This document discusses the rock physics templates (RPT), and how we have implemented the RPT approach in the Hampson-Russell software release HRS-9.

The method is based on theory proposed by Dvorkin and Nur (1996) and Ødegaard and Avseth (2003).

We will show the equations and the concepts behind the theory using graphical methods.

We will also use log and inverted seismic data from the Colony sand of central Alberta to illustrate these methods.
Ødegaard and Avseth (2003) proposed a technique they called the rock physics template (RPT), based on theory by Dvorkin and Nur (1996), in which the fluid and mineralogical content of a reservoir could be estimated on a crossplot of $V_P/V_S$ ratio against acoustic impedance, as shown here.
The Ødegaard/Avseth/Dvorkin/Nur RPT

- The Ødegaard and Avseth RPT involves computing the $V_P/V_S$ ratio and $P$-Impedance of various rock types based on their mineralogy, porosity, fluid type, pressure and grain contacts.

- This involves four steps:
  - Computing the moduli of the dry rock frame for a given porosity/pressure using the Hertz-Mindlin theory.
  - Computing the moduli of the dry rock frame over a range of porosities using the lower Hashin-Shtrikman bound.
  - Performing fluid substitution with the Gassmann equations.
  - Computing the density, P-wave velocity, S-wave velocity for each scenario and cross-plotting $V_P/V_S$ ratio versus $P$-impedance.
Computing the initial dry rock values

The dry rock values can be computed in various ways:

- Using Hertz-Mindlin contact theory.
- Using the Gregory-Pickett method (the Lower Hashin-Shtrikman bound).
- Using Dvorkin and Nur’s cemented sand model.
Hertz-Mindlin contact theory requires a knowledge of:

- The mineral bulk and shear moduli of the dry rock skeleton.
- The pressure acting on the rock.
- The porosity of the rock.
- And the number of contacts per grain, as we assume the rock consists of spherical grains.

An example of the number of contacts per grain, $n$, in a three-dimensional packing of spheres, is shown on the right.
Hertz-Mindlin contact theory

- Using the normal compression of two identical grains shown on the left of this figure, Hertz computed the effective bulk modulus.

- By adding the tangential force shown on the right, Mindlin computed the effective shear modulus.

Normal (left) and tangential (right) displacement in a two-particle system. (From Mavko et al., 2009)
Hertz-Mindlin contact theory

- The final terms needed in Hertz-Mindlin (HM) contact theory are the shear modulus and Poisson’s ratio of the mineral grains, the pressure and the critical porosity, giving:

\[
K_{HM} = \left[ \frac{n^2 (1 - \phi_c)^2 \mu_m^2}{18 \pi^2 (1 - \nu_m)^2} P \right]^{\frac{1}{3}}, \quad \mu_{HM} = \frac{4 - 4\nu_m}{5(2 - \nu_m)} \left[ \frac{3n^2 (1 - \phi_c)^2 \mu_m^2}{2\pi^2 (1 - \nu_m)^2} P \right]^{\frac{1}{3}},
\]

where: \( P \) = confining pressure, \( K_m \) = mineral bulk modulus, \( \mu_m \) = mineral shear modulus, \( \nu_m \) = mineral Poisson's ratio

\( \phi_c \) = high porosity end-member, and \( n \) = contacts per grain.

- We get the mineral bulk modulus by noting:

\[
\nu_m = \frac{3K_m - 2\mu_m}{2(3K_m + \mu_m)} \Rightarrow K_m = \frac{2\mu_m (1 + \nu_m)}{3 - 6\nu_m}
\]
The lower Hashin-Shtrikman bound

- We extend the Hertz-Mindlin values of the frame moduli (for a given critical porosity) to all porosities by using the lower Hashin-Shtrikman bound:

\[
K_{\text{dry}}(\phi) = \left[ \frac{\phi / \phi_c}{K_{\text{HM}} + (4/3)\mu_{\text{HM}}} + \frac{1 - \phi / \phi_c}{K_m + (4/3)\mu_{\text{HM}}} \right]^{-1} - \frac{4}{3}\mu_{\text{HM}}
\]

\[
\mu_{\text{dry}}(\phi) = \left[ \frac{\phi / \phi_c}{\mu_{\text{HM}} + z} + \frac{1 - \phi / \phi_c}{\mu_m + z} \right]^{-1} - z, \quad z = \frac{\mu_{\text{HM}}}{6}\left( \frac{9K_{\text{HM}} + 8\mu_{\text{HM}}}{K_{\text{HM}} + 2\mu_{\text{HM}}} \right),
\]

and \( \phi = \) porosity.

