Amplitude-preserving reverse time migration: From reflectivity to velocity and impedance inversion

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ABSTRACT

Conventional methods of prestack depth imaging aim at producing a structural image that delineates the interfaces of the geologic variations or the reflectivity of the earth. However, it is the underlying impedance and velocity changes that generate this reflectivity that are of more interest for characterizing the reservoir. Indeed, the need to generate a better product for geologic interpretation leads to the subsequent application of traditional seismic-inversion techniques to the reflectivity sections that come from typical depth-imaging processes. The drawback here is that these seismic-inversion techniques use additional information, e.g., from well logs or velocity models, to fill the low frequencies missing in traditional seismic data due to the free-surface ghost in marine acquisition. We found that with the help of broadband acquisition and processing techniques, the bandwidth gap between the depth-imaging world and seismic inversion world is reducing. We outlined a theory that shows how angle-domain common-image gathers produced by an amplitude-preserving reverse time migration can estimate impedance and velocity perturbations. The near-angle stacked image provides the impedance perturbation estimate whereas the far-angle image can be used to estimate the velocity perturbation. In the context of marine acquisition and exploration, our method can, together with a ghost compensation technique, be a useful tool for seismic inversion, and it is also adaptable to a full-waveform inversion framework. We developed synthetic and real data examples to test that the method is reliable and provides additional information for interpreting geologic structures and rock properties.

INTRODUCTION

Imaging is conventionally regarded as a process for seeking subsurface reflectivity. However, when it comes to reservoir characterization, the impedance and velocity embedded in the reflectivity are more closely related to the physical properties of the hydrocarbon-bearing sediments and are the more desirable result.

Assuming that velocity can be separated into a smooth reference model and a perturbation, Jin et al. (1992) develop a weighted least-squares, linear, iterative elastic inversion in which the forward problem is solved with a combination of ray-theory and the Born approximation. The first iteration is essentially a preserved amplitude Kirchhoff migration (Thierry and Lambaré, 1995). They conclude that the success of the inversion depends on the ability of the smooth reference velocity to provide accurate traveltimes. This work was further developed by Forgues and Lambaré (1997) who provide quantitative estimations of the conditioning of a multiparameter inversion. Both of the studies provide numerical examples only. Operto et al. (2000) continue this approach but extend the ray theory plus Born approximation to multiple arrivals and show with a 2D synthetic data case study that impedance perturbations could be imaged following a ray-theory approach. However, as Forgues and Lambaré (1997) note, the method may not work well on real data because of the lack of computation power and low-frequency information in the seismic data at the time due to restrictions in recording bandwidth and the presence of the free-surface ghost in marine data.

With recent developments in wave propagation computations, reverse time migration (RTM) has become the state-of-the-art technique to image and interpret subtle and complex geologic features, not only because it correctly handles complex velocities and propagates waves without angle limitation but also because it takes advantage of a complete set of acoustic waves (reflections,
transmissions, diffractions, prismatic waves, etc.) to image subsurface structures (Zhang et al., 2007; Zhang and Sun, 2009; Xu et al., 2011). Furthermore, new broadband acquisition and processing techniques (Soubaras and Whiting, 2011) have allowed the recovery of low-frequency information crucial for the estimation of acoustic parameters such as impedance. Here, we use a combination of these two developments, RTM and successful retrieval of low frequencies, to build a method for the estimation of impedance and velocity perturbation from seismic data.

Specifically, in this paper, we propose a new method to generate acoustic velocity and impedance perturbations using an amplitude-preserving, ghost-compensated RTM with a modified imaging condition. The context presented here is for marine acquisition, but the basic principles are just as applicable to land/OBC situations. We begin with a description of conventional amplitude-preserving RTM for angle-dependent reflectivity, and then, we discuss how to compensate for the ghost effects within an RTM to reliably retrieve the low-frequency reflectivity information distorted due to free-surface reflections. Once we have retrieved the low frequencies, we derive a modified imaging condition and show that impedance and velocity perturbations can be estimated from near- and far-angles, respectively, of angle domain common image gathers (ADCGIs). This work is a development of the theory developed by Jin et al. (1992) and Forgues and Lambaré (1997). Next, we describe how the method can be combined with full-waveform inversion (FWI), to provide iterative velocity updates. Finally, we show results from synthetic and real data sets: First, we demonstrate that ghost-compensated RTM with a modified imaging condition reliably extracts impedances and velocities. This includes synthetic examples and a real data impedance well match. Second, we show how the RTM-based velocity perturbation estimation can be incorporated into an FWI flow using a real data example.

