Prestack seismic amplitude analysis: An integrated overview

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Abstract

In this tutorial, I present an overview of the techniques that are in use for prestack seismic amplitude analysis, current and historical. I show that these techniques can be classified as being based on the computation and analysis of either some type of seismic reflection coefficient series or seismic impedance. Those techniques that are based on the seismic reflection coefficient series, or seismic reflectivity for short, are called \textit{amplitude variation with offset} methods, and those that are based on the seismic impedance are referred to as \textit{prestack amplitude inversion} methods. Seismic reflectivity methods include: near and far trace stacking, intercept versus gradient analysis, and the fluid factor analysis. Seismic impedance methods include: independent and simultaneous P and S-impedance inversion, lambda-mu-rho analysis, Poisson impedance inversion, elastic impedance, and extended elastic impedance inversion. The objective of this tutorial is thus to make sense of all of these methods and show how they are interrelated. The techniques will be illustrated using a 2D seismic example over a gas sand reservoir from Alberta. Although I will largely focus on isotropic methods, the last part of the tutorial will extend the analysis to anisotropic reservoirs.

Introduction

The amplitude variation with offset (AVO) and prestack inversion techniques have grown to include a multitude of subtechniques, each with their own assumptions. In this tutorial, I make the assumption that AVO techniques are based on the computation and analysis of some type of seismic reflection coefficient series, or reflectivity for short, and that prestack inversion techniques are based on the computation and analysis of some type of seismic impedance. To understand the distinction between impedance and reflectivity, refer to Figure 1, which shows the different ways in which geologists and geophysicists look at their data.

As shown on the left side of Figure 1, the goal of all geoscientists is to understand the subsurface of the earth, which consists of a series of layers of variable structure, lithology, porosity, and fluid content. To do this, the geologist generally analyzes borehole well log measurements of some layer parameter $P$, whereas the geophysicist, when using exploration seismology, analyzes changes in this layer parameter at the interfaces between successive layers, called the \textit{reflectivity} ($R$). The fundamental idea in this tutorial is that there is a relationship between $P$ and $R$, and it can be written

$$R_{Pi} = \frac{P_{i+1} - P_i}{P_{i+1} + P_i} = \frac{\Delta P_i}{2P_i},$$

where $\Delta P_i = P_{i+1} - P_i, \bar{P}_i = \frac{P_{i+1} + P_i}{2}$ and subscript $i$ refers to the $i$th layer. That is, the reflectivity is found by dividing the change in the parameter between two successive layers by twice its average value. The subscript $P$ in the reflectivity $R_P$ indicates that the reflectivity is associated with parameter $P$. I will use this notation throughout the tutorial. This is, of course, an oversimplification, because the seismic trace also contains the source wavelet, amplitude scaling effects, noise contamination due to random noise and coherent noise such as multiples, static time shifts due to topography and near-surface weathering, spatial mispositioning due to structure and subsurface scattering at depth, and so on. A complete discussion of these effects and the techniques of seismic processing which have been developed to remove or ameliorate them is beyond the scope of this tutorial.

Even if our processing is extremely good, we are always left with a band-limited reflectivity which can be modeled using the convolutional model, written in matrix form as

$$\begin{bmatrix} W_0 & 0 & \ldots & 0 \\ \vdots & W_0 & \ddots & \vdots \\ W_{n-1} & \ddots & \ddots & 0 \\ 0 & W_{n-1} & \ddots & W_0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & W_{n-1} \end{bmatrix} \begin{bmatrix} R_0 \\ \vdots \\ R_{m-1} \end{bmatrix} = \begin{bmatrix} S_0 \\ \vdots \\ S_{n+m-2} \end{bmatrix},$$

(2)

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or \( S = WR \), where \( R \) is an \( m \)-sample reflectivity (note that subscript 0 represents zero time), \( W \) is the convolutional matrix in which each column contains an \( n \)-sample wavelet shifted by one sample from the previous column, and \( S \) is the \( n + m - 1 \) sample result, or seismic trace. This technique of convolution is illustrated pictorially in Figure 2, where a reflectivity similar to that of Figure 1 has been convolved with a symmetrical, or zero-phase, wavelet.

We can think of convolution as shifting the wavelet so that it is centered at each reflection coefficient, scaling the wavelet by the amplitude of each reflection coefficient, and then summing the result. We say that this result is “band limited” because, although the spectrum of the reflectivity contains all frequencies from 0 Hz to the Nyquist, the seismic wavelet is typically band limited between approximately 10–60 Hz (although some of the new seismic receivers and acquisition techniques are pushing the low end much lower). It is thus important to note that in subsequent sections when I refer to the particular reflectivity model used in an AVO analysis, this more precisely refers to the band-limited result found from applying equation 2.

A key question is which parameter \( P \) we are interested in. For the geologist, the parameter is usually one that enables him or her to infer the reservoir properties, such as porosity, water saturation, TOC, etc. However, the geophysicist is usually limited to parameters that are related to the propagation and reflection of the seismic signal, such as P-impedance, the product of P-wave velocity and density, and S-impedance, the product of S-wave velocity and density. Once these properties have been estimated, the geophysicist can infer the reservoir properties mentioned above, but in this tutorial we will focus more on the estimation of these impedances themselves and the analysis of the band-limited reflectivity from which we derive the impedances.

The geometry of an idealized common midpoint (CMP) seismic gather is shown in Figure 3, in which traces recorded at different offsets are grouped by increasing offset around a CMP point. Notice that each offset \( X_i \) corresponds to an angle of incidence \( \theta_i \) at the reflector. In Figure 3, the single reflector shown will create a seismic reflection due to the contrast between the density \( \rho \), P-wave velocity \( V_P \), and S-wave velocity \( V_S \) in each layer. As we will discuss, the amplitude of this reflection will change at each offset.

Figure 4 shows a set of real gathers which were acquired and processed over a shallow gas field in central Alberta. This data set will be used to explain many of the methods described in this tutorial. In particular, notice the amplitudes change as a function of offset on the trough and peak of the seismic reflection event below 620 ms. Our goal in this tutorial is a physical understanding of this amplitude change, and how it relates to the gas sand.

### Reflectivity methods

#### Normal incidence reflectivity

Unlike the single incident and reflected raypaths shown in Figure 3, in an elastic earth, we actually

![Figure 2](image-url)

**Figure 2.** Convolution of a reflectivity similar to that in Figure 1 with a zero-phase wavelet, where convolution can be seen as shifting the wavelet so that it is centered at each reflection coefficient, scaling the wavelet by the amplitude of the reflection coefficient, and then summing the result.

![Figure 3](image-url)

**Figure 3.** An idealized CMP gather, where the \( X_i \) are the offsets and the \( \theta_i = 0 \) represent the angles of incidence of each seismic raypath (note that \( X_0 = 0 \) and \( \theta_0 = 0 \)). The single reflector shown will produce a seismic reflection caused by the contrast between the density \( \rho \), P-wave velocity \( V_P \), and S-wave velocity \( V_S \) in each layer.
observe mode converted reflected and transmitted P- and S-raypaths for an incident P-wave raypath at arbitrary incidence angle, as illustrated in Figure 5, which shows the conversion of an incident P-wave at a non-zero angle of incidence into its reflected and transmitted P- and S-components. The P, or compressional, waves are elastic waves that travel by the alternate compression and rarefaction of the earth’s subsurface, whereas S, or shear, waves are created by side-to-side shearing of the earth’s subsurface. Shear waves in an isotropic medium with a polarization (particle motion) in a vertical plane are SV-waves, whereas those with a polarization in the horizontal plane are SH-waves. However, there are other directions of polarization perpendicular to the direction of travel which produce particle motion that is a combination of SV- and SH-waves. For SV-waves from a horizontal surface, the particle motion is in the plane of Figure 5 but for SH-waves, think of the particle motion as coming out of the plane shown in Figure 5. In HTI-fractured media, like some carbonates, a vertically traveling S-wave splits into two S-waves, one has polarization aligned parallel and the other perpendicular to the direction of the fractures with the faster S-wave having a polarization in the direction parallel to the fractures.

Initially, we will only discuss isotropic materials that have the same properties in all directions. In this case, SH- and SV-waves have the same velocity, which is slower than the velocity of the P-waves. Intuitively, we can see that a P-wave that strikes a boundary at right angles will produce only a reflected P-wave, but that a P-wave which strikes a boundary at nonnormal incidence (as in Figure 5) will create reflected P- and S-waves. One last observation, which is of fundamental importance to prestack seismic analysis, is that because P-waves compress the reservoir rocks they are very sensitive to the pore fluid, but because the S-waves do not compress the rocks, they are not as sensitive to the pore fluid.

