then the same window size has been used to produce results in panels (b-h). All grids are displayed using a linear color stretch of a 90th percentile except (b-c) using an 80th percentile.
of a spatial filter (or similarly, the sampling interval of an anomaly grid). Actual values of the mean curvature in Figure 11e range from $-368.4$ to $166.4$ km. For comparison, the mean curvature has also been computed after a $15 \times 15$ mean filter is applied to Figure 11a, and values of the mean curvature now range from $-124.9$ to $140.0$ km. 

Roberts (2001, p. 89) writes: “Careful selection of the color map for display on the screen can often make a tremendous difference to what the eye can see.” All curvature attributes in Figure 11d–11h have a wide range or extreme values and are displayed using a 90th percentile and a linear color stretch. When a different percentile and/or a nonlinear color stretch are set, the same curvature may appear and be interpreted differently. The mean curvature ranges from $-2.4$ to $2.6$ km for a 90th percentile and have values between $-9$ and $10$ km for a 99th percentile.

Calculating curvature of gravity or magnetic anomalies helps enhance subtle features. This observation is consistent with Barnes’ (2014) observations about curvature of reflection seismic data. He concluded that curvature is good for identification of fractures and superior to horizontal derivatives for display on the screen can often make a tremendous difference to what the eye can see. All curvature attributes have a wide range or extreme values and are displayed using a 90th percentile and a linear color stretch. When a different percentile and/or a nonlinear color stretch are set, the same curvature may appear and be interpreted differently. The mean curvature ranges from $-2.4$ to $2.6$ km for a 90th percentile and have values between $-9$ and $10$ km for a 99th percentile.

Ten curvatures as a function of these coefficients are given in Roberts (2001) and are reproduced here. The mean curvature $K_m$ is

$$K_m = \frac{a(1 + c^2) + b(1 + d^2) - cde}{(1 + d^2 + e^2)^{3/2}}. \quad (A-1)$$

The Gaussian (or total) curvature $K_g$ is

$$K_g = \frac{4ab - c^2}{(1 + d^2 + e^2)^2}. \quad (A-2)$$

The principal curvatures are also called the maximum and minimum curvatures, and they are

$$K_{\text{max}} = K_m + \sqrt{K_m^2 - K_g^2} \quad (A-3)$$

and

$$K_{\text{min}} = K_m - \sqrt{K_m^2 - K_g^2}. \quad (A-4)$$

The curvedness $K_n$ is

$$K_n = \frac{K_{\text{max}} + K_{\text{min}}}{2}. \quad (A-5)$$

The shape index $S_i$ is

$$S_i = \frac{2}{\pi} \tan^{-1} \frac{K_{\text{max}} + K_{\text{min}}}{K_{\text{max}} - K_{\text{min}}}. \quad (A-6)$$

The most-positive curvature $K_+$ and most-negative curvature $K_-$ are

$$K_+ = (a + b) + \sqrt{(a - b)^2 + c^2} \quad (A-7)$$

CONCLUSIONS

This work concerns 12 curvature attributes that have been used in exploration geophysics. When the 12 curvatures are applied to the equipotential surface, only seven are meaningful. These seven can be expressed as a function of gravity and curvature gradients. In theory, absolute gravity and absolute gravity gradients should be used. However, in practice, curvature can only be computed using observed relative gradients. The mean curvature of the equipotential surface and the curvature of the gravity field line that find use are just the VGG and the HGA of the gravity anomaly, respectively, divided by the total gravity (980,000 mGal).

Curvature of a gravity quantity is more complex than curvature of its source (a geometric surface) because the gravity is a function of the source geometry as well as the density contrast and the relative distance and position between an observation point and a source. The same curvature of different gravity quantities (potential, field, and gradient) may show different patterns. Properties for curvature of a geometric surface are widely found in publications and are not directly applicable to gravity quantities. Careful checks need to be made before application of a curvature attribute to field data.