**AMPLITUDE-PRESERVING REVERSE TIME MIGRATION**

For a zero-phased and designatured shot record at time \( t, d(x_r, y_r, z_r; t) \), with the shot at \( x_s = (x_s, y_s, z_s = 0) \) and receivers at \( x_r = (x_r, y_r, z_r = 0) \), the RTM (Baysal et al., 1983; McMechan, 1983; Whitmire, 1983) based on the theory of amplitude-preserving migration, in the simplified case of the acoustic, isotropic, and constant density wave equation, can be summarized as forward propagation of the source wavefield \( p_F \):

\[
\begin{align*}
\left\{ \begin{align*}
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p_F(x; t; x_s) &= 0, \\
p_F(x, y, z = 0; t; x_s) &= \delta(x - x_s) \int_0^t f(t') dt',
\end{align*} \right.
\end{align*}
\]

and backward propagation of the receiver wavefield \( p_B \):

\[
\begin{align*}
\left\{ \begin{align*}
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p_B(x; t; x_r) &= 0, \\
p_B(x, y, z = 0; t; x_r) &= d(x, y; t; x_r),
\end{align*} \right.
\end{align*}
\]

where \( v = v(x, y, z) \) denotes the velocity, \( f(t) \) denotes the source wavelet, \( \nabla^2 \) denotes the Laplacian operator, and \( x = (x, y, z) \) denotes the subsurface imaging location. Zhang et al. (2007) and Zhang and Sun (2009) show that the “crosscorrelation” imaging condition,

\[
R(x; \theta) = 4 \int \delta(\theta'(x; x_s) - \theta) \times \left( \int p_B(x; t; x_r) p_F(x; t; x_r) dt \right) d\theta' dx_r,
\]

is a proper choice to obtain amplitude-preserving ADCIGs from a wave-equation-based migration in 2D. In equation 3, \( \theta \) is the reflection angle at the imaging location and the term \( \delta(\theta'(x; x_s) - \theta) \) represents the conversion from shot gather to reflection angle gather. For a 2D application, including details of the conversion of subsurface offset gather to angle gather, please refer to Sava and Fomel (2003).

In 3D, the following imaging condition provides angle- and azimuth-dependent reflectivities when the migration velocity is accurate and the common-image gathers are generated in the subsurface angle domain (Xu et al., 2011):

\[
R(x; \theta; \varphi) = 16\pi \int \int \int \frac{v(x)}{\sin \theta'} \delta(\theta'(x; x_s) - \theta) \times \delta(\varphi'(x; x_s) - \varphi) \left( p_B p_F \right) \times (x; t; x_r) dt d\theta' d\varphi' dx_r,
\]

where \( \theta \) and \( \varphi \) are the reflection and azimuth angles, respectively, at the imaging location and the terms \( \delta(\theta'(x; x_s) - \theta) \) and \( \delta(\varphi'(x; x_s) - \varphi) \) represent the conversion from shot gather to reflection and azimuth angle gather. Xu et al. (2011) also describe how to generate 3D angle gatherers from an RTM.

Here, we use the definition of amplitude-preserving migration (Bleistein, 1987; Hanitsch, 1997), which intends to remove geometric spreading and to recover band-limited angularly dependent reflectivity during the migration process. Our angle-domain, amplitude-preserving RTM consists of proper wave-equation formulations, the boundary conditions, and the imaging conditions, as shown in equations 1–4, and amplitude-preserving angle gather conversion (Sava and Fomel, 2003; Xu et al., 2011). If the migration velocity is a good approximation to the real earth model, our RTM handles multipathing automatically and compensates for geometric spreading. However, such an implementation usually assumes the acquired data is well sampled (no data aliasing) and provides good subsurface illumination in the interested imaging areas. Also, the method we have discussed does not compensate for some of the other physical effects that effect image amplitude, such as transmission losses (Deng and McMechan, 2007), attenuation, and dispersion (Samec and Blandy, 1992).

**REVERSE TIME MIGRATION WITH GHOST COMPENSATION**

In marine acquisition, sources and receivers are placed below the water surface. The source and receiver ghosts due to free-surface reflections are thus always recorded in the seismic data and cause angle-dependent frequency and amplitude distortion. In Figure 1a, a synthetic seismic shot record, generated by dynamic ray tracing, models the wave propagation through an isotropic medium with an angle-independent reflectivity containing five horizontal layers at different depths, with uniform amplitude of one. The shot is in the center of the section, with depth of 10 m, and the receivers are out to an offset of 7500 m on either side, with depth of 15 m. The water
velocity is $1500 \text{ ms}^{-1}$. In addition to the effect of geometric spreading, the wavelet amplitude and spectrum are distorted depending on the propagation angles at the shot and the receivers, as described in equations 5 and 6 below. In particular, the low frequency in the seismic data is greatly attenuated due to the ghosts. If we use the conventional amplitude-preserving RTM formulation in equations 1–3,
stack all the migrated common image shot gathers to generate subsurface offset gathers (Sava and Fomel, 2003) and then convert them to subsurface ADCIGs, we end up with a distortion in the spectrum of the migrated image, as shown in Figure 1b and also the incorrect amplitude variation with angle (AVA) trend in Figure 1c.