Zoeppritz (1919) derived the amplitudes of the reflected and transmitted waves shown in Figure 5 using the conservation of stress and displacement across the layer boundary, giving a forward model with four equations and four unknowns (the amplitudes) which can be solved using matrix inversion. These equations are shown in Appendix A. Although the complete solution for an arbitrary angle leads to a complicated set of equations, it is shown in Appendix A that the normal incidence P-wave reflection coefficient can be determined in a straightforward way from the Zoeppritz equations. This is the quantitative confirmation of the intuitive argument which was given above. Generalizing the two-layer case given in Appendix A to an arbitrary number of layers, we get

$$R_{PPi}(0^\circ) = \frac{\rho_i V_{Pi+1} - \rho_i V_{Pi}}{\rho_{i+1} V_{Pi+1} + \rho_i V_{Pi}} = \frac{\Delta I_{i+1} - \Delta I_i}{\Delta I_{i+1} + \Delta I_i}. \tag{3}$$

where $\Delta I_i$ is the acoustic impedance, or P-impedance, of the $i$th layer. As mentioned previously, the reflection coefficient $R_{AI}$ is written in such a way as to indicate which parameter is associated with the reflectivity; in this case, the acoustic impedance. To see how equation 3 relates to equation 1, note that we can rewrite equation 3 as

$$R_{AI} = \frac{\Delta I}{2\Delta I}. \tag{4}$$

where $\Delta I = \Delta I_{i+1} - \Delta I_i$ and $\Delta I = \Delta I_{i+1} + \Delta I_i$. The subscript notation has been dropped in equation 4, so that $R_{AI}$ can be thought of as the $m$-sample column vector of reflectivity shown in equation 2. Figure 6 from Russell et al. (2011) shows the CMP stack of the gathers shown in Figure 4, with a picked event at the zero crossing which represents the center of the gas sand. The stack is often considered to be a reasonable approximation to

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**Figure 4.** Five CMP gathers from around the well on a seismic line recorded over a shallow gas sand in Alberta, with the time-integrated velocity log overlain at CMP 330. The far-offset of 647 m corresponds to an angle of approximately 30° (Russell et al., 2011).

**Figure 5.** Mode conversion of an incident P-wave into reflected and transmitted P- and SV-waves on the boundary between two elastic layers. P-wave direction of travel and particle motion is along the raypaths. SV-wave direction of travel is along the raypaths, but SV-wave particle motion is at right angles to the raypath.
$S_{AI}$, the convolution of the wavelet with $R_{AI}$. However, the stack is actually an average over all angles, and so does not represent the true normal incidence reflectivity. In the next section, we will discuss a better way to estimate the normal incidence reflectivity.

By a similar analysis, in which we assume that the seismic source creates S-waves and that we record S-wave reflections, we find that

$$R_{SSi}(0) = R_{SI} = \frac{\rho_{i+1}V_{Si+1} - \rho_{i+1}V_{Si}}{\rho_{i+1}V_{Si+1} + \rho_{i}V_{Si}} = \frac{SI_{i+1} - SI_{i}}{SI_{i+1} + SI_{i}},$$

(5)

where SI is the shear impedance of each layer. Do not confuse the reflection coefficient in equation 5 with the converted wave shear reflectivity which, as shown in the Appendix A, is equal to zero at normal incidence. Instead, $R_{SI}$ is the true S-wave reflection coefficient which would be recorded using a S-wave source and multicomponent receivers. (Note that there is also an issue of polarity convention when we compare true S-wave recording with converted S-wave recording [Brown et al., 2002]). As with the acoustic impedance reflectivity, we can rewrite the shear impedance reflectivity from equation 5 as

$$R_{SI} = \frac{\Delta SI}{2SI},$$

(6)

where $\Delta SI = SI_{i+1} - SI_{i}$ and $\bar{SI} = \frac{SI_{i+1} + SI_{i}}{2}$. Again, the reflectivity in equation 6 can be thought of as an $m$-sample vector, and the observed seismic trace could be written $S_{AI}$, the convolution of $R_{SI}$ with the seismic wavelet.

To relate our discussion of P- and S-impedance and reflectivity to the data example shown in Figures 4 and 6, Figure 7 shows the P-wave velocity log, S-wave velocity log and density log recorded over the reservoir zone at CMP location 330. Also shown are logs that have been derived from the velocity and density logs and will be discussed later in this tutorial, such as $V_P/V_S$ ratio, Poisson’s ratio, extended elastic impedance (EEI) at 56°, $\lambda\rho$, and $\mu\rho$.

The logs in Figure 7 have all been integrated to time using the sonic log traveltimes and correlation with the seismic data. This is shown by the insertion of CMP gather 330 from Figure 4, which shows how the well logs correlate with the seismic at the gas sand zone, indicated by the horizontal lines on the plot. Note that this is a shallow gas sand that is characterized by low P-wave velocity, density, and $V_P/V_S$ ratio. Outside
of the gas zone, the S-wave sonic log in Figure 7 has been modeled from the P-wave sonic using the mudrock line (Castagna et al., 1985). Inside the gas zone the S-wave log has been modeled using Biot-Gassmann fluid substitution (Mavko et al., 1998). This modeling could also have been done using the method proposed by Greenberg and Castagna (1992), in which more detailed lithology parameters are used.

**The Aki-Richards approximation**

It has been shown (Bortfeld, 1961; Richards and Frasier, 1976; Aki and Richards, 2002) that, for “small” changes in the P-wave velocity, S-wave velocity, and density across a boundary between two elastic media, the P-wave reflection coefficient for an incident P-wave as a function of incident angle can be approximated by the linearized sum of three terms given by

\[
R_{pp}(\theta) = aR_{VP} + bR_{VS} + cR_D, \tag{7}
\]

where \(a = 1 + \tan^2 \theta\), \(b = -8k_{sat}\sin^2 \theta\), \(c = 1 - 4k_{sat}\sin^2 \theta\), \(R_{VP} = \frac{\Delta V_{VP}}{2V_{P1}}\), \(R_{VS} = \frac{\Delta V_{VS}}{2V_{S1}}\), \(R_D = \frac{\Delta \rho}{2\rho_1}\), \(\bar{\theta} = \frac{\theta_1 + \theta_2}{2}\) is the average of the incident, and refracted angles across a layer boundary, \(\bar{V}_P = \frac{V_{P1} + V_{P2}}{2}\), \(\bar{V}_S = \frac{V_{S1} + V_{S2}}{2}\), and \(\bar{\rho} = \frac{\rho_1 + \rho_2}{2}\) are the average velocity and density values across a layer boundary, \(\Delta V_P = V_{P2} - V_{P1}\), \(\Delta V_S = V_{S2} - V_{S1}\), and \(\Delta \rho = \rho_2 - \rho_1\) are the differences of the velocity and density values across a layer boundary, \(k_{sat} = \left(\frac{V_{Psat}}{V_{P1}}\right)^2\) is the shear-to-compressional wave velocity ratio for the in situ, or saturated, rocks averaged between layers. See Figure 5 for an illustration of a single boundary between two layers, which can be generalized for \(N\) layers.

At a more advanced level, it should be noted that the linearized approximation of Aki and Richards (2002) is based on the minimization of the ray parameter \(p = \frac{\sin \theta}{V_{P1}}\), which leads to a definition of the average angle given by \(\bar{\theta} = \tan^{-1}(\Delta \theta \bar{V}_P/V_{P1})\), where \(\Delta \theta\) is the difference between the incident and refracted angles. Because this expression is almost identical to the average of the incident and refracted angles, the average value is normally used instead. Thus, the terms on the right side of the linearized AVO equation are always computed using the average angle rather than the incident angle. Although there is no firm agreement as to whether the angle on the left side of equation 7 refers to the incident angle or to the average angle, here I assume that this is the incident angle, because the average angle is derived from the incident angle. The same comment applies to plots of reflection amplitude versus angle, where we normally assume the angle on the horizontal axis is the incident angle.

Equation 7 is usually referred to as the Aki-Richards equation. The convention for naming the reflection coefficients adopted in equation 1 has been continued, so we can think of the reflectivities in equation 7 as P-wave velocity, S-wave velocity, and density reflectivities, respectively, which are vectors of \(m\)-sample length. By small changes, we mean that equation 1 is valid where the reflectivity values are in the order of 0.1 or less, although it is still a reasonable assumption for values larger than this. Also, again note that the actual measured seismic signal is the convolution of a wavelet with the reflectivity of equation 7, or \(S_{pp}(\theta) = W_{RPP}(\theta)\), and that the wavelet is angle-dependent, losing its higher frequencies as the angle increases. The loss of high

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**Figure 8.** The intuitive derivation of intercept and gradient, where (a) shows the picked amplitudes at time \(t\) over a 10-trace angle gather and (b) shows the least-squares fit of these picks as a function of \(\sin^2 \theta\) that could be used to interpret the intercept \((R_{AI})\) and gradient \((R_{GI})\).