- When \( \phi = \phi_c \) we get the HM result: \( K_{\text{dry}} = K_{\text{HM}} \) and \( \mu_{\text{dry}} = \mu_{\text{HM}} \).

- When \( \phi = 0 \) we get the mineral value: \( K_{\text{dry}} = K_m \) and \( \mu_{\text{dry}} = \mu_m \).
The cemented sand model

Dvorkin and Nur’s model requires modulus values for the mineral and the cement, the porosity of both components, how the cement is deposited and the number of contacts.

This complex model for cemented sands computes the effective bulk and shear moduli by:

\[ K_{\text{eff}} = \frac{1}{6} n(1 - \phi_c) \left( K_{\text{cm}} + \frac{4}{3} \mu_{\text{cm}} \right) S_n, \text{ and } \mu_{\text{eff}} = \frac{3}{5} K_{\text{eff}} + \frac{3}{20} n(1 - \phi_c) \mu_{\text{cm}} S_\tau, \]

\( S_n \) and \( S_\tau \) = scaling functions based on \( \mu_{\text{cm}}, \nu_{\text{cm}}, \mu_{\text{grain}}, \nu_{\text{grain}} \) and \( \alpha \),

\[ \alpha = 2 \left[ \frac{\phi_c - \phi}{3n(1 - \phi_c)} \right]^{0.25} \text{ for cement at grain contacts, and } \alpha = \left[ \frac{2(\phi_c - \phi)}{3(1 - \phi_c)} \right]^{0.5} \]

for cement deposited evenly on grain surface, \( \phi = \) por. of cemented sand.
The Gregory-Pickett method is a simpler method for the moduli, proposed by Hilterman (1983).

- In this method, the dry rock bulk modulus is estimated at a known porosity by assuming a dry rock Poisson’s ratio.
- Next, the pore space stiffness is computed from:

$$\frac{1}{K_{dry}} = \frac{1}{K_m} + \frac{\phi}{K_\phi}$$

Where $K_{dry} = \text{dry rock bulk modulus}$, $\phi = \text{porosity}$, $K_m = \text{mineral bulk modulus}$ and $K_\phi = \text{pore space stiffness}$.

From the pore space stiffness, the dry rock bulk modulus can be estimated for a range of porosities, and gives the correct value in the mineral case.

This method was modified by Russell (2013) to perform the same computation for the shear modulus.
The Modified Gregory-Pickett Method for Moduli

The Gregory-Pickett method assumes a known dry rock Poisson’s ratio at a given porosity.

This figure shows the dry modulus $M_{dry}$ (either bulk, $K$, or shear, $\mu$) as a function of porosity. $M_m$ is the mineral modulus.

We first compute the modulus at calibrated porosity $\phi_{cal}$.

We then compute it at a new modulus $M_{new}$. 
The Modified Gregory-Pickett Method

This method is used in our AVO modelling code.

If $\phi_{\text{cal}}$ is the same as in the HS- method, this is equivalent to the Reuss bound.

The advantage of the HM/HS- method is that pressure and porosity are accounted for.

The advantage of the modified Gregory-Pickett method is that the starting moduli values are calibrated from the logs.
By considering the pressure effects on samples of a dry and saturated rock, Mavko and Mukerji (1995) succinctly show how the Gassmann equation can be derived.

The compressibility of the rock, $C$, which is the inverse of the bulk modulus $K$, is the change of the volume of the rock with respect to pressure, divided by the volume:

$$C = \frac{1}{K} = -\left(\frac{1}{V}\right)\frac{dV}{dP}, \text{ where: } V = \text{volume}, \ P = \text{pressure}.$$ 

In the above equation, there are two fundamental types of pressure: confining pressure, $P_C$, and pore pressure, $P_P$.

Also, there are three different volumes to consider: the volume of the bulk rock, the mineral and the pore space.
The Gassmann equation

Utilizing these concepts, we can build three simple models of the rock volume, as shown here (Mavko and Mukerji, 1995):

- **A. Mineral case**: In A, we compress the mineral.
- **B. Dry case**: In B we compress the mineral and dry pore.
- **C. Saturated case**: In C the mineral and saturated pore.

In A, we compress the mineral, in B we compress the mineral and dry pore, and in C the mineral and saturated pore.
The Gassmann equation

- Combining cases A and B (mineral and dry) gives:

\[
\frac{1}{K_{dry}} = \frac{1}{K_{min}} + \frac{\phi}{K_\phi},
\]

where:

- \( K_{dry} \) = dry rock bulk modulus,
- \( K_{min} \) = mineral bulk modulus,
- \( K_\phi \) = dry pore space stiffness, and \( \phi \) = porosity.