One approach is to compensate for the ghost effect before the migration: This is a challenging area of research and is often based on a series of deghosting operators in the $r$-$p$ domain (Poole, 2013). On the other hand, because propagation angles for the source and receiver are well defined and are easily accessible during the wave propagation part of a migration, it is a fairly simple process to compensate for the ghost effects within an RTM (Zhang et al., 2012). If we assume a (perfect) free-surface reflection (i.e., the reflectivity is well recovered and the AVA relation is more reliable (Figure 1d)). Also, the normalized peak amplitudes along reflectors converge well, which indicates that the reflectivity is well recovered and the AVA relation is more reliable (Figure 1e).

This ghost compensation technique within RTM provides an alternative to processing solutions developed for towed streamer marine acquisition. Zhang et al. (2012) demonstrate that the method proposed above can retain the low-frequency information in the image that is important for seismic inversion.

**Figure 2.** (a) A shot record and (b) a migrated ADCIG using the amplitude-preserving RTM. The dashed blue lines show the selected 0° and 66° traces displayed in Figure 3.
\[
\left( \frac{1}{v_0^2} \frac{\partial^2}{\partial t^2} - \rho \nabla \frac{1}{\rho} \cdot \nabla \right) p(x; t; x_s) = \delta(x - x_s)f(t). \tag{9}
\]

For initial velocity \(v_0\) and density \(\rho_0\) models, the perturbed wavefield \(\delta p\) satisfies

\[
\left( \frac{1}{v_0^2} \frac{\partial^2}{\partial t^2} - \rho_0 \nabla \frac{1}{\rho_0} \cdot \nabla \right) \delta p(x; t; x_s) \\
\approx \left( \frac{2\delta v}{v_0^2} \frac{\partial^2}{\partial t^2} - \nabla \frac{\delta \rho}{\rho_0} \cdot \nabla \right) p_0(x; t; x_s), \tag{10}
\]

where \(\delta v = v - v_0\) and \(\delta \rho = \rho - \rho_0\) denote the velocity and density perturbations, respectively. Using a similar method developed by Jin et al. (1992) and Forgues and Lambaré (1997), with the detailed derivation in Appendix A, we obtain the ray-based relation between the perturbed geologic models and wavefield:

\[
\begin{align*}
\frac{\delta v}{v_0} + \cos^2 \theta \frac{\delta \rho}{\rho_0} &= - \iiint W \frac{\cos \alpha_x}{v_0(x_0)} \frac{\cos \alpha_x}{v_0(x_r)} A_x A_r e^{-i\omega(t_r - t_x)} \\
&\quad \times \delta \hat{d}(x_0; \omega; x_r) \delta(\theta'(x; x_0; x_r) - \theta) d\omega d\theta' dx_0 dx_r, \tag{11}
\end{align*}
\]

where \(A_x\) and \(A_r\) are the amplitudes of the Green’s function from the source or receiver to the image point, respectively; the amplitude weight \(W = 32\pi v_0(x) \cos^2 \theta'/\sin \theta'\), \(\delta \hat{d}\) is the difference between the seismic data and the predicted data from the forward modeling (using the initial velocity and density); and \(t_r\) and \(t_x\) are the traveltimes from the source or receiver to the image point, respectively.

In the context of RTM defined by Zhang et al. (2007) (equations 6 and 7 therein for 2D one-way, wave-equation migration also valid for RTM and easily extended to 3D propagation), the asymptotic forms of \(\hat{p}_F(\omega)\) and \(\hat{p}_B(\omega)\) for the full-acoustic-wave equation are given in terms of source and receiver traveltimes and amplitudes:

\[
\begin{align*}
\hat{p}_F(\omega) &= \frac{\cos \alpha_x}{v_0(x_0)} A_x e^{-i\omega t_x} \\
\hat{p}_B(\omega) &= \int \frac{\cos \alpha_x}{v_0(x_r)} i\omega A_r e^{-i\omega t_r} \delta \hat{d}(x_0; \omega; x_r) dx_r. \tag{13}
\end{align*}
\]

Substituting equations 12 and 13 into equation 11 and replacing the density perturbation with impedance where

\[
\frac{\delta(vp)}{vp_0} = \frac{\delta v}{v_0} + \frac{\delta \rho}{\rho_0}. \tag{14}
\]

Equation 11 can be rearranged into the same form as the imaging condition given by equation 4:

\[
sin^2 \frac{\delta v}{v_0} + \cos^2 \theta \frac{\delta(vp)}{vp_0} = -32\pi \int \int \frac{\nu_0(x)}{\sin \theta'} \cos^2 \theta' \delta(\theta' - \theta) \hat{p}_B(\omega) \hat{p}_F(\omega) d\omega d\theta' dx_r. \tag{15}
\]

Equation 15 shows that for a common shot RTM, if we output near-surface angle gathers with the above imaging condition, the far-angle images predict the impedance perturbation \(\delta(vp)/(vp_0)\), whereas the far-angle images can be used to estimate the velocity perturbation \(\delta v/v_0\). Therefore, we can separate the effects of velocity and density by outputting ADCIGs.

One may also note that the described process of separating the velocity and density effects based on a modified imaging condition provides a result similar to the well-known traditional amplitude variation with offset (AVO) or AVOA methods. In Appendix B, we show that our approach is consistent with the traditional AVO/AVA methods upon an angle-domain rescaling.