**Figure 9.** A comparison between the full Zoeppritz equations (see Appendix A) and the two forms of the Aki-Richards equation computed over the top of a gas sand, where (a) shows the intercept-gradient-curvature approximation of equation 12 and (b) shows the Fatti et al. (1994) approximation of equation 8. In both cases, the two-term and three-term sums are shown.
frequencies is compounded by the effect of NMO stretch. This approximation is also coupled with the linearized angle-dependent approximation given by equation 7, which starts to deviate from the full Zoeppritz solution between 30° and 35°. However, the approximation is quite good out to much larger angles as long as all three terms are used in the approximation. (In some approximations, the third term is dropped.)

There are several important algebraic rearrangements of equation 7 which will be made use of in subsequent derivations. The first one that I will discuss is by Fatti et al. (1994), which was based on earlier work by Smith and Gidlow (1987) and Gidlow et al. (1992), and is given by

\[ R_{pp}(\theta) = aR_{Al} + bR_{Sl} + c'R_{D}, \]  
(8)

where the first two weighting terms, \( a \) and \( b \), are identical to the weighting terms in equation 7 but the third weighting term is given by \( c' = 4\tilde{k}_{sat}\sin^2 \theta - \tan^2 \theta \). Equation 8 is used as the basis for extracting the reflectivity terms from a CMP gather and for impedance estimation first derived by Peterson et al. (1955), which can be written as

\[ R_p \approx \frac{\Delta \ln P}{2}. \]  
(9)

where the letter \( P \) now refers to any parameter, as in equation 1. Equation 9 follows from equation 1 from the calculus formula given by \( \frac{d \ln p}{dt} = \frac{1}{p} \frac{dp}{dt} \), where we eliminate the \( dt \) term and change the derivative operator \( d \) to the difference operator \( \Delta \). Using equation 9, we find that

\[ R_{Al} \approx \frac{\Delta \ln Al}{2} = \frac{\Delta \ln V_p + \Delta \ln \rho}{2} \approx \frac{1}{2} \left( \frac{\Delta V_p}{V_p} + \frac{\Delta \rho}{\rho} \right) = R_{VP} + R_D, \]  
(10)

and

\[ R_{Sl} \approx \frac{\Delta \ln Sl}{2} = \frac{\Delta \ln V_S + \Delta \ln \rho}{2} \approx \frac{1}{2} \left( \frac{\Delta V_S}{V_S} + \frac{\Delta \rho}{\rho} \right) = R_{VS} + R_D. \]  
(11)

A third rearrangement of equation 7 (and the most common) is given by

\[ R_{pp}(\theta) = a'R_{Al} + b'R_{GI} + c''R_{VP}, \]  
(12)

where \( R_{GI} = R_{VP} - 8\tilde{k}_{sat}R_{VS} - 4\tilde{k}_{sat}R_D, \ a' = 1, \ b' = \sin^2 \theta, \) and \( c'' = \tan^2 \theta \sin^2 \theta \). Equation 12 was initially derived by Wiggins et al. (1983) and it is the basis of much of the empirical AVO work performed in our industry. The term \( R_{GI} \) is called the gradient and is usually written as \( B \) in the geophysical literature. The \( R_{Al} \) term is called the intercept and is usually written as \( A \), and the \( R_{VP} \) term is called the curvature and is written as \( C \). However, the \( A, B, C \) notation obscures the fact that only \( R_{GI} \) (or \( B \)) is a new reflectivity compared with the previous methods, and is itself a weighted sum of the reflectivities given in equation 7. The subscript GI in the gradient reflectivity stands for gradient impedance and anticipates the EEI concept that will be discussed later in the tutorial. Equation 12 has the advantage that an estimate of \( \tilde{k}_{sat} \) is not needed in the weighting coefficients used to extract the three parameters \( R_{Al}, R_{GI}, \) and \( R_{VP} \). Although I have written equation 12 to conform with the earlier notation, this obscures the fact that, if we drop the third term (which is reasonable for angles less than approximately 30°) and write the \( a' \) and \( b' \) coefficients explicitly, we get the usual form of the equation, which is written

\[ R_{pp}(\theta) = R_{Al} + R_{GI}\sin^2 \theta. \]  
(13)

The terms intercept and gradient come from an intuitive way of understanding this method, because if we crossplot the picked amplitudes of an angle gather at a given time as a function of \( \sin^2 \theta \), and fit a straight line to the points, the intercept of the plot gives \( R_{Al} \) and the slope, or gradient, gives \( R_{GI} \). This is shown in Figure 8, where Figure 8a shows the picks on an angle gather and Figure 8b shows the derivation of the intercept and gradient from these picks. I will shortly discuss a more quantitative way of estimating these terms, which can be generalized to all of the techniques discussed in this section.

The three-term approximations from equations 8 (the intercept-gradient-curvature approximation) and 12 (the acoustic impedance-shear impedance-density approximation) are shown in Figure 9a and 9b, respectively, for the top of a gas sand, where \( V_p = 2250 \) m/s, \( V_S = 1125 \) m/s, and \( \rho = 2110 \) kg/m\(^3\) for the overlying shale and \( V_p = 2000 \) m/s, \( V_S = 1310 \) m/s, and \( \rho = 1995 \) kg/m\(^3\) for the underlying gas sand. In Figure 9a, the full three-term form of equation 12 is shown, as well as the truncated two-term form and the full Zoeppritz calculation for comparison. For angles up to 30°, all three curves agree quite well, but after 30°, the two-term truncation (which is often used) becomes less and less accurate. In Figure 9b, the full three-term form of equation 8 is shown, as well as the truncated two-term form and the full Zoeppritz calculation for comparison. All three curves agree quite well out to 60°, which suggests that the two-term truncation is quite a reasonable approximation to the full Zoeppritz calculation, but also tells us that the extraction of the third term, the density term, can be very sensitive to noise. Note that in Figure 9, the Zoeppritz calculations were done using the true angle of incidence and the Aki-Richards calculations were done using the average
of the incident and refracted angle. Either angle could have been used to annotate the axis, but as noted earlier, we have adopted the usual convention and plotted the amplitudes against incident angle.

Equations 8 and 12 are algebraic reformulations of equation 7. Another way of reformulating equation 7 involves transforming to parameters which are nonlinearly related to velocity and density. This involves the use of differentials as well as algebra. For example, Shuey (1985) transformed equation 12 from dependence on $V_p$, $V_S$ and $\rho$ to dependence on $V_I$, $\rho$, and Poisson’s ratio $\nu = \frac{V_S^2 - 2V_p^2}{2(V_p-V_S)}$. Because the first and third terms in equation 12 are not dependent on S-wave velocity, only the second term in equation 12 changes in the Shuey (1985) formulation, and thus the gradient reflectivity term $R_{GI}$ can be written as

$$R_{GI} = R_{VP} - 2(R_{AI} + R_{VP}) \left( 1 - \frac{2\nu}{1 - \nu} \right) + \frac{\Delta \nu}{(1 - \nu)^2}, \quad (14)$$

where $\vec{\nu} = \frac{\nu_i + \nu_{i+1}}{2}$, and $\Delta \nu = \nu_{i+1} - \nu_i$. Equation 14 is written differently from the from the notation used by Shuey (1985) so that it conforms to the notation used in this tutorial. It is tempting to reformulate equation 14 by introducing a Poisson’s ratio reflectivity $R_{\nu} = \frac{\Delta \rho}{2\rho}$, but this would lead to unnecessary complications.

