Joint hydrophone and accelerometer receiver deghosting using sparse Tau-P inversion
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Summary
Removing the receiver ghost from marine towed streamer data before migration provides better low and high frequency response as well as a higher signal-to-noise ratio for preprocessing steps such as multiple suppression and velocity analysis. The combination of pressure data recorded by hydrophones and particle velocity data or acceleration data recorded by motion sensors has the potential to reliably derive a ghost-free wavefield. We present a progressive joint sparse $\tau-p$ inversion method to perform 3D deghosting using pressure data ($P$), the acceleration $z$-component ($A_z$), and the acceleration $y$-component ($A_y$). We demonstrate the effectiveness of this method using a multi-sensor streamer data set from the North Sea.

Introduction
In marine acquisition, the receivers on the cable record both the desired up-going wavefield and the undesired down-going wavefield (the receiver ghost) reflected from the water surface. The interference between up-going and down-going wavefields limits the effective bandwidth of the seismic data. Research studies have been performed to use the pressure data ($P$) and the particle velocity ($V$) to remove the receiver ghost (e.g., Berni, 1984; Ruehle, 1984; Robertsson et al., 2001; Carlson et al., 2007). Unlike the pressure data recorded by hydrophones, the particle velocity is measured by geophones that bear the vertical orientation. The up-going wavefields detected by the geophone and hydrophone are in-phase, and the down-going wavefields (the receiver ghost) are $180^\circ$ out-of-phase. Therefore, these two components are complementary to each other in terms of receiver ghost attenuation.

Vassallo et al. (2010) and Özbek et al. (2010) proposed a Generalized Matching Pursuit (GMP) method, which works in frequency-$x$-wavenumber-$y$-wavenumber domain, for 3D joint deghosting and interpolation using multicomponent data as input, which contains pressure data, $P$, and particle acceleration data, $A_z$ and $A_y$.

We present a progressive joint sparse $\tau-p$ inversion method to perform 3D deghosting using pressure data ($P$), acceleration $z$-component ($A_z$), and acceleration $y$-component. This method overcomes the Nyquist limitation imposed by the large cable spacing through sparse inversion (Herrmann et al., 2000; Trad et al., 2002) as well as extra constraints from up-going and down-going wavefields in all the three components. We tested this method on a multi-sensor streamer data example from the North Sea.

Method
Assuming a ghost-free up-going pressure wavefield (our target), $U_0(t;x_i,y_i)$ (time–$x$-coordinate–$y$-coordinate), is recorded at the surface $(z_i = 0)$, we can transform it into frequency–$x$-slowness–$y$-slowness. $f - p_x - p_y$, domain,

$$U_0(f;p_x^i, p_y^i) = \sum_j U_0(t;x_j,y_j) \phi^{2\pi if(x_j,y_j)} \phi^{2\pi i p_x^i (x_j-x_i)} \phi^{2\pi i p_y^i (y_j-y_i)}, \quad (1)$$

where $i=1,2,...,n$ with $n$ the total number of receivers located at $(x_i,y_i,z_i)$, and $j=1,2,...,m$ with $m$ the total number of slowness ($p_x^i, p_y^i, p_z^i$). We keep the zero term $z_i p_z^i$ in Equation 1 so that it’s easier to understand later how we calculate the derivative with respect to $z$.

The slowness in each direction is bounded by the water velocity $v$:

$$v^{-2} = (p_x^i)^2 + (p_y^i)^2 + (p_z^i)^2. \quad (2)$$

The up-going and down-going pressure data recorded by receivers on the streamer can be written as

$$\begin{cases}
U(f;x_i, y_i) = \sum_j e^{-i2\pi f T_i^z} U_0 \\
D(f;x_i, y_i) = -\sum_j e^{i2\pi f T_i^z} U_0
\end{cases} \quad (3)$$

where $L$ is the reverse $\tau-p$ transform operator,

$$L(f;x_i, y_i; p_x^i, p_y^i) = e^{i2\pi f(x_i,y_i)} \phi^{2\pi i p_x^i}, \quad (4)$$

and $T_i^z$ is the ghost-delay time determined by the known receiver depth $r_i$ and the slowness $(p_x^i, p_y^i)$,

$$T_i^z = 2r_i \sqrt{v^{-2} - (p_x^i)^2 - (p_y^i)^2}. \quad (5)$$

The total pressure wavefield recorded by receivers on the cable, $P(f;x_i, y_i)$, is the summation of the up-going and down-going wavefields,

$$P = U + D = \sum_j R_p U_0, \quad (6)$$

with $R_p$ the reghosting operator for pressure data, $P$,

$$R_p(f;x_i, y_i; p_x, p_y) = e^{-i2\pi f T_i^z} e^{-i2\pi f T_i^y} \phi^{2\pi i p_x^i}, \quad (7)$$
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Using the relationship between pressure and particle acceleration,

\[ \rho(A_x, A_y, A_z) = -(\partial P / \partial x, \partial P / \partial y, \partial P / \partial z), \]  

we can obtain the total acceleration z-component,

\[ A_z(f); x_i, y_i), \]  

\[ A_z = \frac{1}{\rho} \sum_i 2\pi f \rho R_p R_L U_0, \]  

with \( \rho \) the water impedance, \( 2\pi f \) the time-differentiation operator, \( R_p \) the reghosting operator for \( A_z \),

\[ R_p(f; x_i, y_i; p_x, p_y) = e^{-i\pi f t} + e^{+i\pi f t}, \]

and \( v \) the reverse obliquity-correction operator for \( A_z \),

\[ v = \sqrt{1 - (v_p)^2} - (v_p)^2. \]

Similarly, we can obtain the total acceleration y-component, \( A_y(f; x_i, y_i) \),

\[ A_y = -\frac{1}{\rho} \sum_i 2\pi f \rho R_p R_L U_0, \]

with \( v_y \) the reverse obliquity-correction operator for \( A_y \).