Figure 3. (a) The impedance and (b) velocity perturbations for the synthetic example in Figure 2, which present the true perturbations (in blue) and estimated perturbations from the amplitude-preserving RTM (in red).
If we assume that the density function $\rho$ is a constant, equation 15 reduces to

$$\frac{\delta v}{v_0} = -32\pi \iiint \frac{v_0(x)}{\sin \theta'} \cos^2 \theta' \delta(\theta' - \theta)$$

$$\times \frac{\hat{p}_B(\omega) \hat{p}_F(\omega)}{\omega} \, d\omega d\theta' dx_s. \quad (16)$$

Because equation 16 is independent of $\theta$, integrating equation 16 over $\theta$ from zero to $\pi/2$, we have

$$\frac{\delta v}{v_0} = -32\pi \iiint \frac{v_0(x)}{\sin \theta(x; x_s)} \cos^2 \theta(x; x_s)$$

$$\times \frac{\hat{p}_B \hat{p}_F(x; \omega; x_s)}{i\omega} \, d\omega dx_s. \quad (17)$$

In this case, the stacked image from equation 15 collapses to the velocity perturbation only, as shown in equation 17. This is consistent with the earlier results using Kirchhoff migration (Operto et al., 2000) and RTM (Zhang et al., 2012).

In practice, a band-pass filter is applied to seismic data before RTM that matches the effective bandwidth in acquisition, especially the low-frequency content. Our RTM is carefully implemented to ensure amplitude and phase correctness during migration, together with a Laplacian filter (Zhang and Sun, 2009) on the output images to remove backscattering noise. However, the impedance and velocity perturbations found using equation 15 are still band limited due to the bandwidth restriction of seismic data. When the low-wavenumber information of the impedance and velocity is achievable in the initial velocity $v$ and density $\rho$, these band-limited perturbations can be used to invert for more accurate velocity and density models, such as the example in the numerical experiment section using FWI.

**RELATIONSHIP OF AMPLITUDE-PRESERVING REVERSE TIME MIGRATION TO FULL-WAVEFORM INVERSION**

A simple link between RTM and FWI can be seen by considering a true velocity model that is smoothly varying and in which density contrasts generate the reflections. In this case, the forward modeling
produces no reflections and the subsequent FWI residuals contain the
reflections only. Construction of the FWI gradient then becomes
an RTM exercise with an appropriate imaging condition. This gives
rise to the classical statement that “migration is the first iteration of
full waveform inversion” (Mora, 1989).

If we consider marine data with ghosts and ignore the density
variations, then equation 11 with ghost compensation gives the
velocity update as

$$\frac{\delta v}{v_0} = -16\pi^2 \iint \frac{v_0(x)}{\sin \theta(x; x, x, x)} \cos \alpha_x \cos \alpha_y \times A_x A_y \frac{\delta d - \delta d_0}{\tilde{G}_x \tilde{G}_y} \, dx, dy, d\omega. \quad (18)$$

For a conventional FWI using the steepest descent method, the
velocity is updated by the gradient direction of the least-squares cost
function $E = \min \| \delta d \|^2$ (Tarantola, 1984). In the high-frequency
approximation, the gradient calculated in FWI can be expressed
as (see, for example, Tarantola, 1984; Mora, 1987)

$$\frac{\partial E}{\partial v} = -\iint 2\omega^2 G(x; x; x; x) \delta d \, dx, dy, d\omega.$$

Comparing equations 18 and 19, we see that the two formulations
have similar asymptotic expressions, with the same phase
($\omega(\tau_x + \tau_y)$) and amplitude polarity, but they use different ampli-
tude weighting functions. The scaling in the FWI formulation is
frequency dependent ($\omega^2$) and also without the surface ampli-
tude and ghost compensation $\cos \alpha_x \cos \alpha_y / (v_0(x) v_0(x) \tilde{G}_x \tilde{G}_y)$.

Therefore, conventional FWI tends to favor higher frequencies
during a given update. This will slow down the convergence in a time-
domain iterative implementation of FWI, when compared with the
RTM formulation.

Equation 18 gives a direct map from the seismic data to the velocity
perturbation and can also be used to give an iterative update of the
velocity model in a similar manner to standard implementations
of FWI, but by replacing the FWI-derived, unscaled velocity model
perturbation with one computed by our new RTM formulation. This
is achieved using the following workflow:

1) Starting from a smooth velocity model, we compute the cost
function of the perturbed wavefield.
2) We use this same starting velocity model in our new RTM for-
mulation to generate the necessary ADCIGs.
3) We extract the current velocity model perturbation $\delta v / v$ by per-
forming an AVA-like fit of equation 15 to the ADCIGs at each
depth in the velocity model.
4) We compute an optimal scalar to minimize the cost function of
the perturbed wavefield by a step-length calculation on the ex-
tracted current velocity model perturbation (this is a standard
process; see, e.g., Virieux and Operto, 2009).
5) We update the velocity model with this scaled velocity model
perturbation.
6) We iterate until convergence.