More recently, Gray et al. (1999) reformulated equation 7 for two sets of fundamental constants: $\lambda_{sat}$, $\mu_{sat}$, and $\rho_{sat}$ (the first and second Lamé parameters and density, respectively), and $K_{sat}$, $\mu_{sat}$, and $\rho_{sat}$ (bulk modulus, shear modulus, and density, respectively), with the shear modulus being identical to the second Lamé parameter), where the relationships between these elastic constants and the saturated compressional velocity ($V_{Psat}$) and shear velocity ($V_{Ssat}$) are given by

$$V_{Psat} = \sqrt{\frac{\lambda_{sat} + 2\mu_{sat}}{\rho_{sat}}} = \sqrt{\frac{K_{sat} + \frac{4}{3}\mu_{sat}}{\rho_{sat}}}, \quad (15)$$

and

$$V_{Ssat} = \sqrt{\frac{\mu_{sat}}{\rho_{sat}}}. \quad (16)$$

where the subscript sat indicates that the parameters are measured in the in situ, or saturated reservoir. (Adding this subscript is a very important point that is neglected in many articles, as will shortly be discussed). As with Shuey’s work, this reformulation required the use of algebra as well as differentials relating $\lambda_{sat}$, $\mu_{sat}$, and $K_{sat}$ to $V_{Psat}$, $V_{Ssat}$, and $\rho_{sat}$. Russell et al. (2011) derive a generalized equation based on Biot-Gassmann poroelasticity theory which contains both of the Gray parameterizations as special cases. This equation is written

$$R_{PP}(\theta) = a''R_f + b''R_\mu + c''R_D, \quad (17)$$

where $a'' = (1 - \frac{K_{sat}}{K_{dry}})\frac{\omega^2 \theta}{\kappa_{dry}}$, $b'' = \frac{K_{sat}}{K_{dry}} \sec^2 \theta - 4\kappa_{sat} \sin^2 \theta$, $c'' = 1 - \frac{\omega^2 \theta}{2}$, $\kappa_{dry} = (\frac{K_{dry}}{V_{Pdry}})^2$, $R_f = \frac{\Delta \rho}{2\rho}$, and $R_D$ is as previously defined. The key new parameters in equation 17 are the fluid reflectivity $R_f$ and the squared dry rock shear-to-compressional wave velocity ratio $\kappa_{dry}^2$. Although an in-depth discussion of the fluid reflectivity is beyond the scope of this tutorial (the interested reader is referred to Russell et al., 2011), a quick summary is that the fluid term $f$ relates the saturated and dry bulk moduli and first Lamé parameter (that is, $K_{sat} = \kappa_{dry} + f$ and also $\lambda_{sat} = \kappa_{dry} + f$) and is written explicitly as

$$f = a^2 M, \quad (18)$$

where $a = 1 - \frac{K_{sat}}{K_{dry}}$ is the Biot coefficient, $M = \frac{(\frac{4}{3}a^2 + \frac{\phi}{K_{dry}})^{-1}}{2}$ is the fluid modulus, $K_{dry}$ is the mineral bulk modulus, $\phi$ is the porosity of the reservoir, and $K_f$ is the bulk modulus of the fluid in the pores. Thus, $\Delta \rho$ is the change in the fluid parameter and $\Delta \mu$ is the average fluid parameter between layers. The key point to note is that, in the Gray et al. (1999) formulations, it is the reflectivity associated with saturated bulk modulus and first Lamé parameter that are extracted, whereas in the general reflectivities $R_f$ and $R_D$ of equation 17, the fluid and dry parts of reservoir are separated, using the dry rock bulk modulus ratio $\kappa_{dry}$ as a new parameter. As shown in Table 1 of Russell et al. (2011) (which shows the inverse ratio, $V_p/V_S$, squared), the value of $\kappa_{dry}$ allows us to “tune” equation 16 to the two formulations of Gray et al. (1999). If $\kappa_{dry} = 3/4$ (i.e., 1/1.333...) then $R_f$ becomes the bulk modulus reflectivity given by

$$R_f = \frac{\Delta K_{sat}}{2K_{sat}}, \quad (19)$$

and if $\kappa_{dry} = 1/2$, then $R_f$ becomes the lambda reflectivity given by

$$R_l = \frac{\Delta \lambda_{sat}}{2\lambda_{sat}}. \quad (20)$$

For values of $\kappa_{dry}$ less than 0.5, the fluid reflectivity $R_f$ is influenced by the type of fluid and the porosity in the reservoir as they influence Biot’s coefficient $a$ and fluid modulus $M$, which were defined earlier. For example, a value of 3/7 (i.e., 1/2.333...) corresponds to the dry rock velocity ratio squared of a clean, unconsolidated sandstone, and the resulting fluid reflectivity will indicate the fluid content and porosity of the reservoir itself.

Because equations 7, 8, 12, and 17 are all linear sums of three reflectivity terms multiplied by weighting terms, they can be used to extract estimates of the three reflectivities by picking and analyzing the amplitudes over all offsets or angles on a seismic gather at each time sample $t$, as shown in Figure 8a. For an $N$-trace gather, the forward problem can be written as
where the term $R_{PP}(t, \theta_i)$ represents the picked amplitude at time $t$ and angle $\theta_i$, shown as the dots in Figure 8a. Here, $a_i$, $b_i$, and $c_i$ are the primed or unprimed coefficients given in equations 7, 8, 12, and 17, and $R_a$, $R_b$, and $R_c$ are the reflectivities associated with these coefficients. It is not necessary to convert the gathers from the offset to angle domain, as we can use the following relationship between the sine of the angle and the time and offset and velocity (Walden, 1991)

$$\sin \theta = \frac{X V_{\text{INT}}}{V_{\text{rms}}^2},$$

(22)

where $X$ is the offset, $V_{\text{INT}}$ is the interval P-wave velocity, $V_{\text{rms}}$ is the root-mean-square (rms) P-wave velocity, and $t$ is the two-way seismic traveltime.

Equation 21 can be solved using the standard least-squares inversion approach to give the three reflectivity terms, which can be written explicitly as

$$m = (G^T G + \sigma I)^{-1} G^T d,$$

(23)

where

$$d = \begin{bmatrix} R_{PP}(t, \theta_1) \\ R_{PP}(t, \theta_2) \\ \vdots \\ R_{PP}(t, \theta_N) \end{bmatrix}, \quad G = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \vdots & \vdots & \vdots \\ a_N & b_N & c_N \end{bmatrix},$$

$$m = \begin{bmatrix} R_a \\ R_b \\ R_c \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

**Figure 10.** The intercept times gradient section for the gathers shown in Figure 4, where the picked event starting at 640 ms on the left of the section represents the transition from the top to the base of the gas sand. The amplitudes have been normalized between −1 and +1. Note the good definition of the gas sand between CMPs 316 and 346 by the product section, which is positive at the top and base of the sand (Russell et al., 2011). and $\sigma$ is a prewhitening factor. Once these reflectivity terms have been estimated from the prestack data at each gather and time sample, they can be used as the basis for the impedance inversion methods discussed in the next section, or they can be combined in various ways.

The use of equations 12 and 23 allows us to estimate the intercept ($R_{AI}$) and gradient ($R_{GI}$) and, optionally, the curvature ($R_{VP}$) in a more quantitative way than was shown in Figure 8b. Figure 10, from Russell et al. (2011), shows the product of the intercept and the gradient, and clearly shows the gas sand anomaly in red. The reason for this is that, in the type of gas sand shown in Figure 7, called a Class III gas sand, the intercept and gradient are both negative at the top of the gas sand and both positive at the base of the gas sand and thus their product is positive at the top and base of the gas sand, as seen in Figure 10.

Another technique that is commonly used to help locate a gas sand is to crossplot the gradient against the intercept and to identify anomalous outliers on this crossplot (Foster et al., 1993). Figure 11 shows such a crossplot from a 100-ms window centered at the picked event seen in Figure 10, between CMPs 300 and 356. On Figure 11, the red polygon zone captures outliers that have negative intercept and gradient values, and thus correspond to the top of the gas sand; the blue polygon captures outliers that have positive intercept and gradient values, and thus correspond to the base of the gas sand. This is seen in Figure 12, where the colors on the section correspond to time and CMP values captured by the equivalently colored polygon in Figure 11. The roughly elliptical set of points that are uncolored in Figure 11, and trend at an approximate −45° line on the crossplot, correspond to wet sands and shales, as shown by Castagna et al. (1998). This is obviously an oversimplification, because the cluster of points around the origin in Figure 11 is not confined to a single linear trend. More recently, Foster et al. (2010) have shown that, although shale upon shale values occupy this approximate −45° line and go through the origin, shale upon sand values do not go through the origin. Thus, the points seen as the “wet” trend in Figure 11 come from different nongas charged scenarios.