- Combining cases A and C (mineral and saturated) gives:

\[
\frac{1}{K_{sat}} = \frac{1}{K_m} + \frac{\phi}{\tilde{K}_\phi},
\]

where:

- \( K_{sat} \) = saturated rock bulk modulus,
- \( \tilde{K}_\phi = K_\phi + \frac{K_m K_f}{K_m - K_f} \)

is the saturated pore space stiffness, and \( K_f \) = fluid bulk modulus.
The Gassmann equation

- Combining these equations gives the Gassmann equation, a function of porosity ($\phi$) and water saturation ($S_w$):

$$\frac{K_{sat}}{K_m - K_{sat}} = \frac{K_{dry}}{K_m - K_{dry}} + \frac{K_f}{\phi(K_m - K_f)} = dry(\phi) + fluid(\phi, S_w)$$

- The fluid bulk modulus is computed with the Ruess average (normal case) or the Voigt average (patchy saturation):

Reuss: $K_f = \left[ \frac{S_w}{K_w} + \frac{1 - S_w}{K_{hc}} \right]^{-1}$, Voigt: $K_f = S_w K_w + (1 - S_w) K_{hc}$

- To compute new porosity and fluid values with Gassmann, HM and HS- we simply re-arrange the equation to give:

$$K_{sat}(\phi_{new}, S_{w\_new}) = K_m \left( \frac{1}{dry(\phi_{new}) + fluid(\phi_{new}, S_{w\_new})} \right)^{-1}$$
The Gassmann equation

- Once we have computed the dry rock bulk and shear moduli at various porosities, we compute their values at various fluid saturations, using the Gassmann equation.
- Gassmann showed that the shear modulus is independent of fluid content.
- He also showed that the bulk modulus of the various components are related by the following equation:

\[
\frac{K_{sat}}{K_m - K_{sat}} = \frac{K_{dry}}{K_m - K_{dry}} + \frac{K_f}{\phi(K_m - K_f)}, \text{ where:}
\]

\[
K_f = \left[ \frac{S_w}{K_w} + \frac{1 - S_w}{K_h} \right]^{-1} = \text{fluid bulk mod., } K_m = \text{mineral bulk mod.}
\]

- The next page shows this in a more intuitive way.
This figure shows the dry bulk modulus $K_{dry}$ (in red) and the fluid bulk modulus $K_{sat}$ as a function of porosity.

There are two ways to go from the dry high porosity value ($\phi_{dry\_cal}$) to the saturated low porosity value ($\phi_{sat\_new}$):

1. Up to the saturated curve and along the dry.
2. Along the dry curve and up to the saturated one.

That is, as shown in the appendix, the Gassmann equation is a combination of movement along the saturated and dry modulus paths.
Other bounds

- In the previous figures I used the Reuss bound. If we let $M$ stand for either the bulk or shear modulus, the Reuss bound is the harmonic average given by:

$$\frac{1}{M_R(\phi)} = \frac{1 - \phi / \phi_c}{M_{cal}} + \frac{\phi / \phi_c}{M_m}$$

- The Voigt bound is the arithmetic average given by:

$$M_V(\phi) = (1 - \phi / \phi_c)M_{cal} + (\phi / \phi_c)M_m$$

- The Hill bound is the mean of the Voigt and Reuss bounds:

$$M_H(\phi) = (M_V + M_R) / 2$$

- The upper and lower Hashin-Shtrikman bounds are more complex and are described in the appendix.

- The figure on the next page compares all the bounds.
Comparison of the five bounds

The five bounds for the bulk modulus (left) and shear modulus (right) from the mineral value ($\phi = 0$) to the calibrated value ($\phi_c = 0.4$). The heavy lines are the Hashin-Shtrikman upper bound, or HS+, in red, and Hashin-Shtrikman lower bound, or HS-, in blue, and the dashed lines show the Voigt, Hill and Reuss bounds.
Computing the density and velocities

- Once we have the moduli computed, we compute the velocities.
- This also requires that we compute the density at each porosity and saturation, using the formula:

\[ \rho_{sat}(\phi, S_w) = \rho_m (1 - \phi) + \rho_w S_w \phi + \rho_{hc} (1 - S_w) \phi \]

- Having the moduli and density as a function of porosity and fluid saturation allows us to compute the velocities:

\[ V_P(\phi, S_w) = \sqrt{\frac{K_{sat}(\phi, S_w) + (4/3)\mu(\phi)}{\rho_{sat}(\phi, S_w)}}, \quad V_S(\phi, S_w) = \sqrt{\frac{\mu(\phi)}{\rho_{sat}(\phi, S_w)}} \]