NUMERICAL AND REAL DATA EXAMPLES

The first example is designed to demonstrate the reliability of
using RTM after ghost compensation to estimate impedance and
velocity perturbations. Here, we construct a horizontally invariant
geologic model to a 4-km depth. The velocity and density are ran-
domly generated within their dynamic ranges to make the two series
uncorrelated. Figure 2a shows the central shot record from the

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**Figure 5.** (a) The velocity perturbation computed from amplitude-preserving RTM and (b) the true velocity perturbation. The trace location is indicated by the dashed line shown in Figure 4c and 4d.
model, with receivers spread on both sides to a maximum offset of 12 km. The source and receiver ghosts are recorded at a shot and receiver depth of 6 and 8 m, respectively. Setting the background velocity and density to $2000 \text{ ms}^{-1}$ and $2000 \text{ kgm}^{-3}$, respectively, we use the ghost-compensated RTM formulation to propagate the wavefields, apply the appropriate imaging condition to generate subsurface offset gathers (Sava and Fomel, 2003), and then convert them to ADCIGs, as shown in Figure 2b. Based on our theory, the $0^\circ$ trace directly estimates the impedance perturbation and the velocity perturbation can be calculated from any nonzero-degree trace. However, far-angle data will have fewer numerical errors when separating the perturbations. Thus, we select the $0^\circ$ and $66^\circ$ traces for impedance and velocity perturbation, respectively. After simple rescaling, one can see from Figure 3a and 3b that the estimated perturbations (in red) give a good match to the true perturbations (in blue), derived by subtracting the constant background impedance or velocity from the exact impedance or velocity, respectively. The mismatch of the velocity perturbation at depths greater than 3 km is due to the lack of large reflection angles over the fixed surface offset range and the event stretch on the far-angle ADCIG.

Next, we demonstrate the performance of our new RTM formulation to produce velocity perturbations, as described by the modified boundary condition in equations 7 and 8 and the imaging condition in equation 17. Here, we used the Sigsbee2a model (Paffenholz et al., 2002) in Figure 4a to generate the synthetic data set, with the source and receiver depth at 25 ft. In this example, we focus on the area to the left of the salt body, as denoted by the box in Figure 4a. Zooming into this area in Figure 4b, we show the constant gradient background velocity used for migrating the data. The RTM velocity perturbation image is shown in Figure 4c. Figure 4d is the true velocity perturbation, derived by subtracting the migration velocity from the true velocity model. With the ghost compensation, the RTM image does a good job in reconstructing the velocity perturbation. In Figure 5, we choose one imaging location, indicated by the dashed line in Figure 4 and compare the velocity perturbation from RTM (Figure 5a) with the true one (Figure 5b). The overall match of the two is good, except for a scaling difference, which can be calibrated by a linear search.

In the third example, we apply the new RTM formulation to the BP2004 2D model (Billette and Brandsberg-Dahl, 2005). The synthetic data are generated by high-order, finite-difference acoustic modeling using variable velocity and density models, with shot spacing of 50 m, receiver spacing of 25 m, and 8-km maximum offset. Source and receiver ghosts are recorded at a 12.5-m depth. In Figure 6, we compare the impedance perturbation output from our RTM (Figure 6a) using a smooth migration velocity with the true impedance perturbation (Figure 6b), in which the true impedance perturbation is derived by subtracting the product of the smooth migration velocity and an assumed constant density from the true impedance. The estimated and true impedance perturbations generally agree quite well. For detailed investigation, we present the estimated and true impedance perturbations at the location indicated by the dashed line in Figure 6a and 6b. The estimated impedance perturbation does a good job of reproducing the true perturbation, except at the sharply changing impedance boundaries, e.g., at the water bottom or at the top of salt boundary. This is due to the large model perturbation that does not fully satisfy the linearization assumption in deriving equation 11 and the lack of ultralow frequencies in the seismic data. This result demonstrates the
applicability of our theory to geologic models with complex structures and density variations.

Next, we present ghost-compensated RTM images for a shallow water data set from the central North Sea. The source and receiver depths are 6 and 7 m, respectively. The geology of this area shows a fairly flat velocity profile down to the Top Balder horizon at approximately 3-km depth. The velocities then kick strongly here and again in a high-velocity chalk section. These are typical geologic features in this area. Figure 7a shows a stacked reflectivity image, and Figure 7b shows an impedance perturbation section — the low-frequency components (lost due to ghost effects) are compensated in both of the images, which give the sections a more continuous and textured appearance characteristic of broadband data. The inset in Figure 7b compares the estimated impedance perturbation from RTM against impedance the well-log data filtered to the seismic bandwidth at the location given by the red vertical line. It can be seen that the estimated impedance provides a good approximation to the measured data. Figure 7c shows a colored inversion result (Lancaster and Whitcombe, 2000), using the RTM reflectivity of Figure 7a and the well log. Figure 7d shows the RTM impedance perturbation only, obviously generated without the use of the well log, but replotted from Figure 7b with the same color map as the colored inversion in Figure 7c. The filtered well-log impedance is included at the well location to aid comparison here. It is clear that there is good agreement between the colored inversion and the RTM impedance. This shows that the new RTM formulation is not only a technique for achieving images with broad bandwidth, but also a useful tool to estimate band-limited impedance of the real-world subsurface geology, achieved without using well data.