Although the displays shown in Figures 10–12 clearly highlight the gas sand zone, they are qualitative interpretations, and tell us nothing about the physical properties of the gas sand. A more quantitative approach involves extracting the reflectivities given in equation 8 by applying equation 23 with the appropriate weighting coefficients. That is, we estimate $R_{AI}$, the acoustic impedance reflectivity, $R_{SI}$, the shear impedance reflectivity, and optionally $R_{DI}$, the density reflectivity. To do this, we also need an estimate of $F_{sat}$ (the saturated S- to P-wave velocity ratio squared) in the weighting coefficients, usually obtained by smoothing the P-wave velocity estimate from a well log or rms velocity function and dividing this into the smoothed S-wave velocity. The
smoothed S-wave velocity can be estimated from the P-wave velocity using the mudrock equation proposed by Castagna et al. (1985), and given by

\[ V_P = d + eV_S, \] (24)

where \( d \) and \( e \) are either locally derived scaling coefficients or are taken directly from the Castagna et al. (1985) paper, in which case \( d = 1360 \, \text{m/s} \) and \( e = 1.16 \). Note that this equation was also used by Castagna et al. (1998) to show that the cluster of points at roughly \(-45^\circ\) on the crossplot of Figure 11 corresponds to wet sands and shales.

Another important AVO indicator is the fluid factor \( \Delta F \), which is the fluid deviation away from the mudrock line defined in equation 24 (Smith and Gidlow, 1987; Fatti et al., 1994). As shown in equation 25, the fluid factor is the weighted difference between the extracted acoustic impedance and shear impedance reflectivities.

Because this is a new reflectivity, we will call it \( R_F \) (notice the use of capital \( F \) to differentiate this term from the fluid term that came out of equation 17, which we denoted by small \( f \)). In our notation, the fluid factor can be written

\[ R_F = R_{AI} - 1.16 \bar{k}_{\text{sat}} R_{SI}, \] (25)

where the value of 1.16 has been used for the coefficient \( e \) in equation 24.

Figure 13, from Russell et al. (2011), shows the fluid factor section computed from the CMP gathers in Figure 4. The color scale has been normalized between +1 and −1, where red shows an increase in the fluid response and blue shows a decrease. Note the clearly defined fluid anomaly between 620 and 640 ms.

For the extraction of the fluid reflectivity in equation 17, we need an estimate of \( \bar{k}_{\text{dry}} \), the dry S to P-wave velocity ratio squared, which can be obtained deterministically by estimating the dry rock ratio using Biot-Gassmann analysis, or empirically by finding the optimum rotation on a well log crossplot or an inverted \( f\rho - \mu\rho \) crossplot (discussed in the next section). Note that this last approach moves us from reflectivity to impedance, which will also be discussed in the next section. As a final illustration of reflectivity methods, Figure 14 shows the extraction of the fluid modulus from the gathers of Figure 4 using a \( \bar{k}_{\text{dry}} \) value of 1/2.333, or 0.429, which corresponds to a clean, highly porous sand.

**Impedance methods**

The inversion of seismic data to produce an estimate of acoustic impedance was first proposed by Lindseth (1979). The basic concept was to recursively invert equation 3 to give

\[ AI_{i+1} = AI_i \left[ \frac{1 + R_{AI}}{1 - R_{AI}} \right]. \] (26)

If our seismic trace consisted of the full-bandwidth, noise-free reflectivity shown in Figure 1, the recursive application of equation 26 would recover the complete impedance profile shown in Figure 1, assuming that our estimate of the starting impedance was accurate. However, the fact that the seismic trace is band limited with the seismic wavelet as shown in equation 2 means that only the impedance within the seismic bandwidth can be recovered. In many cases, this is sufficient and relative impedance inversion as implemented by equation 26 is commonly applied. It is sometimes preferable to produce a full-bandwidth estimate of the impedance from the seismic volume, which involves incorporating the missing frequency parts of the impedance from some external model. The model can be created by inserting well log impedance profiles at their correct position on a seismic volume and interpolating the imped-

![Figure 11. AVO intercept versus gradient crossplot of a zone of the intercept and gradient sections taken from a 100-ms window around the event picked in Figure 10, between CMPs 300 and 360. The regions captured by the red and blue polygonal zones are shown in Figure 12.](image1)

![Figure 12. The captured red and blue polygonal crossplot zones from Figure 11, where the red zone shows the top of the gas sand and the blue zone shows the base of the gas sand.](image2)
anes while guiding the structure using picked seismic events.

Once the model is built, the simplest approach to extracting the low frequencies is to apply a low pass filter to the model and add these low-frequency impedances to the recursively inverted traces. However, several other approaches have been developed over the years (Oldenburg et al., 1983; Russell and Hampson, 1991), the details of which are outside the scope of this paper. The concept behind all amplitude inversion methods can be captured in Figure 15, in which we constrain some type of seismic volume with the information from a model volume using a particular inversion algorithm.

The fundamental question before starting an inversion are therefore: What model volume and seismic volume should we use? Traditionally, acoustic impedance inversion has been done using the stacked seismic volume, but as explained earlier, this is actually incorrect, because the stack is an average over all angles, whereas the acoustic impedance reflectivity is the normal incidence P-wave reflectivity. Thus, a more accurate input to acoustic impedance inversion would be the band-limited acoustic impedance reflectivity \( S_{AI} = WR_{AI} \) estimated from the seismic gathers using equation 23 with the coefficients from equation 8 as input. Likewise, the other two outputs from this reflectivity extraction, \( S_{SI} = WR_{SI} \) and \( S_D = WR_D \), can be used as inputs for shear impedance and density inversion. The model volumes in Figure 15 would be derived from the P-impedance, S-impedance, and density values from the well log measurements.

Any inversion algorithm can be used in this process, as long as the model volume used in Figure 15 is the parameter to be inverted for and the seismic volume is the band-limited reflectivity which has a physical link to this earth parameter. An alternate approach is to use simultaneous inversion (Buland and Omre, 2003; Hampson et al., 2005) in which equation 8 is still used as the basis for the band-limited reflectivity extraction, but rock physics constraints are included. In this approach, the extraction of the band-limited reflectivities is not performed directly and the inversion input consists of angle gathers, a P-impedance model, and relationships between P-impedance, S-impedance, and density.

Once the impedance inversion has been done, there are several approaches that can be used to interpret the results. A straightforward method is to divide the P-impedance volume by the S-impedance volume to create a \( V_P/V_S \) ratio volume and to crossplot this against P-impedance using the rock physics template (RPT) approach (Odegaard and Avseth, 2003). Anomalous fluid zones can then be picked on the crossplot and highlighted on the seismic volume. Figure 16 shows the P-impedance inversion of the gathers in Figure 4 using a simultaneous inversion approach, and Figure 17 shows the \( V_P/V_S \) ratio that was derived by dividing the P-impedance inversion of these gathers by the S-impedance inversion. Figure 18 then shows the crossplot of the two volumes, where a polygonal zone has been identified which has low P-impedance and \( V_P/V_S \) ratio. Figure 19 shows the region of the seismic stack that corresponds to the polygonal zone in Figure 18. This region correlates very well with the known gas sand.

Once the gathers have been inverted to P- and S-impedance, the results can be transformed to lambda-rho (\( \lambda \rho \)) and mu-rho (\( \mu \rho \)) volumes (Goodway et al., 1997), where

![Figure 13](image1.png)  
**Figure 13.** The fluid-factor (\( \Delta F \)) section for the gathers shown in Figure 4, where the picked event starting at 640 ms on the left of the section represents the transition from the top to the base of the gas sand. As in the intercept times gradient product section of Figure 8, note the good definition of the gas sand between CMPs 316 and 346.

![Figure 14](image2.png)  
**Figure 14.** The fluid reflectivity (\( R_f \)) extraction for the gathers shown in Figure 4, where the picked event starting at 640 ms on the left of the section represents the transition from the top to the base of the gas sand. Note the decrease in the fluid reflectivity at the top of the gas sand, and the good definition of the gas sand between CDPs 316 and 346.