The sign difference between Equations 7 and 10 is caused by the ghost in sensor during the joint inversion. Poole (2014; Trad et al., 2002). We therefore applied a low-rank optimization step to reduce the model space before we performed the full inversion. By doing this, not only do we significantly lower the cost, but we also make the inversion more stable.

Another major challenge for this 3D inversion scheme is that, in marine towed streamer acquisition, the data is sampled in frequency bands for the frequency content. To overcome this sampling issue, along with the low-rank optimization to reduce the model parameters, we started this inversion scheme using high-cut filtered data (e.g., 10 Hz) to get an initial result. We subsequently use this result to guide the inversion for data with higher frequency (Herrmann et al., 2000). This process can be repeated progressively until reaching the desired frequency.

After \( U_0(f; p_x, p_y) \) has been found, an inverse \( \tau - p \) transform and an inverse Fourier transform are applied to obtain the receiver-ghost-free data \( \hat{U}_0(t; x_i, y_i) \). In Equation 13, if we use only pressure data \( P \) to derive the receiver-ghost-free data \( \hat{U}_0 \) and reverse back using the operator for \( A_z \) or \( A_y \), we can generate equivalent acceleration data \( A_z \) or \( A_y \) from pressure data \( P \), which can be used to attenuate the noise in \( A_z \) or \( A_y \) through a cooperative denoise process (Peng and Huang, 2014).

Application to field data

We tested our method on a seismic data set acquired with multi-sensor variable-depth streamers (receiver depth ranged from 10 m to 50 m) that were comprised of hydrophones (\( P \)) and accelerometers for \( z \) and \( y \) directions (\( A_z \), \( A_y \)). The channel spacing was 12.5 m, and the cable spacing was 100 m.

Before we put these three components into the deghosting inversion depicted in Equation 13, we applied a low-cut and \( f - k \) (frequency-wavenumber) dip-filtering to \( P \) and a cooperative denoise process (Peng and Huang, 2014) to \( A_z \) and \( A_y \). You can see from Figure 1 that the majority of the noise in the input data sets (Figures 1a-1c)
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is properly attenuated (Figures 1d-1f).

Figure 2a shows a couple of shot gathers for pressure data before receiver deghosting. The primary events (red arrows) are followed by ghost events (blue arrows) with opposite polarities, which are effectively attenuated by our multi-sensor joint inversion algorithm (Figure 2b). To closely examine the wavelet changes before (2c) and after (2d) receiver deghosting, we zoomed in on the blue boxes in 2a and 2b, respectively. Figure 2e shows the amplitude spectrum comparison before (blue) and after (red) receiver deghosting. The receiver ghost notches up to the fourth-order are effectively in-filled.

Figure 3a presents the Kirchhoff migration stacked image before deghosting. The image after receiver deghosting (Figure 3b) shows better resolution because of the removal of ghost interference. Some shallow events once shadowed by ghost can be clearly seen after deghosting (Figure 3b, red arrows). The deghosted image becomes easier to interpret with better defined geo-bodies (Figure 3b, blue circles). Figure 3c composes the migrated common depth point (CDP) gathers (gather location marked by blue arrows in 3a for data before receiver deghosting). The curving-down ghost events, due to the carefully designed variable-depth streamer geometry (Soubaras, 2013), were effectively attenuated (Figure 3d).

Discussions and Conclusions

We presented a progressive joint sparse $\tau - p$ inversion method to perform 3D deghosting using multi-sensor marine towed streamer data. The resulting deghosted images through multi-sensor inversion have higher resolution and better defined geo-bodies for seismic interpretation and reservoir characterization.

Our deghosting algorithm overcomes the Nyquist limitation of the large cable spacing because of the progressive sparse inversion and extra constraints from six consistent wavefields (up-going and down-going wavefields from three components). Although our multi-sensor joint inversion scheme includes built-in noise suppression due to the cross-check among all the six wavefields, the successful attenuation of most of the noise in $A_z$ and $A_y$ components through cooperative denoise (Peng and Huang, 2014) is a key preconditioning step to mitigate the noise contamination from these two components.

Our deghosting inversion is equally valid for two- or one-component data and simply requires the removal of any one or two rows from Equation 13. Validation has been carried out for $P$-only deghosting as well as for dual sensor ($P,A_z$) deghosting. We found the results of $P$-only and dual sensor deghosting inversions are useful in evaluating the merits of 3-component deghosting.

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Figure 1: (a) Raw $P$. (b) Raw $A_z$. (c) Raw $A_y$. (d) $P$ after denoise. (e) $A_z$ after cooperative denoise. (f) $A_y$ after cooperative denoise.
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Figure 2: (a) Pressure data before receiver deghosting. (b) Receiver ghost-free data through multi-sensor joint inversion. (c) Wiggle view of the blue box in (a). (d) Wiggle view of the blue box in (b). (e) Amplitude spectra for data before (blue) and after (red) receiver deghosting through multi-sensor joint inversion.

Figure 3: Kirchhoff migration stacked images for (a) input pressure data before deghosting and (b) deghosted data through multi-sensor joint inversion. Common depth point (CDP) gathers for (c) input pressure data before deghosting and (d) deghosted data through multi-sensor joint inversion. The location of CDP gathers shown in (c) and (d) is marked by the blue arrows in (a).
REFERENCES


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