Finally, using the same shallow water data set from the central North Sea, we demonstrate how the velocity perturbation can be used in an iterative manner to update the velocity field. Because of the limited dynamic range available in a color map, we choose to zoom into the shallow and deep sections separately. Figure 8a

Figure 7. Central North Sea data example: (a) ghost-compensated stacked RTM reflectivity image, (b) impedance perturbation section with a comparison of filtered well-log impedance (in black) and estimated RTM impedance (in red) at the location of the vertical line, (c) colored inversion generated using the RTM reflectivity from (a) and the well log, and (d) RTM impedance perturbation from (b), but replotted with the same color map as in (c). The filtered well-log impedance is included at the well location to aid comparison here.
Figure 8. Central North Sea data example: (a) shallow section display of the initial velocity overlaid with an RTM image, (b) shallow section display of the inverted velocity overlaid using the proposed iterative method with RTM. Some interesting geologic features at this depth are now present in the RTM-updated velocity model, (c) Deeper section display of the initial velocity overlaid with an RTM image, (d) deeper section display of the inverted velocity overlaid using the proposed iterative method with RTM. Velocity structure within the chalk package is clearly improved by the update; in particular, note the lower velocity Hod formation. (e) Deeper section display of ghost-compensated RTM image migrated with initial (left) and RTM-updated (right) velocity model. The red circles highlight some areas of difference, albeit a subtle improvement.
shows the shallow section for the initial velocity model overlaid with an RTM image. This overlay is done to aid the visual identification between the features in the velocity model and those in the seismic stack section. Figure 8b shows the same shallow section after an iterative application of the velocity perturbations derived from RTM as described earlier. The extra detail in the updated velocity model is clear and correlates with the main geologic features in this shallow section, namely, the contortuaries and dewatering faults. Figure 8c shows the deeper section for the initial velocity model overlaid with an RTM image — the chalk package is clearly identifiable on this section. Figure 8d shows the same deeper section after an iterative application of the velocity perturbations. We now see internal structure in the chalk section, in particular, the lower velocity Hid formation (a typical feature in the North Sea) within the chalk package. Also, the vertical extent of the chalk section is better defined with sharper boundaries. Figures 8e compares ghost-compensated RTM images for the deeper section migrated with the initial (left) and updated velocity (right) model, respectively. Although it is a subtle effect, there is an improvement in the image obtained with the velocity model updated by our RTM velocity perturbations, and this improvement is observed in the areas in which we have the strongest velocity change. Overall, many high-frequency detailed features are formed in our RTM-updated velocity model. These features seem geologically plausible and agree with our knowledge of the area. In summary, the real data impedance and velocity example shown here confirm the applicability and potential uses of our theory to geologic models with complex structures.

CONCLUSIONS

We have developed a theory of amplitude-preserving RTM to delineate the impedance and velocity perturbations of subsurface structures. The near-angle stacked image provides the impedance perturbation estimate, whereas far-angle images can be used to estimate the velocity perturbation. Together with the ghost compensation technique, the method described here can be a useful tool of seismic inversion for marine exploration and used to link and bridge the gap between RTM and FWI. Synthetic and real data examples have shown that the method is reliable and provides additional geologic information for interpretation and inversion.

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APPENDIX A

DERIVATION OF EQUATION 11

We assume that the investigated subsurface area of interest is an isotropic medium with velocity $v$ and density $\rho$ such that the propagation of wavefield $p(x; t; x_i)$ can be modeled by using the acoustic-wave equation and the recorded seismic data $d(x_i; t; x_i)$ at the receiver location $x_i$, and source at location $x_i$. This is consistent with the amplitude-preserving RTM in equations 1 and 2 and can be modeled as follows:

$$
\begin{align*}
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \rho \nabla \frac{1}{\rho} \cdot \nabla \right) p(x; t; x_i) &= \delta(x - x_i) \delta(t), \\
d(x_i; t; x_i) &= p(x_i; t; x_i).
\end{align*}
$$

Introducing small perturbations to the velocity and density profiles using $v + \delta v$ and $\rho + \delta \rho$, respectively, the resultant wavefield $p + \delta p$ and observed data perturbation $\delta d$ must satisfy the same acoustic wave equation A-1, as follows:

$$
\begin{align*}
\left( \frac{1}{(v + \delta v)^2} \frac{\partial^2}{\partial t^2} - (\rho + \delta \rho) \nabla \frac{1}{\rho + \delta \rho} \cdot \nabla \right) (p + \delta p) &= \delta(x - x_i) \delta(t), \\
\delta d(x_i; t; x_i) &= \delta p(x_i; t; x_i).
\end{align*}
$$