![Figure 15](image3.png)  
**Figure 15.** The basic flow for all seismic inversion methods.
\[
\lambda \rho = A I^2 - 2 S I^2, \tag{27}
\]

and
\[
\mu \rho = S I^2. \tag{28}
\]

These volumes can also be crossplotted and anomalous fluid zones picked on the crossplot highlighted on the seismic volume, as in the RPT approach. Theoretically, a crossplot of \(\mu \rho\) against \(\lambda \rho\) should show a vertical separation between the gas sands and the wet sands. That is, gas sands should plot on the low side of the \(\lambda \rho\) axis. A generalization of the \(\lambda \rho\) concept was proposed by Russell et al. (2003), in which a fluid volume is created using a more general form of equation 28 given by
\[
f \rho = A I^2 - \bar{k}^{-1}_{\text{dry}} S I^2, \tag{29}
\]

where \(f \rho\) is the full bandwidth fluid term which is equivalent to the fluid term \(f\) defined in equation 18 multiplied by the density. Notice that, when \(\bar{k}^{-1}_{\text{dry}}\) is equal to 2, equation 29 reduces to equation 27 and \(f \rho\) reduces to \(\lambda \rho\). As in the three-term generalized AVO fluid equation (equation 17), this approach uses Biot-Gassmann poroelasticity theory, so the fluid term \(f \rho\) in equation 29 could be found by inverting the fluid reflectivity \(R_f\) in equation 17 and multiplying this by the inverted density. The advantage of extracting the reflectivity and inverting it is that the inverted results do not need to be squared, as in equation 29. By squaring the impedance data, the noise is also squared, which could result in instability in some data areas. Of course, there are also potential problems with the nonsquared inversion methods, most notably the fact that the low-frequency trend cannot be estimated from the seismic data due to bandwidth limitations and also that the density term is very sensitive to noise and the limited angle range found in many surveys. One approach to improving the density estimate is to use simultaneous inversion (Hampson et al., 2005), in which rock physics constraints are built into the inversion algorithm. A final point to note is that equation 29 also gives us an empirical way to estimate \(k_{\text{dry}}\), by changing this value until the crossplot of \(\mu \rho\) versus \(f \rho\) shows the best vertical separation between the gas sands and the wet sands.

Yet another approach to using the inverted P- and S-impedance results, which also avoids squaring the impedance, was proposed by Quackenbush et al.

![Figure 16](image1.png)

**Figure 16.** P-impedance estimation from the gathers of Figure 4 using the simultaneous inversion approach.

![Figure 17](image2.png)

**Figure 17.** The \(V_P/V_S\) ratio estimation from the gathers of Figure 4 using the simultaneous inversion approach.

![Figure 18](image3.png)

**Figure 18.** Crossplot of \(V_P/V_S\) ratio inversion from Figure 17 with P-impedance inversion of Figure 16 over a 100 ms extraction window around the gas sand zone between CMPs 300 and 360. The polygonal zone highlights the gas sand, which has low impedance and \(V_P/V_S\) ratio.

![Figure 19](image4.png)

**Figure 19.** The red zone shows the extent of the polygonal zone in Figure 18 mapped back onto the seismic section. The wiggle traces show the P-impedance.
Elastic impedance (EI)

So far, we have discussed several different types of impedances: acoustic, shear, Poisson, lambda-rho, and mu-rho, which are derived from acoustic and shear impedance. Connolly (1999) has proposed a new type of impedance which can be thought of as the impedance underlying the angle dependent reflectivity derived by Aki and Richards (2002), which he calls EI and defines as

\[ EI(\theta) = V_p^a V_b^b \rho^c, \]  

(31)

where \( a = 1 + \tan^2 \theta \), \( b = -8k_{sat} \sin^2 \theta \), and \( c = 1 - 4k_{sat} \sin^2 \theta \). Note that the \( k_{sat} \) term in equation 31 is a constant which is averaged over the zone of interest. If the coefficients \( a \), \( b \), and \( c \) look familiar, it is because we first saw them as the scaling factors for the Aki-Richards equation (equation 7). To see why these two equations are related, we first take the logarithm of equation 31, to get

\[ \ln EI(\theta) = a \ln V_p + b \ln V_s + c \ln \rho. \]  

(32)

Next, we take the differentials of the parameters in equation 32 to get

\[ \Delta \ln EI(\theta) = a \Delta \ln V_p + b \Delta \ln V_s + c \Delta \ln \rho. \]  

(33)

Finally, we use the logarithmic approximation of equation 9 (and also a scale factor of 1/2 to keep the reflectivity definitions consistent) to get

\[ R_{EI}(\theta) = \frac{\Delta EI(\theta)}{2EI(\theta)} = a \Delta \ln V_p + b \Delta \ln V_s + c \Delta \ln \rho, \]  

(34)

which gets us back to the original Aki-Richards equation. I have introduced the term \( R_{EI}(\theta) \) instead of \( R_{PP}(\theta) \) to continue making the reflectivity subscript conform to the underlying impedance model. Also note that the EI reflectivity defined in equation 34 has the same form as the acoustic and shear impedance forms seen in equations 4 and 6, which is the difference of the impedance across a boundary divided by twice its average. Regardless of what name we apply to the EI reflectivity, in its band-limited form it corresponds to a trace taken from the seismic angle gather at an angle of \( \theta \). That is, \( S_{EI}(\theta) = WR_{EI}(\theta) \) is the band-limited trace at angle \( \theta \). Thus, to create the inverted EI volume, we use the methodology suggested in Figure 15, where a constant angle stack is the seismic input, the model volume is derived from equation 34, and your preferred inversion algorithm is used.

As pointed out by Whitcombe (2002), the limitation of EI is its variable dimensionality, an observation which was first made by VerWest et al. (2000). That is, as the angle of incidence increases, the value of EI goes down. To correct for this problem, Whitcombe (2002) introduces the following normalized version of EI

\[ EI(\theta) = V_{p0} \rho_0 \left[ \left( \frac{V_p}{V_{p0}} \right)^a \left( \frac{V_S}{V_{s0}} \right)^b \left( \frac{\rho}{\rho_0} \right)^c \right], \]  

(35)

where \( V_{p0}, V_{s0}, \) and \( \rho \) are reference constants that are computed by averaging the P- and S-wave velocity and density over the zone of interest. This scaled version of the EI inversion can be computed by using equation 35 to create the model volume that is input to the EI inversion. Whitcombe et al. (2002) extends this scaling to the concept of EEI. In EEI, the Aki-Richards equation (equation 12) is modified by dropping the third term and transforming the \( b \) coefficient from \( \sin^2 \theta \) to \( \tan \chi \), where \( \chi \) represents a rotation angle in intercept-gradient space. The resulting equation is then multiplied by \( \cos \chi \), giving the following form of the EEI reflectivity

\[ R_{EEI}(\chi) = R_{AI} \cos \chi + R_{GI} \sin \chi. \]  

(36)

The transformation from \( \theta \) to \( \chi \) also allows us to rewrite the scaled EI model (equation 35) as a function of \( \chi \), giving the following EEI model

\[ EEI(\chi) = V_{p0} \rho_0 \left[ \left( \frac{V_p}{V_{p0}} \right)^p \left( \frac{V_S}{V_{s0}} \right)^q \left( \frac{\rho}{\rho_0} \right)^r \right], \]  

(37)

where \( p = \cos \chi + \sin \chi \), \( q = -8k_{sat} \sin \chi \), and \( r = \cos \chi - 4k_{sat} \sin \chi \). EEI inversion can be implemented using the flowchart in Figure 15 by using equation 37 to create the model volume in equation 36 to create the seismic volume. As Whitcombe et al. (2002) show, the interpretational advantage of EEI is that at different \( \chi \) angles it correlates to different physical parameters such as bulk modulus, shear modulus, lambda, \( V_p/V_s \) ratio, etc. Two special angles are 0° and 90°, at which we see from equation 36 that the EEI equation reduces to \( R_{AI} \) and \( R_{GI} \), respectively. Thus, EEI (0°) is identical to acoustic impedance and EEI (90°) can be called gradient impedance, or GI, which motivated the subscript to the gradient reflectivity when it was introduced earlier.

The optimum \( \chi \) angle is found by correlation between EEI (\( \chi \)) and a parameter of interest. As an example of EEI, Figure 20 shows the correlation \( V_p/V_s \) ratio from
the logs in Figure 7 for our Colony example with EEI ($\chi$) at various $\chi$ angles. From Figure 20, we see that the best correlation value is found at an angle of $56^\circ$. Referring to Figure 7, note that the $V_P/V_S$ ratio and EEI ($56^\circ$) have been plotted side by side in that figure and that the two curves look very similar except for the scaling between them.

Figure 21 shows the computation of the EEI reflectivity at the computed angle of $56^\circ$ from the correlation in Figure 20. This is the seismic section that is inverted to give the final EEI result. Finally, Figure 22 shows the computation of $V_P/V_S$ ratio for our gas sand example using the EEI log model from Figure 7 and seismic model from Figure 21 with a rotation angle of $56^\circ$. Because EEI does not scale directly to the parameter of interest, the results in Figure 22 have been scaled to $V_P/V_S$ ratio values.

**Extension to anisotropic media**

So far in this tutorial, we have assumed only the isotropic case. More recently, AVO and prestack inversion have been extended to anisotropic reservoirs, using techniques such as amplitude versus azimuth (AVAz). In this tutorial, I will focus only on vertically transversely isotropic (VTI) and horizontally transversely isotropic (HTI) media (Thomsen, 1993; Rüger, 1997, 1998), but the analysis can also be extended to orthorhombic media (Tsvankin, 1997; Rüger, 2002). Figure 23, from Rüger (1997), shows schematically the difference between VTI and HTI media, where Figure 23a shows that a VTI medium can be modeled by parallel sheets in the horizontal, or $x_1$, direction, and an HTI medium can be modeled by parallel sheets in the vertical, or $x_3$, direction, and Figure 23b shows the two angles needed in the following equations. For VTI media, we will need only the incidence angle $\theta$, but in HTI media we will require the incidence angle $\theta$ and the azimuth angle $\phi$.