- Finally, the \( V_P/V_S \) ratio and the \( P \)-impedance (\( \rho V_P \)) are computed and crossplotted, as shown in the next page.
The Ødegaard/Avseth/Dvorkin/Nur RPT

- The preceding pages gave a general overview of the rock physics template method.
- The default RPT in our software is the one by Ødegaard and Avseth, which involves the following steps:
  - Compute the moduli of the dry rock frame for a given porosity/pressure using Hertz-Mindlin (HM) theory.
  - Compute the moduli of the dry rock frame over a range of porosities using the lower Hashin-Shtrikman bound.
  - Performing fluid substitution with the Gassmann equations.
  - Compute the P-wave velocity, S-wave velocity for each scenario and cross-plot $V_P/V_S$ ratio versus P-impedance.
Here is a plot of the Ødegaard/Avseth RPT for a range of porosities and water saturations, in a clean sand case.
The rock physics template (RPT)

Now return to the figure shown in the Ødegaard and Avseth (2003) paper. Obviously, the gas sands and brine sands shown here can be modeled by the previous theory. We will compare this fit on a shallow Alberta sand.
The rock physics template (RPT)

I will illustrate its applications using the values between 600 and 700 m for a gas sand from Alberta, as shown on the right above. Note the agreement with the previous template for the various lithological units.
The rock physics template (RPT)

Here, we have computed and drawn the rock physics template using the default parameters. The fit is reasonably good but certainly not perfect.
Adjusting the fit

- To get a better fit, we could change the default parameters and visually observe the result.
- However, another approach is to use the computed values of the dry rock bulk and shear moduli and then “tweak” them interactively with the option “User Input”.
- The resulting values at all porosities and saturations are then computed as before, using the lower Hashin-Shtrikman bound and Gassmann substitution.
- As we change the input dry moduli, we can observe the fit to the real data get better or worse.
- The final result is shown in the next figure.
The rock physics template (RPT)

By adjusting the dry bulk and shear moduli, we are able to adjust the template so that the gas sand is predicted very well. However, the wet and cemented sands are not predicted very closely.
On this plot, we have used the cemented sand model and also adjusted the temperature and pressure of the reservoir to get a much better fit.
Finally, we superimpose a shale line by using a similar method but assuming shale type parameters. By adjusting the dry terms, we get a reasonable fit.
Transforming to LMR space

If you prefer mu-rho versus lambda-rho (i.e. $\mu_\rho$ vs. $\lambda_\rho$) as your elastic constants for cross-plotting (or some other combination), the transform simply involves right clicking on the names on each axis. The result is shown here for LMR.
Adjusting the fit

- Next, let’s see how to apply this approach to real data.
- In this example, I will use the inverted data from the 2D line that is intersected by the well just discussed.
- I will also illustrate the new interactive multiple cross plot window option in HRS-9 R2.
- The final result is shown in the next figure.
A pre-stack inversion was done over the seismic volume associated with the gas well, with $V_P/V_S$ ratio shown in color and the traces showing acoustic impedance.
Application to real seismic data

The picked horizons and the crossplot zones corresponding to shale, gas sand and cemented sand are shown here.
Application to real seismic data

Here is the $V_P/V_S$ Ratio versus $P$-impedance crossplot of the three picked zones from the previous cross-section.
Application to real seismic data

The superposition of the rock physics template built from the well logs on top of the seismically derived values, showing a reasonable match.
Future enhancements

- The current implementation of the RPT is largely built around the method proposed by Ødegaard and Avseth, which involves computing the dry rock frame moduli using Hertz-Mindlin theory and the moduli over a range of porosities using the lower Hashin-Shtrikman (HS) bound.
- In HRS-10 we plan to offer the user the option to use all five of the bounds described in this talk: Reuss, Voigt, Hill and both the upper and lower HS bounds.
- This will implement the Gregory-Pickett method in the same way as described by Hilterman (1983) and Russell (2013), in which the Reuss bound was used.
- We also plan to expand the RPT approach to include the theory of Vernik and Kachanov (2010).
In this talk, I discussed the computation of the rock physics template (RPT) and how we have implemented this method in the Hampson-Russell software suite. The method is based on papers by Dvorkin and Nur (1996) and Ødegaard and Avseth (2003), involving Hertz-Mindlin, Hashin-Shtrikman, cemented sand and Gassman substitution theory.

Instead of focusing on the more complicated equations (given in the appendix), I illustrated the concepts behind the theory using graphical methods.

Using log and inverted seismic data from the Colony sand of central Alberta, I illustrated the use of the RPT.
References