By substituting equation A-1 into A-2, applying Taylor’s expansion, and ignoring the higher order terms, we have

$$
\begin{align*}
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \rho \nabla \frac{1}{\rho} \cdot \nabla \right) \delta p(x; t; x_i) &= \left( \frac{2 \delta v}{v^2} \frac{\partial^2}{\partial t^2} - \left( \nabla \frac{\delta \rho}{\rho} \right) \cdot \nabla \right) p(x; t; x_i).
\end{align*}
$$

Although the inhomogeneous part of equation A-3 is quite complicated, the Green’s function $G(x; x'; t - t')$ for solving it is only required to satisfy the homogeneous equation:

$$
\left( \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \rho \nabla \frac{1}{\rho} \cdot \nabla \right) G(x; x'; t - t') = \delta(x - x') \delta(t - t').
$$

Using the boundary condition in equation A-2, the solution of equation A-3 can then be expressed at the receiver locations, $x_i$,

$$
\begin{align*}
\delta d(x_i; t; x_i) &= \int \int \frac{2 \delta v(x) \frac{\partial^2}{\partial t^2} p(x_i; t'; x_i)}{v(x)^3} G(x_i; x; t - t') \, dt' \, dx,
\end{align*}
$$

or equivalently in the frequency domain

$$
\begin{align*}
\delta d(x_i; \omega; x_i) &= \int \frac{2 \omega^2 \delta v(x) p(x_i; \omega; x_i)}{v(x)^3} G(x_i; x; \omega) \, d\omega,
\end{align*}
$$

We denote the traveltimes and amplitudes from a location $x$ to another location $y$ as $t(x; y)$ and $a(x; y)$, respectively. Asymptotically, the pressure and Green’s functions satisfy the following equations:
\[ p(x; \omega; x_s) = a(x; x_s) e^{i \omega \tau(x; x_s)} \]  

(A-7)

and

\[ G(x; x; \omega) = a(x; x) e^{i \omega \tau(x; x)} . \]  

(A-8)

and the gradient of the asymptotic pressure function is given by

\[ \nabla p(x; \omega; x_s) \approx i \omega \nabla \tau(x; x_s) a(x; x_s) e^{i \omega \tau(x; x_s)} . \]  

(A-9)

Substituting equations A-7–A-9 into A-6, the high-frequency approximation of the second term on the right side can be expressed as

\[
\nabla \cdot \left( G(x; x; \omega) \nabla p(x; \omega; x_s) \right) \\
= -\omega^2 \nabla \tau(x; x_s) \cdot \left( \nabla \tau(x; x_s) \right) \\
+ \nabla \tau(x; x_s) a(x; x_s) a(x; x) e^{i \omega \tau(x; x_s) + \tau(x; x)} \\
= -\omega^2 \cos 2 \theta + 1 \frac{1}{v(x)^2} a(x; x_s) a(x; x) e^{i \omega \tau(x; x_s) + \tau(x; x)} \\
= -\omega^2 \frac{2 \cos^2 \theta}{v(x)^2} A(x; x_s; x) e^{i \omega T(x, x_s; x)} ,
\]

(A-10)

where \( T(x, x_s; x) = \tau(x; x_s) + \tau(x; x) \) and \( A(x; x_s; x) = a(x; x_s) a(x; x) \) denote the ray traveltime summation and amplitude product of the wavefield from a source location \( x_s \) to a subsurface location \( x \) and reflected back to the receiver location \( x_s \), respectively, and \( \theta \) is the incident angle at the subsurface location \( x \).

Substituting equation (A-10) into equation A-6, we have

\[
\tilde{\delta}d(x_r; \omega; x_s) = -\left[ \frac{2 \omega^2}{v(x)^2} \left( \frac{\delta v(x)}{v(x)} + \cos^2 \theta \frac{\delta p(x)}{p(x)} \right) \right] \\
\times A(x_r; x_s; x) e^{i \omega T(x, x_s; x)} dx ,
\]

(A-11)

which describes the relationship of the data perturbation in the frequency (time) domain to the perturbations in the model domain. However, for imaging algorithms, we aim for the (inverse) relationship of equation A-11 that describes the model perturbations in terms of the data perturbation. Following Bleistein [1987], we introduce the inverse operator \( B(x; \omega; x_s; x; \theta; \varphi) \), and the model perturbations must satisfy

\[
\frac{\delta v}{v} + \cos^2 \theta \frac{\delta p}{p} = \int_{\Omega} B(x; \omega; x_s; x; \theta; \varphi) \tilde{\delta}d(x_r; \omega; x_s) \\
\times e^{-i \omega T(x, x_s; x)} dx_r dx_s d\omega ,
\]

(A-12)

where \( \varphi \) is subsurface azimuth angle. By substituting equation A-11 into equation A-12, we have

\[
\frac{\delta v}{v} + \cos^2 \theta \frac{\delta p}{p} = -\int_{\Omega} B(x; \omega; x_s; x; \theta; \varphi) \\
\times \int 2 \omega^2 / v(x_s)^2 \left( \frac{\delta v(x)}{v(x)} + \cos^2 \theta \frac{\delta p(x)}{p(x)} \right) \\
\times A(x_r; x_s; x_r) e^{i \omega (T(x, x_s; x_r) - T(x, x_s; x_r))} dx_s dx_d d\omega .
\]