For VTI media, Rüger (1997) expanded previous work by Thomsen (1993) to show that we could write an extended version of equation 12 as

$$ R_{VTI}(\theta) = R_{AI} + \left( R_{GI} + \frac{\Delta \delta}{2} \right) \sin^2 \theta $$
$$ + \left( R_{VP} + \frac{\Delta \varepsilon}{2} \right) \sin^2 \theta \tan^2 \theta, \quad (38) $$

where $\Delta \delta$ and $\Delta \varepsilon$ are the change in Thomsen’s $\delta$ and $\varepsilon$ parameters (Thomsen, 1986) across the layer interface. The presence of VTI anisotropy will affect the AVO reflectivity response and will depend on the magnitude and sign of the change in the anisotropic parameters between layers. However, when the terms in brackets are extracted using equation 21, these anisotropic terms cannot be separated into their isotropic and anisotropic components. Figure 24 shows a set of modeled isotropic and anisotropic AVO curves over Class 1, 2, and 3 gas and wet sands, adapted from Blangy (1994). The three Classes were proposed by Rutherford and Williams (1989), and are separated based on their acoustic impedance contrast with respect to the surrounding shale, where a Class 1 sand has a higher impedance, a Class 2 sand has a near zero change in impedance, and a Class 3 sand has a lower impedance with respect to the shale. Although Rutherford and Williams (1989) focus only on gas sands, Blangy (1994) ex-

![Figure 20](image)

**Figure 20.** The correlation between EEI($\chi$) and $V_P/V_S$ ratio from the logs shown in Figure 7. Notice that the best correlation is at an angle of $56^\circ$.

![Figure 21](image)

**Figure 21.** The application of the rotation angle of $56^\circ$ using equation 36, that is $R_{EEI}(56^\circ) = R_{AI} \cos(56^\circ) + R_{GI} \sin(56^\circ)$. This is the seismic section that is inverted to give the EEI result.

![Figure 22](image)

**Figure 22.** The computation of $V_P/V_S$ ratio for our gas sand example using the EEI log model from Figure 7 and seismic model from Figure 21 with a rotation angle of $56^\circ$. 

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tends the models to wet sands to illustrate the effect of an anisotropic shale above the sand. In each model, the gas sand is isotropic but the shale above the gas sand exhibits VTI anisotropy with \( \delta = 0.15 \) and \( \varepsilon = 0.30 \). Table 1 shows the complete set of parameters for the models.

For the gas case, Figure 24a shows that the effect of anisotropy is to increase the AVO effect more than would be expected. For the wet case, shown in Figure 24b, the effect of anisotropy is to make the wet response look like a gas response. Although it could be argued that these models are slightly unrealistic due to the fact that the P-wave velocity does not change between the wet and gas cases, nonetheless they do show that the effect of an anisotropic shale over an isotropic sand is a situation that should be modeled before undertaking any drilling program based on the AVO results.

For HTI reservoirs, the reflectivity is now dependent on the average incident angle \( \theta \) and the azimuth \( \phi \) (see Figure 23b), as shown in the following equation from Rüger (1997):

\[
R_{\text{HTI}}(\theta, \phi) = R_{\text{AI}} + (R_{\text{GI}} + R_{\text{HTIB}} \cos^2 \phi) \sin^2 \theta + (R_{\text{VI}} + R_{\text{HTIC}} \cos^2 \phi) \sin^2 \theta \tan^2 \theta, \tag{39}
\]

where \( R_{\text{HTIB}} = \frac{\Delta \delta_{(V)}}{2} + 8k_{\text{sat}} \frac{\Delta \phi_{(S)}}{2} \), \( R_{\text{HTIC}} = \Delta \delta_{(V)} \sin^2 \phi - \Delta \delta_{(S)} \), \( \Delta \delta_{(V)} \) and \( \Delta \delta_{(S)} \) are Thomsen’s delta and epsilon parameters in HTI media, \( \gamma \) is Thomsen’s gamma parameter, and \( k_{\text{sat}} = \left( \frac{\rho}{\rho_{\text{sat}}} \right)^2 \). Again, I have changed the normal terminology, in which \( R_{\text{HTIB}} \) is usually called \( B_{\text{ani}} \), to represent the fact that this term is an HTI reflectivity associated with the second, or \( B \), term in the AVO expression.

Although it is theoretically possible to extract \( R_{\text{HTIB}} \) and \( R_{\text{HTIC}} \) if we have a full range of azimuths and incidence angles, in practice only \( R_{\text{HTIB}} \) is within the noise range of typical recorded gathers. Thus, equation 21 can be modified by dropping the third term in equation 39, but dividing the second term into two parts, as shown here:

\[
\begin{bmatrix}
R_{\text{PP}}(\theta_1, \phi_1) \\
R_{\text{PP}}(\theta_2, \phi_1) \\
\vdots \\
R_{\text{PP}}(\theta_N, \phi_M)
\end{bmatrix}
= 
\begin{bmatrix}
1 \sin^2 \theta_1 & \sin^2 \theta_1 \cos^2 \phi_1 \\
1 \sin^2 \theta_1 & \sin^2 \theta_1 \cos^2 \phi_1 \\
\vdots & \vdots \\
1 \sin^2 \theta_1 & \sin^2 \theta_M \cos^2 \phi_M
\end{bmatrix}
\begin{bmatrix}
R_{\text{AI}} \\
R_{\text{GI}} \\
R_{\text{HTIB}}
\end{bmatrix},
\tag{40}
\]

which can be inverted for \( R_{\text{AI}}, R_{\text{GI}}, \) and \( R_{\text{HTIB}} \). As shown by Bakulin et al. (2000), \( R_{\text{HTIB}} \), or \( B_{\text{ani}} \) as they refer to this term, is proportional to the fracture density. Therefore, by extracting the anisotropy gradient in addition to the isotropic intercept and gradient using the inverse of equation 40, we can estimate the fracture density. Note that in equations 39 and 40, we have used \( \phi \) to denote the known azimuth. Another unknown is the angle \( \beta \), which is the angle between the chosen zero azimuth direction and the direction of the fractures themselves. This unknown can be introduced into equation 39 by replacing \( \cos^2 \phi \) with \( \cos^2 (\phi - \beta) \), which makes the equation nonlinear. As shown by Jenner (2002), the angle \( \beta \) can also be estimated by reparameterizing the problem. A similar technique was used by Gray and Head (2000) to estimate fracture direction and density in the Man-

**Figure 23.** Schematic models for VTI and HTI media, where (a) shows VTI medium can be modeled by parallel sheets in the horizontal direction, an HTI medium can be modeled by parallel sheets in the vertical direction, and (b) illustrates that VTI media are defined using only incidence angle \( \theta \) but HTI media require incidence angle \( \theta \) and azimuth angle \( \phi \) (from Rüger, 1997, 1998).

**Figure 24.** Modeled AVO responses for a shale over (a) a gas sand and (b) a wet sand. The solid line shows the response for an isotropic shale and the dashed line shows the response for a VTI shale with \( \delta = 0.15 \) and \( \varepsilon = 0.30 \) (Adapted from Blangy, 1994).
Table 1. The parameters for the Type (or Class, which is the more common terminology) I, II, and III sands for the anisotropic study shown in Figure 24 (from Blangy, 1994).

<table>
<thead>
<tr>
<th>Material properties</th>
<th>( V_p ) (m/s)</th>
<th>( V_S ) (m/s)</th>
<th>Density</th>
<th>( \delta )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overlying shale</td>
<td>3300</td>
<td>1700</td>
<td>2.35</td>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>Type I gas sand</td>
<td>4200</td>
<td>2700</td>
<td>2.35</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Type I water sand</td>
<td>4200</td>
<td>2100</td>
<td>2.45</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Overlying shale</td>
<td>2896</td>
<td>1402</td>
<td>2.25</td>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>Type II gas sand</td>
<td>3322</td>
<td>2215</td>
<td>2.00</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Type II water sand</td>
<td>3322</td>
<td>1402</td>
<td>2.25</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Overlying shale</td>
<td>2307</td>
<td>1108</td>
<td>2.15</td>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>Type III gas sand</td>
<td>1951</td>
<td>1301</td>
<td>1.95</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Type III water sand</td>
<td>1951</td>
<td>930</td>
<td>2.20</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 25. The estimation of fracture strike and density using the inverse of equation 40 (modified by the inclusion of fracture orientation angle \( \beta \)) in the Manderson Field, Wyoming (adapted from Gray and Head, 2000).