Curvature of a gravity quantity is not as good as the HGA technique for delineation of boundaries and edges, but the dip and strike curvatures may be used to reveal ridges and valleys as well as to differentiate between them. Shape identification using curvature of an equipotential surface or another gravity quantity may be possible in theory, but interference in practice can produce too many false indications. Fundamentally, curvature is a function of the second-order horizontal derivatives and thus suffers strong interference from horizontally neighboring sources. The capability of structural delineation and identification using curvature, just like using the potential field itself, diminishes with an increasing source depth.

Several curvature attributes enhance subtle features and may achieve effects similar to some automatic gain control filters. This will be useful in interpretation of gravity gradient or magnetic anomalies that have a great range of wavelengths and magnitudes.

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APPENDIX A

TWELVE CURVATURES OF A SURFACE

In general, one can determine the six coefficients of equations 2 or 3 via solving a least-squares problem for the data with a user-specified window size. Specifically, Roberts (2001) gives the formulae that yield the coefficients from data values of a $3 \times 3$ window. Eleven curvatures as a function of these coefficients are given in Roberts (2001) and are reproduced here. The mean curvature $K_m$ is

$$K_m = \frac{a(1 + c^2) + b(1 + d^2) - cde}{(1 + d^2 + e^2)^{3/2}}. \quad (A-1)$$

The Gaussian (or total) curvature $K_g$ is

$$K_g = \frac{4ab - c^2}{(1 + d^2 + e^2)^2}. \quad (A-2)$$

The principal curvatures are also called the maximum and minimum curvatures, and they are

$$K_{\text{max}} = K_m + \sqrt{K_m^2 - K_g^2} \quad (A-3)$$

and

$$K_{\text{min}} = K_m - \sqrt{K_m^2 - K_g^2}. \quad (A-4)$$

The curvedness $K_n$ is

$$K_n = \frac{K_{\text{max}} + K_{\text{min}}}{2}. \quad (A-5)$$

The shape index $S_i$ is

$$S_i = \frac{2}{\pi} \tan^{-1} \frac{K_{\text{max}} + K_{\text{min}}}{K_{\text{max}} - K_{\text{min}}}. \quad (A-6)$$

The most-positive curvature $K_+$ and most-negative curvature $K_-$ are

$$K_+ = (a + b) + \sqrt{(a - b)^2 + c^2} \quad (A-7)$$
and
\[ K_\perp = (a + b) - \sqrt{(a - b)^2 + c^2}. \] (A-8)

The dip (profile) curvature \( K_{\text{dip}} \) is
\[ K_{\text{dip}} = \frac{2(ad^2 + be^2 + cde)}{(d^2 + e^2)(1 + d^2 + e^2)^{3/2}}. \] (A-9)

The strike (tangential) curvature \( K_s \) is
\[ K_s = \frac{2(ae^2 + bd^2 - cde)}{(d^2 + e^2)(1 + d^2 + e^2)^{1/2}}. \] (A-10)

The contour (plan) curvature \( K_c \) is
\[ K_c = \frac{2(ae^2 + bd^2 - cde)}{(d^2 + e^2)^{3/2}}. \] (A-11)

The differential curvature is not used in Roberts (2001) but is particularly useful for interpretation of the gravity potential. It is the difference between the maximum curvature and the minimum curvature
\[ K_d = K_{\text{max}} - K_{\text{min}}. \] (A-12)

The shape index is dimensionless, the Gaussian curvature has the dimension of \( 1/\text{length}^2 \), and the other 10 curvatures have the dimension of \( 1/\text{length} \).

Roberts (2001) applies equations A-1–A-11 to depth or time, which implies a vertical-axis downward coordinate system. The gravity or magnetic anomaly corresponds to a vertical up system. We need to reverse the sign of the anomaly when equations A-1–A-12 are applied to gravity or magnetic anomalies. This consideration has already been given when curvature of the equipotential surface is expressed as a function of the gravity and gravity gradients, i.e., equations 6–12.

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