(A-13)

Changing the integration variable from the surface coordinates, \((x_s, x_r)\), to the subsurface angles, \((\theta', \varphi')\), equation A-13 can be approximated as

\[
\frac{\delta v}{v} + \cos^2 \theta \frac{\delta p}{p} \\
\approx -\int_{\Omega} B(x; \omega; x_s; x; \theta'; \varphi') \delta (\theta - \theta') \delta (\varphi - \varphi') \\
\times \int 2 \omega^2 / v(x_s)^2 \left( \frac{\delta v(x)}{v(x)} + \cos^2 \theta \frac{\delta p(x)}{p(x)} \right) \\
\times A(x_r; x_s; x_r) e^{i \omega (T(x, x_s; x_r) - T(x, x_s; x_r))} dx_s dx_d d\omega' .
\]

(A-14)

The approximation in equation A-14 is only valid when the high-frequency asymptotic approximation \( \omega(T(x_1; x_s; x) - T(x, x_s; x)) \approx \frac{k(x' - x)}{C} \) (Bleistein, 1987). The Jacobian in equation A-14 includes the Beylik determinant (Beylkin, 1985) and provides the connection between local direction vectors at an image location and the actual acquisition geometry. In the current case of isotropic media, it is derived in Xu et al. [2011] as follows:

\[
\left| \frac{\partial (k, \theta', \varphi')}{\partial (x_r, x_s, \omega)} \right| = 2(4\pi)^4 \frac{\alpha v^2 \cos^2 \theta}{v(x_s)} \frac{1}{v(x)} ,
\]

(A-15)

where \( \alpha_s \) and \( \alpha_r \) (again) denote the source and receiver propagation angle with respect to the vertical direction, respectively.

To make equation (A-14) valid, we require the inverse operator \( B(x; \omega; x_s; x; \theta; \varphi) \) to satisfy

\[
\frac{2 \omega^2}{v^2} A(x; x_s; x) B(x; \omega; x_s; x; \theta; \varphi) \left| \frac{\partial (x_r, x_s, \omega)}{\partial (k, \theta', \varphi')} \right| = \left( \frac{1}{2 \pi} \right)^3
\]

(A-16)

By substituting equation (A-15) into equation A-16, we have

\[
B(x; \omega; x_s; x; \theta'; \varphi') = -W \frac{\cos \alpha_s \cos \alpha_r}{v(x_s)} A(x; x_s; x) ,
\]

(A-17)

where \( W = 32 \pi v(x) \cos^2 \theta' / \sin \theta' \). Substituting equation (A-17) into equation A-12, we find the desired relationship of equation 11 as
\[ \frac{\delta v}{v_0} + \cos^2 \theta \frac{\delta \rho}{\rho_0} = -\int \int W \cos \alpha_x \cos \alpha_x A(x; x; \omega; x) \delta \theta(x; \omega; x) \times \delta(\theta - \theta')e^{-i\omega T(x; x; \omega)} \kappa dx_0 dx_0 d\theta'. \quad (A-18) \]

### APPENDIX B

#### RELATION TO AMPLITUDE VARIATION WITH OFFSET

The AVO studies are often based on approximations to the Zoeppritz equations. These equations describe the reflection and transmission coefficients of a plane-wave incident at a planar interface. The approximations are valid for small changes in elastic properties across the interface and, depending on the approximation used, for incidence angles up to 30° or so. In the acoustic case, the approximation to the P-wave reflectivity derived by Aki and Richards (1980) for an incident plane P-wave is given by

\[ R_{pp}(\theta_a) = \frac{1}{2} \left[ \frac{\delta(V_P \rho)}{V_P \rho} \right] + \frac{1}{2} \tan^2 \theta_a \left[ \frac{\delta V_P}{V_P} \right]. \quad (B-1) \]

where \( V_P \) and \( \theta_a \) are the P-wave velocity and the mean of the incident and transmission angles, respectively. Equation 15 indicates that the image from an amplitude-preserving RTM provides

\[ I(\theta) = \cos^2 \theta \left[ \frac{\delta(V_P \rho)}{V_P \rho} \right] + \sin^2 \theta \left[ \frac{\delta V_P}{V_P} \right]. \quad (B-2) \]

To reconcile these two formulas, we notice that an angle-dependent scaling of \( \cos^2 \theta \) is applied to the ADCIGs from an RTM. Dividing equation B-2 by this factor gives the Aki and Richards AVO approximation, so equation B-1, which is commonly used in AVO analysis (Debski and Tarantola, 1995; Li, 2005), is consistent with our proposed RTM in equation 15.

### REFERENCES


Pooe, C., 2013, Pre-migration receiver deghosting and re-datuming for variable depth streamer data: 83rd Annual International Meeting, SEG, Expanded Abstracts, 4216–4220.