As can be seen by the discussion in the previous paragraph, it is impossible to separate the quality of the prestack analysis results from the quality of the acquisition and processing of the data itself. At the acquisition phase, it is important to get the largest angular range possible, as well as a good range of azimuths if AVAz is the goal. At the processing stage the key steps to consider are true amplitude preservation, good random and coherent noise elimination, high-frequency signal recovery, and an imaging technique that is not only kinematically correct but also preserves amplitudes. For very highly structural data, this last step may be unobtainable and the use of prestack AVO for inversion in very complex areas like subsalt plays can be problematic. Of course, each of the topics just mentioned could lead to its own tutorial.
Also, there is no clear consensus on how to achieve many of these processing goals, so the interpreter is encouraged to communicate closely with the processor, and produce the necessary QC plots after every step.

Finally, the importance of modeling cannot be overstressed. There is no point in getting excited about a red “blob” on your AVO results if you have no idea what that red blob means. It is important to perform many modeling steps, including fluid replacement with reasonable rock physics parameters, synthetic modeling with the option to add events other than just primaries (e.g., converted waves and multiples), and realistic wavelet generation. In this tutorial, we have focused on isotropic Biot-Gassmann fluid replacement modeling, but there are many different techniques available to the interpreter.

The key thing that I have tried to emphasize in this tutorial is that virtually all of the techniques used in prestack amplitude analysis are derived from a three-(or two-) term linearized approximation to the Zoeppritz equations, and that they can be classified as either reflectivity methods or impedance methods. The advantage of staying in the seismic reflectivity domain is that this is the “natural” domain in which seismic data is recorded. However, for a geologic analysis of earth properties, the low-frequency trend of the various earth parameters is often preferred, and this involves the inversion of the seismic reflectivity. If inversion is your preferred method of analysis, it is important to understand how the low-frequency component is being incorporated into the final result and also to understand the possible pitfalls of each different method. Every method discussed in this tutorial has value, and it is only by comparing the methods on your particular data set, and calibrating the results against models built from well logs, that you will be able to make an informed decision on a particular method’s usefulness when compared to all the others.

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Appendix A

The Zoeppritz equations and normal incidence

Zoeppritz (1919) showed how to compute the amplitudes of the reflected and transmitted waves shown in Figure 3 using the conservation of stress and displacement across the layer boundary. This gives a forward model with four equations and four unknowns (the amplitudes) which can be arranged in matrix notation as

\[
\begin{bmatrix}
\sin \theta_1 & \cos \phi_1 & \sin \theta_2 & \cos \phi_2 \\
\cos \theta_1 & \sin \phi_1 & \cos \theta_2 & -\sin \phi_2 \\
\sin \phi & \frac{V_{S1} \cos 2\phi}{\rho_{1} V_{P1}} & \sin \phi & \frac{V_{S1} \cos 2\phi}{\rho_{1} V_{P1}} \\
-\cos \phi & \frac{V_{S1} \sin 2\phi}{\rho_{1} V_{P1}} & \cos \phi & \frac{V_{S1} \sin 2\phi}{\rho_{1} V_{P1}}
\end{bmatrix}
\times
\begin{bmatrix}
R_{PP} (\theta_1) \\
R_{PS} (\theta_1) \\
T_{PP} (\theta_1) \\
T_{PS} (\theta_1)
\end{bmatrix}
= 
\begin{bmatrix}
\sin \theta_1 \\
\cos \theta_1 \\
\sin 2\theta_1 \\
\cos 2\phi_1
\end{bmatrix}.
\]

(A-1)

Inverting the matrix form of the Zoeppritz equations in A-1 gives us the exact amplitudes as a function of angle

\[
\begin{bmatrix}
R_{PP} (\theta_1) \\
R_{PS} (\theta_1) \\
T_{PP} (\theta_1) \\
T_{PS} (\theta_1)
\end{bmatrix}^{-1} =
\begin{bmatrix}
-\sin \theta_1 & -\cos \phi_1 & \sin \theta_2 & \cos \phi_2 \\
\cos \theta_1 & -\sin \phi_1 & \cos \theta_2 & -\sin \phi_2 \\
\sin 2\theta_1 & \frac{V_{S1} \cos 2\phi}{\rho_{1} V_{P1}} & \sin 2\theta_1 & \frac{V_{S1} \cos 2\phi}{\rho_{1} V_{P1}} \\
-\cos 2\phi_1 & \frac{V_{S1} \sin 2\phi}{\rho_{1} V_{P1}} & \cos 2\phi_1 & -\frac{V_{S1} \sin 2\phi}{\rho_{1} V_{P1}}
\end{bmatrix}
\times
\begin{bmatrix}
\sin \theta_1 \\
\cos \theta_1 \\
\sin 2\theta_1 \\
\cos 2\phi_1
\end{bmatrix}.
\]

(A-2)

Note that the matrix in equations A-1 and A-2 is a simplification of the general solution given in Aki and Richards (2002) (equation 5-37, page 141) in which all possible combinations of input P- and SV-plane waves are solved for. Solving equation A-2 will lead to a complicated set of equations, and we discuss in the tutorial a linearized form of this inversion for the R_{PP} term, called the Aki-Richards approximation. However, the normal incidence case leads to the following simplification, given as follows

\[
\begin{bmatrix}
R_{PP}(0^\circ) \\
R_{PS}(0^\circ) \\
T_{PP}(0^\circ) \\
T_{PS}(0^\circ)
\end{bmatrix} =
\begin{bmatrix}
0 & -1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & V_{S1} & 0 & \frac{V_{S1} V_{P1}}{\rho_{1} V_{P1}} \\
-1 & 0 & \frac{V_{S1} V_{P1}}{\rho_{1} V_{P1}} & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
0 \\
1 \\
0 \\
1
\end{bmatrix},
\]

(A-3)

with solution

\[
\begin{bmatrix}
R_{PP}(0^\circ) \\
R_{PS}(0^\circ) \\
T_{PP}(0^\circ) \\
T_{PS}(0^\circ)
\end{bmatrix} =
\begin{bmatrix}
0 \\
1 \\
0 \\
1
\end{bmatrix}.
\]
\[
\begin{bmatrix}
R_{PP}(0^\circ) \\
R_{PS}(0^\circ) \\
T_{PP}(0^\circ) \\
T_{PS}(0^\circ)
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{\rho V_{P2}}{\rho V_{P2} + \rho V_{P1}} & \frac{\rho V_{P1}}{\rho V_{P2} + \rho V_{P1}} & 0 \\
\frac{\rho V_{S1} V_{P1}}{\rho V_{S1} + \rho V_{S2}} & 0 & 0 & \frac{-\rho V_{S1} V_{P1}}{\rho V_{S1} + \rho V_{S2}} \\
\frac{\rho V_{S2} V_{P1}}{\rho V_{S1} + \rho V_{S2}} & 0 & 0 & \frac{-\rho V_{S2} V_{P1}}{\rho V_{S1} + \rho V_{S2}} \\
\frac{\rho V_{S1} V_{P2}}{\rho V_{S1} + \rho V_{S2}} & 0 & 0 & \frac{-\rho V_{S1} V_{P2}}{\rho V_{S1} + \rho V_{S2}}
\end{bmatrix}
\begin{bmatrix}
0 \\
1 \\
0 \\
1
\end{bmatrix}.
\]

(A-4)

The normal incidence reflection and transmission coefficients are therefore given as

\[
R_{S0} = T_{S0} = 0, \quad R_{P0} = \frac{\rho V_{P2} - \rho V_{P1}}{\rho V_{P2} + \rho V_{P1}},
\]

\[
T_{P0} = \frac{2\rho V_{P1}}{\rho V_{P2} + \rho V_{P1}} = 1 - R_{P0}.
\]

References


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Brian Russell received a B.S. (1975) from the University of Saskatchewan, an M.S. (1978) from Durham University, and a Ph.D. (2002) from the University of Calgary, all in geophysics. He is vice president, software for Hampson-Russell, A CGG Company, a CGG Fellow, and an adjunct professor in the Department of Geoscience at the University of Calgary. His research interests include rock physics, seismic inversion, and seismic attribute analysis. He is a past president and honorary member of SEG and CSEG, a member of EAGE, and received the SEG Cecil Green Enterprise Award in 1996.